Vagueness, Gradability and Typicality

A Comprehensive Semantic Analysis

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by

Galit Weidman Sassoon

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Any comments on the contents of this dissertation are most welcome (gala@post.tau.ac.il; weidmans@bgu.ac.il).
Abstract

The analyses of predicates (and in particular nouns) in semantics and psychology focus on separated sets of facts. This fact reduces the adequacy of the theories in both disciplines.

On the one hand, semantic theories usually associate adjectives, but not nouns, with a gradable structure (mapping of entities to degrees along ordering dimensions). However, the last forty years of research in cognitive psychology have established beyond doubt that the concepts that nouns denote do possess a gradable structure. The relevance of these facts to semantics is demonstrated by some novel linguistic data. For example, I show that nouns occur more freely than adjectives in one type of comparison statements, whose semantic interpretation is standardly assumed to be mediated by degrees. This fact supports the view that the semantic analysis of nouns ought to involve mapping to degrees.

On the other hand, Kamp and Partee (1995) have analyzed nouns as gradable, but empirical and theoretical considerations suggest that their theory is inadequate. Furthermore, psychological theories, which treat nouns as gradable and multi-dimensional, fail to explain important semantic contrasts: First, gradable adjectives, but not nouns, are compatible with a variety of expressions whose meanings relate to degrees. Second, in multi-dimensional adjectives, but not in nouns, the ordering dimensions can be accessed and quantified over. By looking at the entire set of facts, a line of explanation suggests itself, hinging upon distinctions in the type of graded structure of nominal and adjectival concepts. I define the notion of a dimension-set in a precise and formal way, and I provide an improved account for the linguistic contrasts between nouns and adjectives. Finally, cognitive psychologists view many of their finding as refuting logical rules which form the basis of semantic theories. By showing that these findings are compatible with, and to some extent motivated by, semantic and pragmatic rules, I pave the way to bridging the gap between semantics and psychology. I propose that mechanisms that are advanced by psychological theories can and should be embedded within a semantic model that represents knowledge and its gradual growth ("a learning model"), to allow for a more adequate representation of psychological and semantic facts.

Finally, prominent semantic gradability theories are concerned with accounting for polarity effects (differences between positive and negative predicates like tall and short), and with providing a recursive semantic derivation for comparison statements (like Dan is taller than every boy is). I propose a detailed formal theory concerning the degree functions that positive and negative predicates denote and concerning the derivation of semantic interpretation for comparison statements. I show that my theory is superior to previous theories in terms of the set of facts it adequately predicts. In addition, though this theory enriches the interpretation of predicates with a new element (a transformation value), it simplifies the interpretation of predicates by removing from it several other elements (like degree relations, minus operations and supremum operations). Thus, my proposal is more economic in these respects.
Part I
Introduction and Data
1 INTRODUCTION

1.1 Basic notions

Categorization is the ability to sort objects into classes. Concepts are the mental representations (knowledge-structures) by virtue of which we can employ this ability in new situations, times, and states of background knowledge. Within model-theoretic semantics, the semantic interpretation of expressions in the language is modeled through the notion of intension (Montague, 1974). The intension of an expression \( \alpha \) is a function from worlds \( w \), times \( t \), knowledge states \( c \), etc., into the extension (denotation) of \( \alpha \) in \( w,t,c \ldots \), \([\alpha]_{w,t,c}\ldots\) (Indices are omitted whenever they are not relevant for the discussion.) Predicates (namely, expressions like, fruit, red, non-red, red-fruit, things to take from home in case of fire, etc.) can be viewed as names of concepts: They are expressions which have categories as their intensions, namely, functions from worlds, times, knowledge states, etc., into classes of entities, the extensions, or contextually given denotations.

This dissertation will focus on linguistic-concepts, namely concepts that tend to have names in natural languages (unlike, for instance, all kinds of dot-pattern categories). Some of the linguistic-concepts may lack a name in some language or other, yet they may play a role in the speakers' conception of the world and of the other linguistic-concepts. Accordingly, let us call the set of names of the linguistic-concepts that a speaker possesses, (which consists of, at least, all the predicates in the speaker's language), CONCEPT. The dissertation discusses the interpretation of the elements in this set.

1.2 Structure and goals

The interpretation of predicates is an important subject for research in both psychology and linguistics, but researchers in these two disciplines focus on radically different sets of facts. The primary goal of this dissertation is developing a model for the interpretation of predicates that will improve upon its predecessors with respect to both linguistic and psychological adequacy. With this goal in mind, in chapter 2 of part I I am juxtaposing the two sets of facts (they are reviewed in sections 2.1 and 2.2). I describe the discrepancies between the central psychological and linguist facts, but I also emphasize a number of points, whereby the knowledge that has been accumulated in the former field can shed light on central problems in the latter, and vice versa. In part 2 (chapters 3-4) I describe the standard theories about predicate interpretation (the structure of concepts) in linguistic (formal semantics) and psychology (the research of concepts). These theories deal with vagueness (partial information about a predicate or concept), gradability (the ordering of entities along dimensions in adjectives) and typicality (gradability effects in nominal concepts). I argue that standard positions in both disciplines need to be adapted. In part 3 (chapters 5-9) I describe my own proposal of a new model for the interpretation of predicates. I support it by showing that it solves the problems that were discussed in the preceding chapters.

1.2.1 Chapter 2: Vagueness, Gradability and Typicality, Two sets of facts

On the one hand, the facts which are the primary focus of investigation in linguistics (reviewed in section 2.1) seem to show that adjectives are typically vague and gradable, but nominal predicates (like bird and apple) are typically neither vague nor gradable. Adjectives like tall are classified as vague because in most contexts there are abundant entities about whom we cannot tell whether they are tall or not. Most adjectives, but not nouns, are classified as gradable, because the former but not the latter are compatible with comparatives,
equatives, superlatives, and other degree modifiers. For example, we can say about entities that they are taller, less tall, the tallest, etc. Conversely, with nouns, we cannot say so (# more bird; equally bird; #birdest). In addition, in multi-dimensional adjectives (adjectives with several ordering dimensions), but not in nouns, the ordering dimensions can be accessed and quantified over. For example, we can say about entities that they are healthy with respect to blood pressure, healthy in some, most or every respect, generally healthy, overall healthy, or healthy except for blood pressure. Conversely, with nouns, we cannot say so (# bird with respect to flying; # bird in some / most / every respects / # generally / overall a bird; # bird except for flying).

On the other hand, the facts which are the primary focus of investigation in psychology (reviewed in section 2.2) seem to show that also nominal predicates are typically both vague and gradable. The last forty years of research in cognitive psychology have established beyond doubt that speakers consider certain entities to be better examples of nouns than others (e.g., robins are often considered more typical birds than ostriches) and that speakers associate nouns with ordering dimensions (like feathers, flying, nesting, singing, etc.) It has been shown that these dimensions do not stand for categorization criteria (Wittgenstein 1953; Rosch 1973; Hampton 1979), but rather, the mean degree of the more typical entities on these dimensions is bigger than the mean of the less typical entities. So, indeed, these dimensions function as ordering dimensions. It was also found that the more typical entities are acquired earlier by children and adults, they are remembered best, retrieved faster, etc. (Mervis and Rosch 1981).

The gap between the two sets of facts results in sharp discrepancies between the standard theories in the two fields. Obviously, this reduces the adequacy of the theories in both fields.

### 1.2.2 Chapter 3: The linguistic perspective on Vagueness, Gradability and Typicality

Chapter 3 is divided to three sub-sections. In 3.1 I review theories about vagueness, in 3.2 I review theories about gradability in adjectives, and in 3.3 I review theories about gradability effects in nouns (typicality).

In central semantic theories of vagueness (van Fraassen 1969; Kamp 1975; Fine 1975; Veltman 1984; Landman 1991), linguistic expressions are interpreted relative to information states (contexts). Contexts, unlike worlds, may consist of partial information. Most importantly, in partial contexts, adjectives like tall are associated with a vague interpretation that consists of a positive denotation (the set of entities that are known to be tall in the context), a negative denotation (the set of entities that are known not to be tall in the context), and a gap (the remaining entities). The interpretation of nouns like bird is assumed to be completely known (except for rare borderline cases) – Nouns have virtually no gap. Furthermore, semantic theories associate adjectives like tall with a gradable structure that consists of a degree function that maps entities to degrees along a dimension (e.g. height). As the most important linguistic test for gradability is felicity in the comparative, linguistic theories of gradability are usually theories about comparison statements. There are numerous classical puzzles in the analysis of gradability and comparison. I focus on the following questions.

First, is vagueness related to gradability, and how? Some theories attempt to reduce gradability to vagueness (examples include Lewis 1970; Lewis 1979; Fine 1975; Kamp 1975; Klein 1982, etc.) The main problem with these theories is that, while they allow for an elegant account for judgments of ordering among instances whose membership is unknown, they fail to correctly predict judgments of ordering in category members and non-members, and in non-vague predicates. In chapters 5 and 8, I propose that gradability reflects the order of removal of vagueness (not vagueness per se). This proposal captures the gradability effects in category
members and non-members, and in non-vague predicates (including the typicality effects in nominal concepts).

Second, does grammar link gradable predicates with full fledged numerical degrees? There are two main approaches, which I call the ordinal scale analysis (Kamp 1975; Cresswell 1976; Klein 1980; Moltmann 2006) and the interval scale analysis (Russell 1905; Bartsch and Wunderlich 1970; Vennemann 1972; von Stechow 1984; Kamp and Partee 1995; Kennedy 1999; Schwarzschild and Wilkinson 2002; Landman 2005; Rotstein and Winter 2005). The ordinal approach assumes that predicates map entities to degrees that represent the ordering between entities, but not the precise distance between them. The interval approach assumes that predicates map entities to degrees that represent both (to full fledged numerical degrees, for which the difference operation is defined). The interval approach provides more natural accounts for the fact that adjectives like tall allow for numerical degree modification (as in 2 meters tall and two meters taller), for the semantics of comparison (more is analyzed as a difference operation), etc. Yet, this approach leaves many other questions open. For example, if grammar links adjectives with numerical degrees, it is not clear why in most cases (for instance, in adjectives like happy, healthy, beautiful, etc.), there is indeterminacy concerning the mapping of entities to degrees, numerical modifiers are infelicitous, etc. In chapters 5 and 9, I propose that all the natural language gradable predicates map entities to full fledged numerical degrees. Indeterminacy concerning the mapping of entities to degrees and infelicity of numerical modifiers are explained as indeterminacy concerning the units and other parameters of the degree function.

Third, in most semantic gradability theories, the degrees in a given predicate P are ordered along some predicate specific ordering relation. For instance, a relation, \( \succ_{\text{tall}} = \succ \) (the "bigger than" relation of the real numbers), may order the degrees of tall, and its converse relation, \( \prec_{\text{short}} = \prec \) (the "smaller than" relation of the real numbers), may order the degrees of short. These theories do not explicate how the degree ordering is fixed in predicates of the form "P with respect to (wrt) Q" (like healthy wrt blood pressure), where the connections between the Q degrees and their ordering in P are not trivial. The degrees are ordered neither by the standard "bigger than" relation of the real numbers, nor by its converse relation. In chapters 5 and 7, I propose a theory that does not associate predicates with predicate specific degree ordering relations. Systematic strategies for producing and modifying degree functions are described, such that the degrees of all the predicates are naturally ordered by the standard "bigger than" relation of the real numbers. This proposal explains more facts while being more economic.

Fourth, negative predicates (like not-tall and short) produce a variety of puzzling polarity effects. They usually do not allow for numerical degree modification (\# 2 meters short), except in the comparative (2 meters shorter), they cannot co-occur with the positive predicates in comparison statements (\# Dan is taller than he is not tall), their felicity with ratio modifiers like twice or four times is degraded (for instance, twice as short is less acceptable than twice as tall), etc. I describe central accounts (Seuren 1978; von Stechow 1984; Kennedy 1999, 2001, etc.), and their problems (e.g. the mentioned accounts produce wrong predictions regarding comparisons like shorter than two meters, they wrongly predict that clausal comparatives are downward entailing, etc.). In chapters 5, 7 and 9 I analyze negative predicates as denoting reversed and transformed degree functions. This analysis captures a long line of polarity effects, without producing the wrong predictions.

Fifth, much attention in the analysis of comparison statements is focused on assigning semantic interpretations that will correctly predict the interactions of comparative morphemes with quantifiers and intensional operators. This task is far from trivial. The most efficient compositional analysis is the (yet unpublished) Supremum theory (Landman 2005), which is based on insights from Schwarzschild and Wilkinson (2002). In chapter 9, I show that my
analysis of negative predicates as denoting reversed functions simplifies the Supremum theory (it makes the same predictions without stipulating a minus and a supremum operation in the interpretation of predicates).

Sixth, some adjective pairs can co-occur in between-predicate comparisons (as in *The table is longer than it is wide*), but many other pairs produce incommensurability effects. They cannot co-occur in between-predicate comparisons (*#The table is longer than it is heavy*). The most prominent analysis for these effects (Kennedy 1999) suggests that comparison requires that the predicates share the dimension. The main problem for this analysis is that nouns with different ordering dimensions can freely co-occur in between-predicate comparisons (as in *more a table than a chair, more a car than a house*, etc.) In chapter 7, I propose that between-predicate comparisons are comparisons by normalization, and hence usually compatible with nominal, but not with adjectival, meanings.

Finally, almost all semantic theories share the assumption that the interpretation of nouns does not involve any mapping of individuals to degrees. Rather, these predicates are directly associated with a set of instances, the positive denotation, and a set of non-instances, the negative denotation. Every element of the domain is directly linked to one of these sets. This explains the incompatibility of nouns with the operations denoted by equatives, comparatives, superlatives and degree modifiers (cf. 2.1). Despite the advantages of this assumption, the phenomena in the second set of facts in chapter 2 (cf. 2.2) remain unexplained if nouns are assumed to be non-gradable (including some purely linguistic facts). Can we capture the facts in the second set (cf. 2.2), while maintaining the assumption that nouns are not gradable in the usual linguistic sense? This is precisely what Kamp and Partee's (1995) influential analysis, the supermodel theory, has attempted to do. Given its central status in linguistics, I dedicate section 3.3 to showing that this analysis is inadequate. Namely, if we are up to a correct analysis, we should give up the assumptions that nouns are non-vague and non-gradable. Nouns should be linked with a gradable structure, the way standard psychological theories suggest (as described in chapter 4). The infelicity of nouns in comparative structures (*#more bird*) should be predicted by some more subtle features of this structure.

1.2.3 Chapter 4: The psychological perspective on Vagueness, Gradability and Typicality

Chapter 4 reviews the basic psychological perspective on concepts and their structure. According to the standard theories in cognitive psychology, the concepts that nouns denote are gradable, and they are linked with a set of dimensions. Murphy's (2002) seminal handbook about concepts and categorization demonstrates well the wide range and richness of the empirical findings, whose discovery was triggered by the cognitive approach. At the same time, this book emphasizes the diversity of theoretical models in the field. According to Murphy, more efforts need to be dedicated to the challenge of capturing generalizations within the data and theories. With this challenge in mind, chapter 4 is organized around the cognitive structures or mechanisms that contemporary theories assume to be involved in categorization tasks. Among the most common and important structures is the set of dimensions and the similarity degree function. The main idea in cognitive dimension models, "The Weighted Mean Hypothesis", is that the degree of typicality in (or similarity to) a category (or an entity) is given by a weighted mean in the category's (or entity's) dimensions (Wittgenstein 1968 [1953]; Medin and Shepherd 1978; Mervis and Rosch 1981, etc.) Experimental results strongly support this hypothesis. Yet, there is still little agreement about the way dimensions ought to be represented. Dimension-sets are represented as lists (Hampton 1979-1997), vectors in conceptual spaces (Gardenfors 2004), theories (Murphy and Medin 1985), frames (Smith et al. 1988), networks (Murphy and Lassaline 1997), and so on and so forth. In addition, researchers still hardly concur about the ways entities' degrees in the
dimensions are determined, about the ways dimensions are chosen and assigned attentional weights (Armstrong, Gleitman and Gleitman 1983; Murphy 2002), about the precise averaging method and about the precise categorization criterion (Ashby and Maddox 1993).

From the linguistic perspective, the main problem of the psychological theories is that they blur important linguistic distinctions between different predicate types. That is, gradable adjectives, but not nouns, are compatible with comparative, equatives and degree modifiers. In addition, in multi-dimensional adjectives, but not in nouns, the dimensions can be accessed and quantified over (cf. 2.1). These facts remain unexplained under the assumption that nouns are gradable and multi-dimensional. In chapter 5 and 7, I propose an explanation for the entire set of facts by pointing out distinctions in the type of dimensions and graded structures which characterize nominal and adjectival concepts.

In addition, a common view in psychology regards truth-conditional semantic theories as inherently incompatible with psychological findings. In particular, substantiated findings from the research of conceptual combination (the typicality judgments in complex predicates) are viewed as refuting logical rules, such as the intersection rule for the interpretation of conjunctions and modified nouns. In chapters 7 and 8, I argue that the intersection-rule is compatible with effects of concept combination, and that it provides explanations for certain facts (for failures of inheritance of dimensions from the parts to modified noun, for the emergence of new dimensions in modified nouns, and for conjunction fallacies). Finally, I explain non-intersective effects in nominal concepts (which psychologists call overextension effects) by analogy to linguistic accounts of similar effects in adjectives. Pragmatic considerations explain these effects in both nouns and adjectives.

Another problem in current psychological theories is that they do not fully represent (if at all) the partiality and context dependency of our knowledge about concepts and their dimensions (Wisniewsky and Medin 1994; Murphy 2002). Conversely, formal semantic theories of vagueness and partial information allow a representation for effects of context and of general knowledge on the semantic interpretation of expressions in the language (cf. 3.1). These insights fall beyond the scope of psychological theories, which tend to reject truth conditions altogether. In chapters 5-8, I show that psychological theories can and should be embedded within a formal-semantic model, to allow a fuller representation of the effects of context and general knowledge on the typicality judgments, of the dimension-sets and their selection, etc.

1.2.4 Chapter 5-9: My proposal

In chapter 5, I summarize the motivations for the new proposals which are described in chapters 6-9. I briefly and informally describe the gist of my new perspective on adjectives, nouns, gradability, and partial information concerning gradable structures (degree functions and dimension-sets). In chapter 6, I present in detail the contexts and recursive semantic rules in a model that represents partial interpretation about gradable structures. In chapter 7, I present and motivate a typology of predicates sorted by the type of their degree function. In chapter 8, I discuss the learning principle, which explains a variety of facts including the acquisition of degree functions in simple and complex predicates. Finally, in chapter 9, I propose an account of the main polarity effects (differences between positive and negative predicates), and of the distribution of comparative and ratio morphemes with different predicate types.

My theory diverges from a standard linguistic theory of gradability, in that, in my theory, all the predicates are associated with a gradable structure, including the so-called sharp nouns. In chapter 7, I propose that there are relatively few types of simple operations, which apply to degrees of given degree functions and produce degrees for new functions. For example, the
degrees of negative predicates are produced by reversed functions. A reversed function takes a degree 'n' of a positive predicate, and returns a degree of the form '– n', or 'Tran – n' (where Tran is a real number, the transformation value, as explained below). The bigger n is, the smaller '–n' is. Thus, a reversed function produces reversed orderings, by virtue of its use of the difference operation '–'. A function of a predicate of the form "P wrt Q" (healthy wrt blood pressure) takes a Q degree 'n', and returns a P degree of the form '—|Value – n|', where Value is the ideal blood pressure value. So the smaller the distance between n and this value, |Value – n|, the bigger one's degree of health. I propose that since this function uses the difference operation twice, healthy is grasped as positive (in sick wrt blood pressure, the number of reversals is odd). Finally, nominal functions take a set of degrees n₁…nₘ and values Value₁…Valueₘ, and they return the (reversed) weighted mean of the distances |Value₁ – n₁|…|Valueₘ – nₘ| (so the smaller the distances are of, e.g., a bird, from the ideal values for birds on the bird dimensions, the bigger its degree in bird is). I show that these proposals make correct predictions concerning the graded structure of different predicate type, while they spare the need of stipulating a degree relation <p in the interpretation of predicates.

Concerning dimensions, I propose that they are simply predicates of the same arity as the predicates of which they are dimensions. In chapter 7, I propose that we have two ways to process dimension-sets. We can process the dimension-set either as a set of typicality dimensions, or as a set of rules (necessary or sufficient conditions for membership in the denotation). I propose that the noun-adjective distinction functions as a cue that tells us how to process the dimensions. If a predicate is morphologically marked as a noun, the dimension set is, by default, processed as a typicality-set. The dimensions are combined by a mean function, as psychological theories propose. Yet, I propose that if a predicate is morphologically marked as an adjective, the dimension set is, by default, processed as a set of rules (necessary conditions which are jointly sufficient for membership in the denotation). I propose that there are two types of adjectives, disjunctive and conjunctive. The categorization rule for conjunctive adjectives (like healthy) is the requirement to reach a threshold in every dimension. The categorization rule for disjunctive adjectives (like sick or different) is the requirement to reach threshold in some dimension. So the dimensions are combined by a Boolean operation, disjunction (in disjunctive adjectives) or conjunction (in conjunctive adjectives), or equivalently, they are bound by a universal quantifier (in conjunctive adjectives) or existential quantifier (in disjunctive adjectives). For example, imagine that the noun bird is associated with the dimensions flying and singing and that the adjective healthy is associated with the dimensions blood pressure (bp) and pulse. The denotation of bird is indicated by the mean degrees of entities on the two dimensions, not by dimension-intersection (it is not the case that for any x, x is a bird iff x flies and / or sings). Conversely, the set of healthy entities is given by the intersection of the two dimensions (for any x, x is healthy iff x is healthy wrt bp and x is healthy wrt pulse), and the set of sick entities is fixed by the union of the two dimensions (for any x, x is sick iff x is sick wrt bp or x is sick wrt pulse).

I provide supporting evidence for this proposal, using a corpus study, and by showing that it accounts for the linguistic contrasts between nouns and adjectives.

First, it is well known that except phrases are licensed by universal, but not by existential quantifiers. I show that except-phrases often apply to conjunctive multi-dimensional adjectives and operate on their dimension sets. This is predicted by the proposal that their interpretation (categorization rule) involves an implicit universal quantification over the dimensions (x is healthy iff x is healthy in every respect). I also show that except-phrases are hardly ever used with disjunctive multi-dimensional adjectives (whose dimensions are existentially bound), except when they are negated (because a negated existential statement is
equivalent to a universal statement). In addition, except-phrases are hardly ever used with nouns whose dimensions are combined with mean operations.

Second, I show that with-respect-to (wrt) phrases are licensed with a predicate P (as in \( P \operatorname{wrt} F_1 \)) iff the dimensions of either the predicate or its negation are regarded as necessary conditions for membership (such that it makes sense to say that x is P wrt \( F_1 \)) but not P wrt \( F_2 \)). Thus, only adjectives with at least two dimensions (like healthy) or their nominalizations (e.g. health) license wrt arguments. As a result, it is possible to access the adjectival dimensions (grammatical operations can bind the wrt-argument, as in healthy in some respects), but not the nominal dimensions (as in # bird wrt flying / in some respect).

Third, I show that this proposal captures a variety of facts concerning the licensing of comparative morphemes with nouns and adjectives. First, more can be used to compare degrees in two different predicates (in between-predicate comparisons, as in more a bird than a horse and more a table than a wall) when the degree functions of the two different predicates can be normalized so as to be comparable. Since nominal functions are normalized in the first place (for the purpose of averaging), they occur more freely in such comparisons. I show that the felicity of between-adjective comparisons improves when the range of the adjectives' degree function has a maximum and a minimum degree, as this allows for normalization. Second, when more is used to compare two degrees in one predicate (in within-predicate comparisons), it is licensed iff the predicate is a one-dimensional adjective. It is licensed in multi-dimensional adjectives, only because they can be (implicitly) modified by a wrt-phrase, which turns them one-dimensional (reduces the dimension set to a singleton). I.e. comparative relations like healthier are interpreted as healthy in every / most / some respect. Conversely, nouns do not license wrt-phrases, so they remain multi-dimensional. As a consequence, they fail to be licensed in within-predicate comparisons. In conclusion, more is inherently Boolean. Since the use of averaging versus Boolean operations (in combining the predicate dimensions) is precisely the thing that distinguishes adjectives from nouns in the current proposal, it is directly predicted that more in within-predicate comparisons will be freely licensed with adjectives, but not with bare nouns. Finally, this proposal predicts that more, in within-predicate comparisons, will not be able to take an and-conjunction as its predicative argument, because conjunctions are inherently multi-dimensional (they have at least two necessary conditions for membership: The conjuncts) and they do not combine with wrt phrases (the same applies to disjunctions). An experimental examination of the interpretation of more when its predicative argument is an and-conjunction or a disjunction supports this prediction. It shows that the connective and (or or) tends to take a wide scope. The most common interpretation for, for instance, more bald and tall, is "balder and taller", where only one dimension occurs in each conjunct.

Fourth, in the past, theories failed to represent the contribution of the typicality dimensions to truth conditions (Cohen 1999: 11) or to our representation of the external world. Following my proposals, I give a semantic analysis of typicality statements. In chapter 7 and 8, I propose that statements like flying is typical of birds are true iff flying is an element of the dimension-set of birds, and that statements like Tweety is typical of a bird are true iff Tweety reaches the standard in all the typicality dimensions of bird. The adverb typically produces flip flop of the arguments. For example, typically, birds fly is true iff flying is typical of birds is true (though typically has more readings depending on the topic-focus structure and the mapping of material to the restriction and nuclear scope). This account sheds light on the connections and differences between typicality statements and generic statements, though this topic is beyond the scope of this thesis (for a short discussion see section 8.9).

My proposal embeds the psychological mechanisms within a formal context structure (a vagueness model). I show that within such a model it is possible to represent the context
dependency of the typicality dimension-sets (and of the interpretation of each dimension),
their gradual learning, their relations to domain general knowledge structures, etc.

As for the connections between vagueness and gradability, given both linguistic and
psychological considerations, in chapter 8, I propose that gradability is intimately related not
to vagueness per se, but to the order in which vagueness can be removed (the order in which
entities are learnt to be denotation members or non-members, whether directly or by
inference, in contexts and their extensions). This observation is formalized as the learning
principle. First, I show that this principle is empirically supported by robust psychological
findings (the learning order effects), and that it allows a natural account for the acquisition of
dimension-sets. Second, I show that it solves severe problems that characterize previous
vagueness-based gradability theories. Third, given that gradability reflects gradual learning, it
can characterize relatively sharp predicates, including the (so-called) sharp nouns (providing
that the membership of different entities in their denotations can be learnt, or inferred,
gradually). Fourth, I show that the compositional predictions of the learning principle are
intuitive. In order to empirically test the predictions of the vagueness removal view (the
learning principle) concerning complex predicates, I have composed a general questionnaire
that collected judgments about the entity-ordering and dimension-sets in conjunctions,
disjunctions and negations. The subjects were 35 Hebrew native speakers. The conclusions
concerning speakers’ judgments are described throughout this dissertation, wherever they are
relevant. The original questionnaire is found in the appendix to chapter 2.

According to my proposal, the mean degrees of entities on the dimensions do not help in
building adjectival denotations or degree functions. For example, an entity d\textsubscript{1} which scores
well in \textit{b.p}, but has a very low health level wrt \textit{pulse}, may not reach the threshold level in
\textit{pulse} and fall outside the denotation of \textit{blood pressure and pulse} (and hence outside the
denotation of \textit{healthy}). At the same time, an entity d\textsubscript{2} with a lower overall mean in these
dimensions, but a higher level in \textit{pulse} might reach both standards and enter these
denotations. Thus, the mean in the adjectival dimensions cannot indicate the denotation. Yet, I
show that, together with the learning principle, my proposal helps to explain facts concerning
the gradable structure of conjunctive and disjunctive expressions. I demonstrate that the
learning principle poses very weak compositional constraints on derived comparatives of
conjunctive and disjunctive concepts. More often than not, even in a context of complete
knowledge concerning the ordering relation that holds between two entities in P and in Q, it is
impossible to infer anything about their ordering in "P and Q" or "P or Q". The facts
concerning the ordering in "P and Q" and "P or Q" need to be directly given. I propose that in
order to resolve vagueness, speakers resort to averaging on the constituents (or dimensions).
This option is allowed only whenever it does not violate the compositional constraints and it
is supported by contextual information (as to the relative importance of the constituents).

Finally, in chapter 9, I account for the polarity effects (the differences between negative
and positive predicates). What do we know about the degree function of negative predicates
like \textit{short}? We know (or we have very strong intuition) about the entity ordering of \textit{short} that
it is reversed compared to that of \textit{tall} (Dan is taller than Sam iff Sam is shorter than Dan).
Thus, the degrees are reversed (if Dan is mapped to a higher degree in \textit{tall} Sam is mapped to a
higher degree in \textit{short}). But, crucially, that is about all that we know about these degrees. In
other words, we know that they are produced by a reversed function, but we do not know
which reversed function. There are many candidates. If \(f_{\text{tall}}\) is the function that is linked with
\textit{tall}, for any constant Tran\(\in\mathbb{R}\), a function \(f_{\text{Tran}}\) that assigns any d the degree (\(\text{Tran} – f_{\text{tall}}\)) can do
the job of reversing the degrees. Do we have intuitions that tell us that \(\text{Tran}_{\text{short}}\) is zero (in any
actual context c)? Well, I do not think so. This can be tested by checking our intuitions
concerning the value of individuals with zero height. Does \textit{short} map them to 0? I do not
know. Maybe they are mapped to a number that approximates infinity? This view is endorsed
by some well-known semantic theories (cf. von Stechow 1984; Kennedy 1999). But if so, then the mapping function of short is a function that transforms height quantities by a non-zero constant, $\text{Tr}_{\text{short}}$. To be honest, we do not know anything about the constant. It may be any real number from zero to infinity. It should be represented as a value that varies between contexts, and that may be unknown in a partial context. I illustrate that by representing this fact, we can explain the polarity effects.

First, roughly, if, for instance, tall maps an entity $d$ to 2 meters, short maps $d$ to $\text{Tr}_{\text{short}} - 2$, where the value $\text{Tr}_{\text{short}}$ is unknown. This produces indeterminacy concerning the number set of short (the values its degree function assigns to entities), which is felt in the fact that numerical-degree modifiers (such as two meters) cannot be used with its positive form. In the lack of knowledge about $\text{Tr}_{\text{short}}$, we can never say which entities are mapped to two meters short, and for that reason statements like Dan is two meters short are infelicitous.

Second, if $d_2$ has a double length compared to $d_1$, tall maps $d_1$ to $n$ (say, 2 meters), and $d_2$ to $2n$ (say, 4 meters). Given that short reverses the degrees, in the given context short maps $d_1$ to $n' = \text{Tr}_{\text{short}} - n$ (e.g. $\text{Tr}_{\text{short}} - 2$ meters), and $d_2$ to $m' = \text{Tr}_{\text{short}} - 2n$ (e.g., $\text{Tr}_{\text{short}} - 4$ meters). But $m'$ is not two times $n'$ (unless $\text{Tr}_{\text{short}}$ is set to zero). Thus, $d_2$ has a double length compared to $d_1$ iff $d_2$ is twice as tall, but not iff $d_2$ is twice as short. As a consequence, twice as short is less acceptable than twice as tall.

Third, when degree-differences are computed (as in Dan is n meters taller / shorter than Sam) the transformation values of the two degrees cancel one another. For instance, $d_2$ has $n$ meters more length compared to $d_1$ iff $d_2$ is n meters taller (iff tall maps $d_2$ to $m$ and $d_1$ to $m - n$) and iff $d_2$ is n meters shorter. I.e. short maps $d_2$ to $\text{Tr}_{\text{short}} - m$ and $d_1$ to $\text{Tr}_{\text{short}} - (m - n)$, and the difference between these two degrees is still $n$ (The difference is negative, $(\text{Tr}_{\text{short}} - m) - (\text{Tr}_{\text{short}} - (m - n)) = -n$, because $d_1$ has a higher degree in short – it is shorter). For that reason statements like Dan is two meters shorter than Sam are perfectly felicitous, etc.

I conclude chapter 9 by presenting formal properties that distinguish between one-dimensional and multi-dimensional predicates. I give an explanation for the fact that comparative-morphemes in within-predicate comparisons cannot directly combine with multi-dimensional predicates (with predicates that denote mean functions or Boolean functions).

In sum, I define the notion of a dimension-set in a precise and formal way, and I provide an improved account for the fact that comparative-morphemes in within-predicate comparisons cannot directly combine with multi-dimensional predicates (with predicates that denote mean functions or Boolean functions).

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2 FROM NATURAL CATEGORIES TO PREDICATE TYPES, TWO SETS OF FACTS

2.1 The first set of facts: A linguistic typology of predicates

2.1.1 Vague predicates

When we look at natural languages, we see that for some predicates, like *even number*, it is determined for each individual in each situation whether it falls under the predicate or not. There is no denotation gap containing individuals that one does not know if they fall under the predicate or not. These predicates are called *non-vague, or sharp*. Other predicates do have a gap. For example, in a certain domain of entities (say, the students in a certain class), we are usually able to positively classify some entities as clearly *tall*, and some other entities as clearly *not-tall*. However, for yet another set of entities, whose height falls in between that of the clearly tall and the clearly short entities in the domain, we are often not able to decide whether they are tall or not (i.e. whether they fall within the positive denotation of *tall*, [[tall]]^+ or the negative denotation of [[tall]]^−). These predicates are called *vague*. Linguists usually assume that, for all practical purposes, it is good enough to consider as sharp also predicates like *bird* or *apple*, which are "by and large" sharp, i.e. which admit only few borderline cases, that are thought to be relevant in only very rare situations.

(1) a. Vague predicates (like tall, bald, large, hot, cool) have a denotation gap, [[P]]^2. In certain contexts c, some entities are neither in [[P]]^+ c nor in [[P]]^− c.
b. Non-vague (‘sharp’) predicates (like even number, bird or apple) have (almost) no denotation gap. By and large, everything is in [[P]]^+ c or in [[P]]^− c in every context c.

2.1.2 Gradable predicates

The term *gradable* is used in linguistics to refer to predicates, like *tall, bald, old, large, good, healthy, and clever*, which are distinguished by the following characteristics.

2.1.2.1 Compatibility with comparative, equative, superlatives, and degree modifiers

First, and most importantly, gradable predicates can felicitously combine with comparative morphemes (2a), equatives (2b), superlatives (2c), and degree modifiers (2d). A few adjectives (for instance, *extinct, even, married and nuclear*), and all the nouns in languages like English (for instance, *bird, apple and chair*), are classified as *non-gradable*, because they cannot felicitously combine with comparison and degree morphemes, as demonstrated in (3a-d).

(2) a. Dan is more / less clever than Sam is  
b. Dan is as tall as Sam is / Dan and Sam are equally tall  
c. Dan is the cleverest / the least clever  
d. Dan is very / fairly / enough / too clever

(3) a.* Tweety is more / less (a) bird than Tan is  
b.* Tweety is as (a) bird as Tan is / Tweety and Tan are equally bird(s)  
c.* Tweety is the birdest / the least bird  
d.* Tweety is very / fairly / enough / too (a) bird
a. Gradable predicates (like tall, bald, large, hot, cool, old, clever) can combine with comparatives (more P; less P) equatives (equally P), superlatives (the most P) and degree modifiers (very, fairly, much, well, enough, too, so).


The semantic distinction that is assumed by linguists to underlie the linguistic contrasts in (2)-(3) is that only for gradable predicates it is the case that entities can possess the properties that they denote to different extents (or degrees). Entities are judged to be instances of these predicates iff the extent to which they satisfy the relevant gradable property, which is usually called the ordering dimension of the predicate, is within the norm, that is, iff they reach the standard for membership under that predicate.

2.1.2.2 Grammatically accessible dimensions

Second, semanticists standardly assume that the meaning of gradable adjectives like tall and bald includes an ordering-dimension, such as height in the case of tall (Kennedy 1999). In fact, most of the gradable adjectives may be linked with several ordering-dimensions (Kamp 1975). For example, the adjective healthy can be ordered on a number of dimensions, such as blood pressure, pulse, fever and lung functions.

The range of dimensions in the interpretation of adjectives is highly context-dependent, mostly in the multi-dimensional ones. This type of vagueness with respect to the ordering-dimension(s) is usually called indeterminacy.

The contextually relevant dimensions of a multi-dimensional adjective can be overtly specified as part of the argument structure of the adjective, using a "with-respect-to" (wrt) prepositional phrase, as demonstrated in (5a). In addition, grammatical operations can access the dimensions of multi-dimensional adjectives and operate on them (Bartsch 1986; Landman 1989). For example, we can quantify over these dimensions or respects, as in the examples in (5b-d).

(5) a. Maria is healthy with respect to blood pressure
   b. Maria is healthy in every respect
   c. Maria is generally healthy
   d. Maria generally knows English

The oddness of the examples in (6) is likely due to the fact that tall is a one-dimensional adjective.

(6) a.? Maria is tall in every respect
    b.? Maria is generally tall
    c.? Maria is tall with respect to height

As for nouns, experimental results established that speakers characterize them by a rich set of dimensions. For example, the noun bird is characterized by dimensions like feathers, small size, flying, singing, perching, eating insects, etc. (Rosch 1973). However, nouns differ from gradable adjectives in that their dimensions cannot be accessed by grammatical operations, like wrt-operators or quantifiers, as demonstrated in (7).
(7) a.# Tweety is a bird with respect to flying / size
    b.# Tweety is a bird in every / some respect
    c.# Tweety is generally a bird

Nouns can be modified by a wrt phrase iff they are derived from (or are systematically connected to) adjectives. Examples include nominalizations of adjectives (health, success, similarity, etc.) and animate nouns (like an Italian, which seem to mean something like "An Italian person"; thus the noun is intimately connected to the adjective Italian). These nouns behave like adjectives in many other respects (for further discussion see chapters 5 and on).

2.1.2.3 Failures of intersective entailments ('drop' and 'permutation')

Third, sentences with modified-nouns in predicate position usually entail the sentences resulting from dropping some of the constituents or changing the constituent ordering (constituent-permutation). For example, (8a) entails (8b) and (8c). Following Landman (2000), I call this inference-pattern drop. In addition, (8a) is equivalent to (8d). Following Landman (2000), I call this inference-pattern permutation. Such entailment-patterns form the basis for the intersective analysis of modified-nouns and conjunctions. According to this analysis, modified-nouns and conjunctive predicates, P∧Q, denote the intersection of their constituents' denotations, as formally stated in (9). The intersection-rule in (9) directly predicts the fact that an item is classified as, for instance, a four legged animal or an animal which is four legged, iff it is classified as an animal and it is classified as four legged.

(8) a. Tweety is a four legged animal
    b. Tweety is four legged
    c. Tweety is an animal
    d. Tweety is an animal and is four legged

(9) \forall w \in W: \[[P∧Q]_w = [[P]]_w \cap [[Q]]_w\]

The same basic facts seem to hold in the verbal domain, too. For example, the entailments from (10a) to (10b)-(10e) are instances of the drop and permutation inference patterns of modified verbs. These additional facts form the basis for the Davidsonian intersective analysis of modified verbs.

(10) a. Dan ate quickly with a knife
    b. Dan ate with a knife
    c. Dan ate quickly
    d. Dan ate
    e. Dan ate with a knife quickly

The drop and permutation entailment patterns tend to fail in constituents which are gradable predicates like tall or mature. For example, (11a) does not entail (11c), and, consequently, it does not entail (11d). The nouns, again, do not tend to cluster with the gradable predicates. For example, (11a) indeed entails (11b).

(11) a. Dan is a mature child
    b. Dan is a child
    c. Dan is mature
    d. Dan is a child and is mature
These non-intersective effects in gradable adjectives seem to occur because these adjectives are interpreted relative to a context-dependent local domain (a comparison-class; Klein 1980), and the standard for membership in them is adjusted for this domain. For example, in (11a), mature is interpreted relative to a domain which is restricted by the noun to children. In (11c) the noun is dropped, so this restriction is ignored, resulting in entailment failures (Kamp and Partee 1995).

As a consequence, these failures characterize modified-nouns, and they disappear in and-conjunctions and in disjunctions. For example, (12a) entails (12b-d). Similarly, while (13a) is a contradiction (Dan is claimed to be an instance of two non-overlapping sets), (13b) and (13c) are completely coherent, because the head noun restricts the local domain, and the adjective's membership-standard is adjusted to this domain (Kamp and Partee 1995).

As a consequence, these failures characterize modified-nouns, and they disappear in and-conjunctions and in disjunctions. For example, (12a) entails (12b-d). Similarly, while (13a) is a contradiction (Dan is claimed to be an instance of two non-overlapping sets), (13b) and (13c) are completely coherent, because the head noun restricts the local domain, and the adjective's membership-standard is adjusted to this domain (Kamp and Partee 1995).

(12)  
a. Dan is fat and bald  
b. Dan is fat  
c. Dan is bald  
d. Dan is bald and fat

(13)  
a. Dan is midget and giant  
b. Dan is a midget giant  
c. Dan is a giant midget

2.1.2.4 'For' arguments

Fourth, in many gradable adjectives, the local domain can be overtly realized by a for phrase (14a-b). Numerical degree modifiers like two meters cannot occur together with a for phrase, as demonstrated by the contrast between (14c) and (14d). In (14c), the required degree in the predicate is fixed by the modifier two meters, not by the predicate's standard, and hence the for phrase (which ought to help fixing the standard by constraining the local domain) plays no role.

(14)  
a. Dan is mature for a child  
b. Dan is healthy for an 80 year old man  
c. Dan is two meters tall  
d. Dan is two meters tall for a basketball player

Nouns are again different. They cannot be modified by a for phrase (14c-d).

(15)  
a. # Tweety is a bird, for a nocturnal animal  
b. ? Tweety is a bird, for a chocolate animal

2.1.2.5 Vagueness is associated with gradability

Fifth, it has been observed that vague adjectives are usually gradable. For instance, tall is both vague and gradable. Other adjectives, which are not vague, are often (but not always) not gradable. Examples include adjectives like even (number), which are clearly not-vague (they have no denotation gap), and are bad in the comparative (*more even). Nouns are usually sharp and, perhaps as a result, non-gradable (Kamp 1975). But some nouns are clearly vague. An example is the noun chair. Out of context, it is impossible to determine whether entities (armchairs, stools, seats without a back, which are not used as a seat, etc.) are chairs or not. Still, crucially, the noun chair, in languages like English, is not gradable in the sense defined above (Kamp & Partee 1995). For example, it is incompatible with the comparative
morpheme (* more chair). Thus, in adjectives vagueness goes together with gradability, but not nouns. The noun meaning seems to be inherently non-gradable.

2.1.2.6 Intermediate conclusions: Nouns as inherently non gradable

In conclusion, by and large, linguistic data show that grammar links adjectives, but not nouns, with grammatically accessible ordering-dimensions and with vague and gradable properties. Nouns usually behave as if they are non-vague and non-gradable. Section 2.2 describes psychological data about concepts, like the ones which nouns denote. A rather different picture of the structure of nominal concepts emerges from this data.

2.1.3 The nature of the degrees in different gradable predicates

What are the degrees into which gradable predicates presumably map entities?

Degree modifiers such as two meters (as in Dan is two meters tall) seem to show that degrees are numbers (or tuples consisting of a number, a unit, a predicate, etc.) However, many languages (like Hebrew) only allow numerical degree modifiers in comparative statements (as in two meters taller), not as modifiers of positive predicates (as in two meters tall). Furthermore, within the languages that, in principle, allow degree modifiers in both contexts, the set of positive predicates which do actually allow this modification varies considerably, and many predicates (like happy, beautiful and intelligent) are excluded from this set (Moltmann 2006).

Kamp and Partee (1995) observe that, given a certain scale (number set), there is much indeterminacy in the mapping of individuals to numbers. This is certainly true of predicates like happy. Given the set of real numbers between 0 to 1, why would a certain person have a degree 0.25 rather than say 0.242 in happy? Furthermore, which set of numbers represent the degrees of predicates like happy or beautiful? Moltmann (2006) says that, given this indeterminacy of the number set, the assumption that numerical degrees play a role in the semantics of predicates and comparatives creates a meaning intention problem. How do speakers know the meaning of their utterances when they are using expressions which (presumably) denote degrees, such as (16a)?

Finally, Moltmann (2006) observes that adverbial modifiers of gradable adjectives, like the ones in (16b), do not denote properties of numbers. For example, numbers which represent beauty degrees can be neither strange nor visible, but particular instantiations of the property beautiful in individuals may be both strange and visible. According to Moltmann (2006), this speaks against the idea that the gradable adjective beautiful maps individuals into numerical degrees, or denotes a relation between individuals and such degrees.

(16) a. Dan is as happy as Sue is. Bill isn't that happy
b. Strangely beautiful; visibly happy; fatally weak; deliberately silent

2.1.4 Positive versus Negative predicates: Polarity effects

Some predicates are felt to be negative. This feeling may arise due to overt marking with a negative morpheme (not tall, non-birds, unnecessary, irrelevant, dissimilar, etc.), but it may also occur in the lack of such overt marking (short, different, etc.) Many pairs of predicates that differ in polarity, like tall and short, or tall and non-tall, stand in the antonym relation. This relation plays a crucial role in the interpretation of gradable predicates.
2.1.4.1 A shared dimension

First, intuitively, predicates and their antonyms are interpreted relative to the same dimension (gradable property). For example, both tall and short seem to denote measures of height.

2.1.4.2 Non-overlapping denotations

Second, the positive predicate in the pair can be described as denoting entities with high degrees in the relevant property, and the negative predicate can be described as denoting entities with low degrees in that property. The positive denotations of predicates and their antonyms usually do not overlap. Rather, the positive denotation of a predicate \( P \) is a subset of the negative denotation of its antonym \( P^\text{ant} \) (\( \forall c: [[P]_c^+ \subseteq [[P^\text{ant}]_c^- \text{ and } [[P^\text{ant}]]_c^+ \subseteq [[P]]_c^- \)), as demonstrated in (17).

\[
(17) \quad \text{In every } c, [[\text{tall}]_c^+] \subseteq [[\text{short}]_c^- \text{ and } [[\text{short}]]_c^+ \subseteq [[\text{tall}]]_c^-).
\]

2.1.4.3 Reversed ordering relations

Third, the ordering in a predicate and in its antonym can be said to be reversed, as demonstrated in (18) (but see also the discussion on more and less below).

\[
(18) \quad \text{Dan is shorter than Sam iff Sam is taller than Dan}
\]

2.1.4.4 The licensing of numerical degree modifiers

Fourth, only positive predicates can be modified by numerical degree modifiers such as two meters, as demonstrated by the contrast in (19a-b). However, the derived comparatives of positive and negative predicates alike can be modified by numerical degree modifiers

\[
(19) \begin{align*}
\text{a. } & \text{Dan is two meters tall} \\
\text{b. } & * \text{Dan is two meters short} \\
\text{c. } & \text{Dan is two meters shorter}
\end{align*}
\]

2.1.4.5 The licensing of ratio modifiers

Fifth, the felicity of ratio modifiers like twice as ADJ as or half as ADJ as is often degraded in negative predicates, compared to their positive antonyms. The contrast in felicity between (20a) and (20b) demonstrates this.

\[
(20) \begin{align*}
\text{a. } & \text{Dan is twice as tall as Sam} \\
\text{b. } & ? \text{Dan is twice as short as Sam} \\
\text{c. } & \text{The table is twice as long as the sofa} \\
\text{d. } & ? \text{The table is twice as short as the sofa} \\
\text{e. } & \text{The table is twice as big as the chair} \\
\text{f. } & ? \text{The table is twice as small as the chair} \\
\text{g. } & \text{Dan is twice as fast as Sam} \\
\text{h. } & ? \text{Dan is twice as slow as Sam}
\end{align*}
\]

The use of ratio modifiers like twice as ADJ as with negative predicates is not completely ruled out, but it occurs significantly more often with positive adjectives than with their
negative antonyms. Table 1 presents the number of entries of the form "twice as ADJ as" and "half as ADJ as" with positive and negative antonyms, which were found in a google search (and the ratio between these numbers). In 75% of the cases (12 of 16 adjective pairs), the use of products like twice is more frequent in positive adjectives than in their negative antonyms (long-short; tall-short; fast-slow; big-small; true-false; safe-unsafe; healthy-sick; good-bad; happy-unhappy; likely-unlikely; similar-different; similar-dissimilar). In two pairs, the pattern is reversed (intelligent-stupid; safe-dangerous). In yet other 2 pairs, the pattern with twice and with half are different (beautiful-ugly; right-wrong).

TABLE 1: THE USE OF TWICE AND HALF WITH ADJECTIVES AND THEIR ANTONYMRS

<table>
<thead>
<tr>
<th></th>
<th>twice as ADJ as</th>
<th>Ratios</th>
<th>half as ADJ as</th>
<th>Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>similar</td>
<td>2,740</td>
<td></td>
<td>similar</td>
<td>647</td>
</tr>
<tr>
<td>dissimilar</td>
<td>0</td>
<td>0.0%</td>
<td>dissimilar</td>
<td>0</td>
</tr>
<tr>
<td>likely</td>
<td>591,000</td>
<td>0.3%</td>
<td>likely</td>
<td>98,800,000</td>
</tr>
<tr>
<td>unlikely</td>
<td>1,660</td>
<td>0.3%</td>
<td>unlikely</td>
<td>12,400,000</td>
</tr>
<tr>
<td>similar</td>
<td>2,740</td>
<td></td>
<td>similar</td>
<td>647</td>
</tr>
<tr>
<td>different</td>
<td>9</td>
<td>0.3%</td>
<td>different</td>
<td>2</td>
</tr>
<tr>
<td>long</td>
<td>1,210,000</td>
<td>1.2%</td>
<td>long</td>
<td>246,000</td>
</tr>
<tr>
<td>short</td>
<td>14,400</td>
<td>1.2%</td>
<td>short</td>
<td>5,460</td>
</tr>
<tr>
<td>true</td>
<td>170</td>
<td></td>
<td>true</td>
<td>3,210</td>
</tr>
<tr>
<td>false</td>
<td>2</td>
<td>1.2%</td>
<td>false</td>
<td>2</td>
</tr>
<tr>
<td>fast</td>
<td>1,300,000</td>
<td>2.7%</td>
<td>fast</td>
<td>66,300</td>
</tr>
<tr>
<td>slow</td>
<td>35,200</td>
<td>4.6%</td>
<td>slow</td>
<td>4,300</td>
</tr>
<tr>
<td>happy</td>
<td>13,800</td>
<td></td>
<td>happy</td>
<td>697</td>
</tr>
<tr>
<td>unhappy</td>
<td>632</td>
<td>4.6%</td>
<td>unhappy</td>
<td>308</td>
</tr>
<tr>
<td>big</td>
<td>307,000</td>
<td>6.0%</td>
<td>big</td>
<td>74,300</td>
</tr>
<tr>
<td>small</td>
<td>18,300</td>
<td></td>
<td>small</td>
<td>3,550</td>
</tr>
<tr>
<td>tall</td>
<td>63,400</td>
<td>6.0%</td>
<td>tall</td>
<td>18,100</td>
</tr>
<tr>
<td>short</td>
<td>14,400</td>
<td>22.7%</td>
<td>short</td>
<td>5,460</td>
</tr>
<tr>
<td>good</td>
<td>184,000</td>
<td>24.6%</td>
<td>good</td>
<td>347,000</td>
</tr>
<tr>
<td>bad</td>
<td>45,200</td>
<td></td>
<td>bad</td>
<td>103,000</td>
</tr>
<tr>
<td>healthy</td>
<td>7,810</td>
<td>24.6%</td>
<td>healthy</td>
<td>2,750</td>
</tr>
<tr>
<td>sick</td>
<td>2,550</td>
<td>32.7%</td>
<td>sick</td>
<td>1,420</td>
</tr>
<tr>
<td>safe</td>
<td>712</td>
<td>33.8%</td>
<td>safe</td>
<td>1,630</td>
</tr>
<tr>
<td>unsafe</td>
<td>241</td>
<td>33.8%</td>
<td>unsafe</td>
<td>3</td>
</tr>
</tbody>
</table>

Less uses in the positive adjective, compared to the negative antonym:

<table>
<thead>
<tr>
<th></th>
<th>twice as ADJ as</th>
<th>Ratios</th>
<th>half as ADJ as</th>
<th>Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>intelligent</td>
<td>1,880</td>
<td>17.4%</td>
<td>intelligent</td>
<td>1,630</td>
</tr>
<tr>
<td>stupid</td>
<td>10,800</td>
<td></td>
<td>stupid</td>
<td>8,280</td>
</tr>
<tr>
<td>safe</td>
<td>712</td>
<td></td>
<td>safe</td>
<td>1,630</td>
</tr>
<tr>
<td>dangerous</td>
<td>3,310</td>
<td>21.5%</td>
<td>dangerous</td>
<td>5,990</td>
</tr>
</tbody>
</table>

Opposite patterns for twice and half:

<table>
<thead>
<tr>
<th></th>
<th>twice as ADJ as</th>
<th>Ratios</th>
<th>half as ADJ as</th>
<th>Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>beautiful</td>
<td>15,200</td>
<td></td>
<td>beautiful</td>
<td>736</td>
</tr>
<tr>
<td>ugly</td>
<td>792</td>
<td>5.2%</td>
<td>ugly</td>
<td>2,980</td>
</tr>
<tr>
<td>right</td>
<td>10</td>
<td></td>
<td>right</td>
<td>1,370</td>
</tr>
<tr>
<td>wrong</td>
<td>656</td>
<td>1.5%</td>
<td>wrong</td>
<td>266</td>
</tr>
</tbody>
</table>
Some adjectives (like 'typical' and 'atypical') seem to be completely incompatible with ratio modifiers, even in the positive form. Finally, half as ADJ as is generally less frequently used, perhaps because fractions are harder to process, or for other additional reasons.

In sum, the licensing of ratio modifiers seems to be dependent on a combination of several factors, one of which is the polarity of the predicate.

2.1.4.6 The licensing of negative polarity items

Sixth, negative polarity items (NPIs) like any and ever (items whose distribution is limited to 'negative' contexts) can only occur in the scope of negative predicates (21a-b). According to Ladusaw (1979), NPIs are licensed in a linguistic context C iff C reverses the direction of entailment (C is downward entailing). For example, for any argument x, x drives fast in Rome asymmetrically entails x drives in Rome (21c). The direction of entailment in the case of the positive predicate safe is the same (20d), but in the case of the negative predicate dangerous it is reversed (21e).

\[(21)\]
\[
a. \quad \text{It is strange/* typical that any of those papers were accepted} \\
b. \quad \text{It would be foolish/* clever of her to even bother to lift a finger to help.} \\
c. \quad x \text{ drives fast in Rome} \quad \Rightarrow \quad x \text{ drives in Rome} \\
d. \quad \text{It is safe to drive fast in Rome} \quad \Rightarrow \quad \text{It is safe to drive in Rome} \\
e. \quad \text{It is dangerous to drive fast in Rome} \quad \Leftarrow \quad \text{It is dangerous to drive in Rome}
\]

2.1.4.7 Incommensurability

Finally, according to Kennedy (1999), it is hard to accept the co-occurrence of a predicate and its antonym in one comparative statement (22a), unless the two predicates are interpreted positively, as measuring deviations from a midpoint to different directions. For example, in (22b), the midpoint is the correct time (Kennedy 1999; see also the discussion on incommensurability below). However, as Buring (2007) and Landman (2005) note, (22c), with the negative predicate in the matrix clause and the positive in the main clause, is perfectly felicitous.

\[(22)\]
\[
a. \quad \# \text{ The house is taller than the ladder is short} \\
b. \quad \text{My clock is faster than yours is slow} \\
c. \quad \text{The ladder is shorter than the house is high}
\]

2.1.5 Contrasting categories

In addition to the antonym, a set of contrasting categories \(K_p\) may play a role in the interpretation of predicates. For example the set \{mammal, insect,\ldots\} plays a role in the interpretation of bird, and the set \{pink, white, orange, yellow,\ldots\} plays a role in the interpretation of red.
Contrasting categories are mutually exclusive, as stated in (23a), and together they cover a local domain, \( D_{K_p} \), as stated in (23b-c). For instance, the domain of apples (\( D_K = \text{[[apple]]}_c \)) can be partitioned by color type, to four contrasting categories (\( K_{\text{red-apple}} = \{ \text{red, green, yellow, brown} \} \)).

\[
(23) \quad \forall c \in C, P \in \text{PRED}:
\begin{align*}
\text{a. } &\forall Q_1, Q_2 \in K_P: \text{[[Q}_1]_c \cap \text{[[Q}_2]_c = \emptyset} \\
\text{b. } &P \in K_P \\
\text{c. } &\cup \{\text{[[Q]]}_c \mid Q \in K_P\} = D_{K_p}
\end{align*}
\]

The antonym relation may be a sub-case of the contrasting category relation, in which \( K_P \) consists of only two predicates (\( P \) and its antonym).

Given that the denotations of members of \( K_P \) are mutually constrained (23), the orderings of entities (their degrees) in contrasting categories must also be mutually constrained. We already saw that this is the case with antonyms (the ordering relation of a predicate and of its antonym are reversed). However, linguistic theories usually disregard the relations between the denotations and orderings of contrasting categories. Most of the predicates, which have more than one contrasting category, are nouns, and nouns are treated by linguists as non-gradable. In 2.2 and in chapter 4, I discuss some relevant psychological findings and theories which explain the ways contrasting categories might affect the ordering relation of a predicate. Note also that future investigation is needed in order to establish the felicity status and correct interpretation of comparisons with two contrasting categories like (24).

\[
(24) \quad \begin{align*}
\text{a. } &\text{This rod is more red than that rod is blue} \\
\text{b. } &\text{This Thai dish is more sour than sweet}
\end{align*}
\]

2.1.6 Between-predicate comparisons

Certain statements (which are often called sub-deletion comparatives) involve comparisons of degrees in two different predicates. In order to abstract away from theoretical considerations, I will call these statements between-predicate comparisons (25a). Conversely, I will call statements that involve comparisons of two degrees in one predicate (such as \( \text{The table is longer than the sofa is wide} \)) within-predicate comparisons. Pairs of predicates may occur in between-predicate comparisons iff their degrees are comparable, as demonstrated by the contrast in (25a-b).

\[
(25) \quad \begin{align*}
\text{a. } &\text{The table is longer than the sofa is wide} \\
\text{b. } &\# \text{The table is longer / more long than the sofa is heavy} \\
\text{c. } &\# \text{Dan is taller / more tall than Ram is intelligent} \\
\text{d. } &\# \text{John is taller / more tall than he is not} \\
\text{e. } &\# \text{John is taller / more tall than he is short}
\end{align*}
\]

The infelicity of (24b) demonstrates the incommensurability of the degrees associated with \( \text{long} \) and the degrees associated with \( \text{heavy} \). In particular, a predicate and its negated form or a predicate and its antonym can never co-occur in between-predicate comparisons, where the antonym is in the \( \text{than} \) clause (Buring 2007; Landman 2005), as demonstrated in (22).

However, under certain circumstances, predicates which are incommensurable can nonetheless occur in sub-deletion statements.
2.1.6.1 Comparisons of deviations from a midpoint

First, they can do so when the two predicates are interpreted as measuring deviations from a midpoint to different directions. Such an interpretation is called comparison of deviation (Kennedy 1999). For instance, the midpoint in (26) is the correct time.

(26) My clock is faster than yours is slow

2.1.6.2 Meta-linguistic interpretations

Second, according to Kennedy, examples like (25c) above can receive a metalinguistic interpretation. That is, they can be used ironically to convey the information that Ram is not intelligent. According to Klein (1991), in this use, (25c) is an answer to the question *is Ram intelligent* rather than to a question like *how tall is Dan* or *how clever is Ram*.

2.1.6.3 Comparisons of relative position

Third, between-predicate comparisons may sometimes be felicitously used in a 'normal' interpretation, that is, so as to compare the relative positions of entities in two different scales (though the availability of this interpretation may be heavily dependent on contextual information).

(27) a. My 14 year old son is also an Aug 24 Virgo. I find that he is more typical of a Leo in the sense that he is outgoing and easy to get along with. Don't get me wrong, he also has Virgo type behavior
b. The program began with Debussy's rarely heard Fantasies. In this three-movement, artfully integrated piece – a “concerted” work rather than a completely developed concerto - the extensive piano part is more of a first among equals than a showcase for a virtuoso soloist
c. When Dan comes home from school and I come home from work, we are always hungry and tired. But usually, Dan is more hungry than tired, while I am more tired than hungry.
d. Dan is tired and Mira is hungry. Take care of them. Dan is more tired than Mira is hungry, so take care of him first.

For example, the speaker in (27a) asserts that her son is more typical of a Leo than of a Virgo, despite the fact that the typicality ordering criteria for Leo and for Virgo are different. The writer of (27b) compares the degrees of a piano part in two different categories. These examples need to be interpreted neither as comparisons of deviation nor meta-linguistically in the above sense. (27b) can answer the question how much is the piano part (of) a showcase for a virtuoso soloist? In (27c), Dan's being more hungry than tired is understood as a Dan having a stronger preference to eat than to sleep. That is, the extents to which the hunger and the tiredness disturb Dan are compared. Similarly, in a context whereby one adult has to take care of two children, Dan is more hungry than Mira is tired in (27d) can be understood as stating that Dan's hunger and Mira's tiredness require urgent action and as comparing the extents to which the action is urgent.
2.1.6.4 Comparisons of contrast categories

According to my informants, between-predicate comparisons seem to improve, in both English and Hebrew, if the adjectives in question are interpreted as contrasting categories (cf. 2.1.5).

(28) a. This rod is more red than it is blue
b. This Thai dish is more sour than sweet (or anything else)

2.1.6.5 Nouns freely occur in between predicate comparisons

Finally, nouns are again different.

We saw that nouns differ from adjectives in that adjectives are typically felicitous in within-predicate comparisons, while nouns are typically not felicitous in such comparisons. Surprisingly, nouns differ from adjectives also in that, unlike most of the adjectives, they freely occur in between-predicate comparisons. I have found numerous examples like (29a-c) in a simple google search for "more a".

(29) a. Bling Bling says "tweet" (I'm convinced he's more a bird than a cat).
b. ... giving me three bits of furniture which she didn't want anymore (a coat rack, chair, and stool thing which is really more a table than anything else)
c. The "wall" was rolling backward until it come to a horizontal position, now being more a table than a wall
d. Chevy is more a car than a truck
e. The ostrich is more a bird than the platypus is a mammal
f. A bat is no more a bird than a whale is a fish

Since linguists usually consider nouns not to be gradable, this data was overlooked in the past.

2.1.7 Within-predicate comparisons

One of the most central linguistic tests for predicate gradability is compatibility with the comparative morphemes in within-predicate comparisons. Consequently, many linguistic theories of gradability focus on the semantics of the comparative morphemes, and the way the meaning for statements of within-predicate comparisons is derived compositionally from the meanings of the parts. I will now briefly survey some of the relevant linguistic data.

2.1.7.1 The morphological complexity of comparative versus positive predicates

The first important finding is the following (Kamp 1975; Klein 1991). Cross linguistically, the morphological forms of positive predicates (e.g. tall) are always less complex than those of the comparative and equative relations (taller; less tall; as tall as). The latter are derived from the former. This is surprising. Intuitively, the meaning of a comparative (the partial ordering of entities by the extent to which they satisfy a property), appears to be a conceptual primitive, or at least to be conceptually simpler than (the meaning of) the positive predicates. Intuitively, in order to determine whether an entity $d_1$ falls under a positive predicate $P$, we need to compare $d_1$'s degree in $P$ and another degree, standard$_P$ ($P$'s contextual standard of membership). $P$'s standard may vary between contexts and may be unknown. Conversely, decisions about membership in the comparative predicate more $P$ than $d_2$ require no
information about P’s standard. The items which are being compared are directly given, as demonstrated in (30). If implicit comparison is part of the syntax, semantics, or processing of P, and P’s standard is not part of the syntax, semantics, or processing of the comparison morpheme, why is P’s form cross linguistically simpler than that of the comparison morpheme?

\[(30)\]
\[a. \quad [\{P\}]_c = \{d \in D \mid d \geq_P \text{ standard}_P \text{ in } c\}\]
\[b. \quad [\{\text{more } P\}]_c = \{<d_1,d_2> \in D^2 \mid d_1 \geq_P d_2 \text{ in } c\}\]

2.1.7.2 The comparative morphemes

The semantics of the comparative should correctly predict the semantic dependencies between more, less and as comparatives.

With regard to more and less, the entailment pattern in (31a) seems to hold for every pair of entities in the local domain of a gradable predicate, as demonstrated in (32b) for the predicate tall. With regard to a predicate P and its antonym P\text{ant}, the entailment pattern in (31c) seems to hold for every pair of entities in the local domain, as demonstrated in (31d-e) for the predicates tall and short (or non-tall). In other words, at least within a common local domain, the scales of a predicate P and its antonym, P\text{ant} (for instance non-P), seem to be inversely related. Intuitively, these inverse relations hold also between the scales of a predicate P and its direct negation, not-P, but this is harder to verify directly, given that the negated predicate is not quite felicitous in the comparative (31f).

\[(31)\]
\[a. \quad \forall d_1,d_2 \in D_P: \quad d_1 \geq_P d_2 \iff d_2 \leq_P d_1\]
\[b. \quad \text{Dan is taller than Sam is} \iff \text{Sam is less tall than Dan is}\]
\[c. \quad \forall d_1,d_2 \in D_{P,\text{Pant}}: \quad d_1 \geq_P d_2 \iff d_2 \geq_{\text{Pant}} d_1\]
\[d. \quad \text{Dan is taller than Sam is} \iff \text{Sam is shorter than Dan is}\]
\[e. \quad \text{Dan is taller than Sam is} \iff \text{Sam is more non-tall than Dan is}\]
\[f. \quad \text{Dan is taller than Sam is} \iff \# \text{ Sam is more not tall than Dan is}\]

In certain predicates, the entailment pattern in (31c) seems to fail, as demonstrated in (32a) (Rotstein and Winter 2005). In particular, pairs of healthy entities can stand in the healthier relation, but not in the relation sicker. The latter can only felicitously apply to sick entities, as demonstrated by the contrast between (32b) and (32c).

\[(32)\]
\[a. \quad \text{Dan is healthier than Sam is} \iff \# \text{ Sam is less sick than Dan is}\]
\[b. \quad \text{Dan and Sam are sick, but Sam is healthier than Dan is}\]
\[c. \quad \# \text{ Dan and Sam are healthy, but Sam is sicker than Dan is}\]

The entailment pattern in (31c) fails in less comparatives in as many cases as it fails with more comparatives, and interestingly, maybe in more cases. In particular, it becomes harder to apply the less healthy relation to pairs of sick entities, as demonstrated by (33).

\[(33)\]
\[a. \quad \text{Dan is less healthy than Sam is} \iff \# \text{ Sam is less sick than Dan is}\]
\[b. \quad \# \text{ Dan and Sam are sick, but Sam is less healthy than Dan is}\]
\[c. \quad \# \text{ Dan and Sam are healthy, but Sam is less sick than Dan is}\]

In addition, every predicate which is combined with a comparative morpheme is affected, at least to some extent, by the presupposition that the ordered entities fall under the predicate. To demonstrate this, I have asked 35 Hebrew speakers to judge whether they agree, yes or no,
with statements in Hebrew, which directly translate into the statements in (34). While 95% of these speakers agreed with (34a), only (65%) agreed with (34b) and only 55% agreed with (34c). Why? These speakers were reluctant to order birds by typicality in non-bird. Some of them have explicitly said so. That is, their no answer has to be interpreted as stating that it is not possible to felicitously apply the comparative more/less typical of a non-bird to arguments which fall outside of the domain of typical of a non-bird (namely – to birds).

This presupposition seems to be stronger in statements with a positive comparative morpheme (more) than in statements with a negative one (less). More speakers were willing to accept (34b) than (34c), despite the fact that (34c) involves but one negation (non), while (34b) involves two negative elements (less and non). Thus, (34b) may be harder to process and, hence, less likely to be accepted. The presupposition that the ordered entities fall under the ordering adjective turns statements which compare non-members inappropriate. This effect seems to be strong enough to turn the more complex statement more acceptable than the simpler (but less appropriate) one.

(34)  
a. A robin is more typical of a bird than an ostrich  
 More P(r, o)  
95% Yes
b. A robin is less typical of a non-bird than an ostrich  
 less P_{\text{non}}(r, o)  
65% Yes
c. An ostrich is more typical of a non-bird than a robin  
 More P_{\text{non}}(o,r)  
55% Yes

In conclusion, more and less comparatives seem to differ in the extent to which they raise the expectation that the ordered entities are predicate members. This expectation may also be stronger for the subject or for the object. These issues call for further empirical investigation. With regard to the equative morpheme as, its semantics seem to be incompatible with degree modifiers such as two meters (35a) (Schwarzschild and Wilkinson 2002). In addition, the truth value of equative statements, like the one in (35b), may change when more fine grained units of measurement (say, millimeters) replace less fine grained ones (centimeters).

(35)  
a. * Dan is two meters as tall as Sam
b. Dan is as tall as Sam

Finally, the comparative morphemes interact with quantifiers and intensional operators in different ways. For a discussion see Kennedy (1999), Rullmann (1995) and Schwarzschild and Wilkinson (2002). For a fuller review of puzzles and theories related to gradability and comparison see Klein (1991) and Kennedy (1999).

2.1.7.3 'At least', 'at most' and 'exactly' readings

With regard to the equative as, the indeterminacy which characterizes numerals characterizes also this comparative morpheme (Schwarzschild and Wilkinson 2002). Like the numeral in (36a), as in (36b) is interpreted as meaning at least as. Like the numeral in (36c), as in (36d) is interpreted as meaning at most as. Like the numeral in (36e), as in (36f) is interpreted as meaning exactly as.

(36)  
a. Parents of 2 children get in free of charge (at least 2)
b. If you are as tall as Dan, you can reach the basket (at least as tall as Dan)
c. I can take 4 persons in this car (at most 4)
d. If your car is as tall as Dan's car, it will have no problem passing under that bridge. (at most as tall as Dan's)

e. How many children do you have? 2. (exactly 2)

f. How tall is Dan? He is as tall as Peter. (exactly 2 as tall as Peter)

As in Kadmon's (1987) treatment of the semantics of numerals, the semantics of the equative morpheme (and probably also the comparative morpheme, as discussed in ongoing chapters; cf. Landman 2005) should be one that can be modified by at least, at most and exactly. The pragmatic theory should ultimately determine when and how these modifier meanings show up.

Note that the same indeterminacy characterizes also numerals in comparative predicates. We can demonstrate this fact by construing examples like (37a-e).

(37)

a. If your car is two centimeters taller than Dan's car, it will not pass under that bridge (at least 2 more)
b. If your car is two centimeters taller than Dan's car, it will have no problem passing under that bridge (at most 2 more)
c. How tall is Dan? He is 2 centimeters taller than Peter (exactly 2 more)

2.1.7.4 Extent versus Quantity interpretations of comparative statements

The semantics of the comparative should be compatible with its cross-categorical nature. In adjectives, the comparative form compares the extents to which the entities manifest the property. For example, (38a) is interpreted as stating that the extent to which Dan is happy exceeds the extent to which Sam is happy.

In nouns, the comparative form compares quantities, not extents. In plural nouns, it compares number of entities. For example, (38b) is interpreted as stating that the number of boys that danced exceeds the number of girls that danced (| [[boys]] ∩ [[danced]] | > | [[girls]] ∩ [[danced]] |). In mass nouns, the comparative form compares amounts of stuff. For example, (38c) is stating that the amount of pepper in the dish exceeds the amount of rice in it, though the measure of amount seems to be indeterminate here: it may be number of units, volume, weight or even effects on the dish's taste. At any rate, (38b) and (38c) involve no reference to extents in the properties boy(s), girl(s), pepper and rice (degrees of boy-hood, girl-hood, etc.), as demonstrated in (38d-e).

(38)

a. Dan is happier/ more interesting than Sam is
b. More boys than girls danced
c. There is more pepper than rice in this dish
d. * This Chevy is more (a) car than that Chevy
e. * This powder is more pepper than that powder

The lack of extent interpretation for more in nouns cannot be explained by the availability of the quantity interpretation (that interferes). This explanation is problematic because the quantity interpretation is not available in singular nouns. Singular nouns only have singular individuals in their denotation and, therefore, combinations like more a boy than or more a car than do not have the quantity interpretation. Nonetheless, in English, it is not felicitous to say that someone is more a boy than someone else, or that something is more a car than something else (cf. (38d-e)). The extent reading is not readily available (we need to say more of a boy and more of a car). Thus, the availability of the quantity reading cannot explain the
incompatibility of the noun meaning with the extent reading of *more* in within predicate comparisons.

Verbs usually cluster with nouns (Klein 1991), in that the comparison is not understood to be between extents to which entities manifest the verbal property. Rather, the comparative applies to one of the verb's arguments. For example, (39a) is interpreted as stating that the temporal duration of Dan's dancing exceeds that of Sam's dancing, or that the spatial path of Dan's walk exceeds that of Sam's walk. However, experience verbs are exceptions – they cluster with the adjectives (Aya Meltzer p.c.). For example, (39b) is interpreted as stating that the extent to which today's film interested Dan exceeds the extent to which yesterday's film did. Finally, adverbs and prepositions may also occur in the comparative, as demonstrated in (39c). The adverbs seem to cluster with the adjectives.

(39)  
a.  Dan walked / danced more than Sam did  
b.  Today's film interested Dan more than yesterday's film did  
c.  more slowly, nearer, before, after

The extents which adjectival comparisons like *happier* compare are sometimes denoted by a corresponding nominal property (e.g. *happiness*). They cannot be denoted by a corresponding verbal property (e.g. *being happy*). For example, (40a) and (40b) are interpreted very much along the same lines, while (38c) is completely infelicitous (Moltmann 2005).

(40)  
a.  Dan is happier than Sam is  
b.  Dan's happiness is greater than Mary's happiness  
c.  * Dan's being happy is greater than Mary's being happy

2.1.7.5 Phrasal versus clausal comparatives

Comparatives which surface as relations between two noun phrases, like (41a), are often called phrasal comparatives. Comparatives which surface as relations between a noun phrase and a clause, like (41b), are often called clausal comparatives.

(41)  
a.  Phrasal comparatives:  *Dan is happier than Sam*  
b.  Clausal comparatives:  *Dan is happier than Sam is*

Intuitively, phrasal and clausal deletion comparative types do not seem to differ in meaning. Yet a closer examination shows that there are differences between them. First, given the syntactic differences between them, the compositional derivation of their truth conditions is different (Kennedy 1999).

Second, extraction is possible from phrasal comparatives, but not from clausal ones (for a discussion see Kennedy 1999).

(42)  
a.  You finally met somebody you are taller than  
b.  * You finally met somebody you are taller than (he) is  
c.  Which planet is Neptune as bright as?  
d.  * Which planet is Neptune as bright as is?

Third, negative polarity items (as opposed to free choice ones) can only occur in the scope of the clausal comparatives, as demonstrated in (43a). Consequently, some authors argue that this is a downward entailing context (Hoeksema 1983). Others show that *than-
clauses in clausal comparatives cannot be downward entailing (Schwarzschild and Wilkinson 2002: 3-8; Landman 2005). For example, given the assumption that Mary is a girl, \( x \) teased every girl entails that \( x \) teased Mary. The direction of entailment is supposed to be reversed if these two expressions are embedded within a downward entailing context. Indeed, (43b1) and (43b2) entail (43b3) (Every boy who teased every girl has also teased Mary; hence, by (41b1), he was sent to the headmaster). This entailment pattern shows that every is indeed downward entailing on its first argument. However, (43c1) and (43c2) do not entail (43c3). Thus, the than-clause in a clausal comparative does not seem to be a downward entailing context. The occurrence of negative polarity items in these contexts calls for an alternative explanation (for further discussion see Landman 2005).

(43)  
\[ \begin{align*}
\text{a.} & \quad \text{It is hotter in New Brunswick today than it ever was in LA} \\
\text{b.} & \quad 1. \, \text{Every boy who teased Mary was sent to the headmaster} \\
& \quad 2. \, \text{Mary is a girl.} \\
& \quad 3. \, \text{Every boy who teased every girl was sent to the headmaster} \\
\text{c.} & \quad 1. \, \text{John is more famous than Mary is} \\
& \quad 2. \, \text{Mary is a girl.} \\
& \quad 3. \, \text{John is more famous than every girl is}
\end{align*} \]

Fourth, according to Kennedy (1999), clausal comparatives differ from phrasal comparatives, in that the former but not the latter are ambiguous in ellipsis. For example, the second conjunct in (44a) is unambiguous, having only the interpretation in (44c). The second conjunct in (44b) is ambiguous, having either the interpretation in (44c) or that in (44d).

(44)  
\[ \begin{align*}
\text{a.} & \quad \text{The table is longer than the rug is wide, and the rug is longer than the desk} \\
& \quad \text{is.} \\
\text{b.} & \quad \text{The table is longer than the rug is wide, and the rug is longer than the desk is.} \\
& \quad \text{c.} & \quad \ldots \text{the rug is longer than the desk is long.} \\
& \quad \text{d.} & \quad \ldots \text{the rug is longer than the desk is wide.}
\end{align*} \]

Finally, clausal comparatives are ungrammatical whenever the "missing" material in the than clause is contained in an extraction island, i.e. in a linguistic context from which extraction is impossible, as demonstrated in (45) with WH-word extraction (Chomsky 1977). The corresponding grammaticality failures in comparatives are demonstrated in (46a-b) (WH-islands); (46c) (complex NP island) and (46d) (adjunct island).

(45)  
\[ \begin{align*}
\text{a.} & \quad * \text{Who did Sam wonder whether Dan kissed?} \\
& \quad * \text{Who did Sam know who said that Dan kissed?} \\
& \quad * \text{Who did I see a paper that said that Dan kissed?} \\
& \quad * \text{Who were we amazed when Dan kissed?}
\end{align*} \]

(46)  
\[ \begin{align*}
\text{a.} & \quad * \text{Mercury is closer to the sun than I wondered whether it was} \\
& \quad * \text{Mercury is closer to the sun than I knew who said it was} \\
& \quad * \text{Mercury is closer to the sun than a paper that said it would be} \\
& \quad * \text{The solar flares were more energetic than we were amazed when the aurora Borealis was}
\end{align*} \]
2.2 The second set of facts: Psychological evidence for vagueness and gradability in nouns

We will begin by reviewing symptoms of vagueness and gradability in nouns, which occur repeatedly in the empirical research of the basic structure of the concepts denoted by natural language nouns. These symptoms are usually called the typicality effects. For an early but extensive review about typicality and categorization see Mervis and Rosch (1981). Throughout the representation of the psychological facts, I also present purely linguistic facts, showing that vagueness and gradability systematically occur in nouns. Most of these facts were observed in the past, but their implications for the semantics of nouns were not worked out. They appear to be incompatible with the conclusions that were drawn based on the facts in 2.1. A complete analysis should clarify exactly how these two sets of facts go together. By putting all these phenomena together, I wish to emphasize that we need to consider the linguistic evidence for gradability when we work on the semantics of nouns.

2.2.1 A denotation gap in nouns

Experimental findings suggest that quite often speakers are uncertain about the membership of entities in nominal categories. For example, tomatoes fall between fruit and vegetables and three-legged seats with a small back fall between chairs and stools. While speakers rarely (under 3% of the time on average) change their minds about the category-membership of clear instances of nouns, they do so above 20% of the time (on average) in borderline cases like curtains for furniture or avocado for vegetables (McCloskey and Glucksberg 1978).

2.2.2 Ordering relations between entities in nouns

2.2.2.1 Psychological evidence for entity orderings in nouns

The last forty years of research in cognitive psychology have established beyond doubt that speakers consider certain entities as better examples than others of categories such as those which natural language nouns denote. For example, a robin is often considered more typical or representative of a bird than an ostrich or a penguin. In addition, a bat or a butterfly is often considered more related or more similar to a bird than a dog or a cow. When subjects are asked to rate an item as to "how good it is as an example of a given category" on, say, a seven-point scale, their ratings are highly similar (the extent of variance between subjects is identical to the extent of variance within different tests of one and the same subject (McCloskey and Glucksberg 1978).

These basic typicality judgments are connected to numerous processing effects. Most importantly, typicality correlates with online categorization times. For example, when robins are considered more typical birds than ostriches, verification time for sentences like a robin is a bird is faster than for sentences like an ostrich is a bird (Rosch 1973; Rips, Shoben and Smith 1973; Rosch, Simpson and Miller 1976; Roth and Shoben 1983; Armstrong, Gleitman and Gleitman 1983).

Furthermore, two variables form the associative strength of the link between (the name of) an instance and (the name of) a category: Category dominance is the frequency that a category is produced in response to the name of an instance. Instance dominance is the frequency that the instance is produced in response to the name of a category (Batting and Montague 1969). Typicality ratings often correlate with these two variables, and mostly with instance dominance.
In addition, in recalling memorized lists of category exemplars, people tend to reorder them, such that the more typical items are produced earlier (Rosch 1973; Rosch, Simpson and Miller 1976). In accordance, Rosch (1973) writes:

Retrieval of category instances from long term memory may be performed by means of serial, fixed-order, self terminating search which begins with the best examples of the category (Rosch 1973: 140-141).

Accordingly, the link between a categorical anaphor (namely, a definite expression like *the fruit*) and its antecedent (an expression like *the apple* or *the banana*) is easier when the antecedent denotes a highly typical member of the category (Garrod and Sanford 1977; Roth and Shoben 1983). Conversely, in passages describing a protagonist selecting a category member and not selecting another member of variable typicality (for instance, in a passage of the form *Dan Just bought a mango but not an apple / kiwi*), the process of anaphora resolution (i.e. reading time of, e.g. *he ate the fruit*) is longer following a negated expression which denotes a typical instance, compared to a negated expression which denotes an atypical instance. Thus, in the activation of *fruit*, typical instances are more accessible. It is harder to suppress them and to access atypical instances instead. Consequently, the negation of highly typical examples is harder to process (Levine 2002). This effect disappears only when the member selected by the protagonist is highly typical (*Dan Just bought a banana but not an apple / kiwi*), due to the closer link between categorical anaphors and highly typical antecedents (Levine 2002).

Finally, the prominence of the typical instances was supported also by studies in the priming paradigm. Judgments of *same* or *different* for pairs of instances were facilitated (primed) by hearing the category name only in pairs of typical instances. Atypical instances were not primed (Rosch, Simpson and Miller 1976).

2.2.2.2 Linguistic evidence for entity orderings in nouns

These ordering effects have several purely linguistic reflexes. First, in order to make the noun *bird* gradable, one can use the modifier *typical*, or only add the particle *of* to the comparative morpheme, as in (47a). Second, nouns turn easily into adjectives, by adding a morpheme like 'y', as in *birdy*, and the resulting adjective is gradable (47b).

\[(47)\]
\[
a. \quad \text{A robin is more (typical) of a bird than an ostrich} \\
b. \quad \text{The noun activity is 'nounier' / less 'nouny' than the noun bird (Ross 1973)}
\]

Third, sometimes, degree-modifiers (48a) and scalar modifiers (48b) can combine with nouns (Alexander Grosu, personal communication). Fourth, as noted in 2.1.6.5, nouns occur freely in between predicate comparisons (48c).

\[(48)\]
\[
a. \quad \text{This is pretty much a chair} \\
b. \quad \text{This is almost a chair} \\
c. \quad \text{This is more a chair than a table}
\]

These facts are hard to explain if nouns are non-gradable.

In conclusion, sometimes nouns are susceptible to degree-modification (and, as we saw in 2.1.6.5, at least one structure that is conventionally viewed as mediated by degrees ("more P than Q") licenses (bare) nouns more freely than it licenses adjectives. Thus, we have evidence
for both conceptual gradability (graded judgments of membership) and morphological gradability (compatibility with morphemes like 'more' and 'much', whose semantics relate to degrees) in nouns.

In addition to these basic facts, typicality is also known as affecting the acceptability of certain reinforcements and hedges. Their acceptability in sentences of the form *an x is P* depends on whether the argument is a typical member (49a-c), an atypical member (49d-f), or a non member which, nonetheless, can be considered typical (49g) (Lakoff 1972).

(49)

a. A robin / sparrow / parakeet is a true bird  
   b. # Technically speaking a robin / sparrow / parakeet is a bird  
   c. # A robin is virtually a bird  
   d. Technically speaking a chicken / duck / goose is a bird  
   e. A penguin is virtually a bird  
   f. # A chicken / duck / goose is a true bird  
   g. Loosely speaking a bat / butterfly / moth is a bird

The acceptability of sentences of the form *X is virtually Y* depends on placing in the X slot the argument denoting the less typical item, as demonstrated in (50) (Lakoff 1973). Similarly, the acceptability of sentences of the form *X is like / similar to Y* depends on placing in the X slot the argument denoting the less typical item (51) (Tversky and Gati 1978). Given this finding about the asymmetry of the similarity relation, Gleitman, Gleitman, Miller and Ostrin (1996) observed (and empirically established) that many linguistic relations, including *equal* and *identical*, are asymmetrical, in a way which depends on the arguments’ typicality.

(50)

a. A penguin is virtually a robin  Much better than:  
   b. # A robin is virtually a penguin

(51)

a. Mexico / Canada is similar to USA  Much better than:  
   b. USA is similar to Mexico / Canada

2.2.3 The noun dimensions

2.2.3.1 Psychological evidence for the association of nouns with dimensions

Experiments have established that speakers associate concepts such as those which natural language nouns denote, with sets of dimensions (for reviews see Rosch and Mervis 1981; Murphy 2002). For instance the noun *bird* is usually associated with dimensions like *feathers, flying, nesting, singing, eating insects, small size*, etc.

The association of nouns with dimensions is also connected to unconscious processing effects. Most importantly, during categorization, speakers pay more attention to dimensions previously rated as more related to the noun (Glass and Holyoak 1975). For example, when *red* is rated as *more important* than *round* for the noun *apple*, speakers decide that *apples are red* faster than that *apples are round*. In addition, the mean ranked importance (or weight) of each dimension in a group of subjects, correlates with the frequency with which the dimension is generated when the subjects are asked to provide a list of dimensions for the noun (Hampton 1987: 59). This correlation is low, though, since certain rather important dimensions which are very general or are presupposed by the noun (like *animate* for *a bird*) are not usually produced by subjects.

There is ample neuropsychological evidence for the cognitive reality of both the typicality entity ordering relations and the dimension sets.
Faster online reaction time in categorization tasks for typical compared to atypical examples is found in healthy adults, elder subjects and Broca aphasic patients, but not in Wernicke aphasic patients, who generally, suffer most from deficits in language interpretation. They make the highest number of errors on the task and are also significantly slower to respond (Kiran and Thompson 2003a). Evidence for the importance of the noun dimensions comes from rehabilitation methods for aphasic patients with naming deficits due to damage in brain areas underlying the representation of lexical categories. Treatment emphasizing the typicality dimensions is a successful approach for training naming, resulting in stronger training effects than other training techniques such as picture-word matching (Kiran and Thompson 2003: 783). In addition, the extensive brain imaging research in normal adults shows that nouns with different characterizing dimensions are stored in different brain areas. Nouns denoting artifacts are identified by their use (e.g. chair is identified by a property like is used to sit on) and hence semantic information about these predicates is stored near brain areas responsible for motor control and action planning. Nouns denoting animate beings are identified by sensory (usually visual) properties. Hence, semantic information about these nouns is stored in areas in which the relevant sense is implemented (Susan Bookheimer 2002).

2.2.3.2 Linguistic evidence for the association of nouns with dimensions

Finally, these effects have a linguistic reflex, too. Like the felicity of nouns in the comparative, also wrt-modification and quantification over the dimensions, which are impossible in bare nouns (52a) (cf. 2.1.2.2), become possible if the nouns are slightly modified (52b). This fact is hard to explain if nouns are not associated with grammatically accessible ordering-dimensions.

(52)  
  a. # Tweety is a bird in every respect / # generally a bird / # a bird wrt flying  
  b. Tweety is a typical bird in every respect / generally typical of a bird / typical of a bird wrt flying

Note that it is not the case that the adjectival dimensions are gradable and the noun dimensions binary. For example, the typicality ordering in bird can be represented by gradable dimensions like size and degree of ferocity, and the ordering in adjectives like healthy can depend on 'binary' (yes / no) dimensions like has cancer. In addition, I do not see why wrt-modification and quantification would be available for gradable dimensions, but not for binary dimensions. Consequently, this distinction fails to explain the differences between the nouns and the adjectives.

2.2.3.3 Lack of clear cut membership criteria in nouns

What do the dimensions which speakers associate with concepts like bird (e.g. feathers, flying, nesting, singing, etc.) stand for? The classical view, which has been prevalent since antiquity, considered these dimensions to be definitional: Necessary and sufficient conditions for membership in the denotation (Aristotle, Apostle 1980: 619-620; Locke 1968 [1690]; Searle 1958; Fodor and Katz 1963; Katz and Postal 1964; Jackendoff 1972). This view was rejected on the basis of philosophical and empirical grounds. Wittgenstein (1968 [1953]) and Fodor et al. (1980) show that the idea of definitions is rarely if ever met in natural categories. For example, which properties can define games? Maybe involve physical activity? But this is not valid for chess. Maybe compete against opponents? But solitaire does not involve any opponents, etc. For each dimension of games, some entities
exist which do not satisfy it, but nevertheless fall under *games*. Thus, a member of a natural category may share a slightly different set of properties with every other member of the category. In other words, the nominal dimensions do not stand for necessary conditions for membership in the denotation. Nor do they stand for sufficient conditions for membership. In many concepts, some entities exist, which satisfy their presumed necessary conditions, but do not fall under them. For instance, famous counter-examples to the definition of *bachelor* are the popes and homosexuals that have been living with a partner for many years. They are adult males that were never married, but are they bachelors? Thus, even if some dimensions form necessary conditions for membership in a category, there is nothing to tell us what a sufficient set of conditions should be. This fact renders the degree of similarity of entities to prototypical examples of the category crucial in determining their status in the category (Hann and Chater 1997).

Empirical studies (Hampton 1979 and 1995) experimentally established that, indeed, satisfying the dimensions is usually neither necessary, nor sufficient for categorization (membership in the denotation) in many different types of nouns (*fish, vegetable, sport, transportation, uncle, grandmother,* etc.) For example, the dimension *horse-genotype* is intuitively thought to form a necessary condition for membership under *horse*. Yet, these experiments show that creatures that violate this dimension, but are highly typical in other *horse*-dimensions, are often judged to be *horses*. In Hampton (1979), the noun *bird* was an exception in that no serious exceptions to its important dimensions were found. Yet, one can easily imagine a situation, whereby a mutation in a certain type of *bird* produces a new type, which is, say, *feather-less*. It is not clear at all that this type will be automatically classified as falling outside the domain of *bird*. This shows that *has feathers* is not really 'necessary' for birds. In a sense, only by accident it does not yet have any exceptions.

Certain attempts were made to weaken the classical theory (see, for instance, Searle 1958), in order to reconcile it with the counter-examples, for instance, by treating the set of criteria as a disjunction, requiring only one of the criteria to be satisfied in each case. Such attempts have finally led to the abandonment of the classical theory.

### 2.2.3.4 The noun dimensions as ordering dimensions

What do the dimensions which people link with a category like *bird* stand for, if not definitions? Empirical studies (Rosch and Mervis 1975) illustrated that when the dimensions are satisfied, they are raising the similarity of entities to prototypical examples. As a consequence, they are raising the typicality and likelihood of categorization of entities in the category. Thus, the noun dimensions constitute ordering dimensions, which together help to measure the typicality (and membership likelihood) of entities in the category.

Following Wittgenstein (1968 [1953]), typicality is equated with *family resemblance* among category members. It was found that the highest typicality (family resemblance) scores belong to items with the largest set of shared dimensions. For example, in experiment 5 in Rosch and Mervis (1975), the stimuli were artificial categories of letter strings. The weight of each dimension (letter type) was indicated by the number of strings in the category in which it occurred. The sum of weights of the letters in each string represented its family resemblance score. Rosch and Mervis found that these scores correlated with subjects' typicality ratings. Thus, the weighted mean of an entity in the dimensions common in a category (*within-category similarity*) is a good indicator of the entity's typicality in the category. This was also found to hold true of more natural categories like *bird* and *furniture* (in Rosch and Mervis 1975 correlations ranged between 0.84 and 0.91). In these categories, a dimension was defined to be a property which many subjects link to the category, and the weight of each
dimension was given by its frequency within the category members (the number of members which subjects described by it; cf. Rosch and Mervis 1975: 579-80).

Thus, these studies established that the entities which are rated as more typical in the category are also rated as more typical in the category dimensions (Rosch 1973); e.g. overall (on the average), a robin scores better than an ostrich (which is a less typical bird) in small, flies, sings, perches in trees and so on. Accordingly, the standard cognitive theory models typicality by average on the dimensions. For instance, according to Tversky's famous (1977) analysis, the contrast model, the similarity of a robin to a canary or to a bird (and, accordingly, the typicality of a robin in bird), depends on dimension matching, i.e. on the number (and weights) of the dimensions common to both concepts, as opposed to the number (and weights) of the dimensions distinct to each concept, which are taken into account. Again, typicality is represented in terms of the sum of the entities' weighted degrees in the noun dimensions, that is, by averaging. A detailed discussion of different dimension models and averaging methods is found in chapter 4.

2.2.4 Contrast categories

Researchers like Rosch and Mervis (1975) found also that the weighted mean of entities in the dimensions which are common in contrasting categories (similarity between-categories) is inversely related to typicality in the category. For example, if two items are equally good in the bird dimensions, the one which is less good in the dimensions of other animal types, such as mammals or reptiles, is regarded as more typical in bird. Rosch and Mervis (1975) found that correlations between typicality and between category dissimilarity for the categories cars and chairs ranged between 0.67 and 0.86. Put differently, violating the dimensions of other categories, i.e. satisfying their negations, is regarded as typical of each category. The set of animal types which are taken into account in calculating typicality in bird is usually called the contrast set (Rosch and Mervis 1975: 591). This notion is related to the notion of a comparison class in the analysis of gradable adjectives in linguistics (cf. 2.1 and 3.1).

It was therefore established that the entities which are rated as more typical in the category are also less typical in (the typicality dimensions of) other related categories (the contrast set).

It was also concluded that the weight of a dimension (how diagnostic it is of the category) depends on the extent to which its frequency of occurrence in the concept's instances is high and on the extent to which its frequency of occurrence in contrasting concepts is low. This probabilistic view concerning the selection of dimensions and its limitations are discussed in chapter 4.

2.2.5 The tight connections between the entity ordering and the denotation

In this section, I present evidence for the following generalizations. First, there are no relations between the typicality of an item, and the item's mere frequency of occurrence in our experiences of the world. Second, often there are only loose relations between the typicality of an item, and the item's frequency of occurrence as a category member. Third, there are indeed tight relations between the typicality of an item, and the subjective judgments of speakers with regard to the item's frequency of occurrence as a category member. Thus, tight relations seem to exist between the entity ordering and the denotation in each predicate, as they are mentally represented in speakers' minds. Let us shortly review the evidence for these generalizations.
2.2.5.1 Typicality and frequency of occurrence

First, typicality cannot be identified with mere frequency of occurrence. For example, in North America and Europe, where most of the research on typicality is conducted, sparrows (which are considered typical birds) are seen far more often than penguins (which are considered atypical birds). However, chickens are seen more often and are talked about more frequently than other bird types, such as the oriole or catbird, which are nonetheless considered equally typical. Similarly, handball is less popular in the media compared to racing, but it is taken to be a more typical type of sport (Rosch 1975).

2.2.5.2 Typicality and frequency of occurrence of items as category members

Second, is typicality related to the frequency of occurrence of an item as category members? In experiment 2, Rosch, Simpson and Miller (1976) controlled for the frequency of occurrence of items in the training sessions of artificially constructed categories of several types. Crucially, participants observed the items which were better in the category dimensions (in categories of letter strings or stick figures) or in overall resemblance to a prototype example (in dot patterns categories), in fewer training sessions. Typicality ratings, verification time of category membership, order of production and error rate were all correlated with the category structure (average in the dimensions or overall resemblance to the category prototype), not with frequency. Homa, Dunbar and Nohre (1991) obtained similar results. However, exemplar theorists have shown that, under certain circumstances, when one item occurs more than others, it is considered to be more typical and it is categorized faster than less frequent items (Nosofsky 1988; Estes 1994). Most importantly, these results are normally obtained for very small artificially invented categories (Murphy 2002), and before learning proceeds to be near perfect (Erickson and Kruschke 1998; For a discussion of other conditions see Estes 1994: 211-217).

If item frequency plays a role, the question is – What do we actually count? Is each occurrence of an item regarded as a different category type (types are being counted) or as a different entity occurrence (occurrences are being counted)? Barsalou, Huttenlocher and Lamberts (1998) presented two groups of subjects with the exact same stimuli during learning. However, they told one group that they may see some stimuli multiple times and they told the other group that each stimulus was unique. This manipulation had virtually no effect under most conditions. The very frequent item had a strong effect when it was interpreted as occurring often and when it was interpreted as having many instances.

In fuzzy models (Zadeh 1965; Lakoff 1973; Osherson and Smith 1981), typicality is identified with 'objective' membership probability. The notion of objective probability is best demonstrated by the restriction that for no item, the probability of membership in two categories (or in an intersective category like brown-apple) will be greater than the probability of membership in but one of the categories (brown or apple). Similarly, for no item can the probability of membership in a category (like bird) be greater than the probability of membership in one of its subordinates (ostrich), as demonstrated in (53). If typicality stands for objective membership probability, it should conform to these principles. However, speakers' judgments about membership-likelihood and typicality do not conform to them. First, speakers often believe in (54a), namely, that some items (like brown-apples) are more likely to be brown-apples than just apples. This belief is called the conjunction-fallacy (Tversky and Kahneman 1983). Second, speakers often believe in (54b), namely that some items (the brown-apples again) are more typical in brown-apple than in apple. This is called the conjunction-effect (Smith et al. 1988). Third, speakers hold that some items (ostriches) are more typical in ostrich than in bird (54c). Let us call this the subtype-effect.
Objective Probability rules:

NO items are more likely brown-apples (brown and apples) than apples.

NO items are more likely ostriches than birds.

(54) a. The conjunction-fallacy:

Some items are more likely brown-apples than apples.

b. The conjunction-effect:

Some items are more typical in 'brown-apple' than in 'apple'.

c. The subtype-effect:

Some items are more typical in 'ostrich' than in 'bird'.

In certain replications of Tversky and Kahneman's (1983) experiment, participants were guided to bet on one of three choices of the form $P$, $P \land Q$, and $P \land \neg Q$. They were told that they will receive the money only if their bet does indeed turn out to be the case up to a certain date. The $P \land \neg Q$ condition was explicitly added so as to eliminate the possibility that $P$ is interpreted as $P \land \neg Q$, in which case there is no fallacy at all. For example, subjects had to divide 7 Euros between the following three bets (the emphasis in bold is mine):

(55) a. In order to reduce traffic fatalities,

the government will launch a publicity campaign.

b. In order to reduce traffic fatalities,

the government will launch a publicity campaign \textbf{and} penalize more harshly dangerous traffic violations.

(Both events must happen for you to win the money placed on this bet).

c. In order to reduce traffic fatalities,

the government will launch a publicity campaign \textbf{and not} penalize more harshly dangerous traffic violations.

(Both events must happen for you to win the money placed on this bet).

The pattern of judgments reported above was replicated even in this improved paradigm (Bononi, Tentori and Osherson 2004). All sixty participants allocated money to some $P \land Q$ at least once, and likewise for $P \land \neg Q$. The mean average of $P$ was 2.10 instead of 7. The mean average for $P \land Q$ was reliably higher.

Note that the names conjunction-fallacies or -effects are problematic, given that the phenomena to which they refer do not characterize conjunctions any more than they characterize modified nouns (like "brown apple") or lexical nouns (like 'ostrich' and 'bird'), and they are only fallacies given the notion of objective probability.

Given these findings, speakers' probability judgments are often called subjective.

2.2.5.3 Typicality and subjective judgments of membership probability

Typicality often couples with subjective membership-probability. Hampton (1998) found a very strong coupling between typicality and membership-probability, in the typicality and membership ratings of about 500 items in 18 categories (as published by McCloskey and Glucksberg 1978). When deviations occurred, they were highly systematic. The deviations from the general pattern of correlation between typicality and subjective judgments of category membership are discussed in chapters 4 and on.

Hampton's (1997) findings show how strongly typicality may affect subjective judgments of membership in a category. Only a third of the subjects in his experiment classified as a


zebra, the offspring of two zebras, which, given a special diet, began to look and behave like a horse. Non-essential dimensions overrode the important ones, even in a categorization task. Examples like this show that typicality is tightly connected to subjective judgments of membership (or likelihood of membership) in a category (see also Hampton 1987, 1998, 1998a, 1997a, Costello 2000).

Effects of items' actual frequencies on their typicality (Nosofsky 1988; Estes 1994) can be attributed to the fact that subjective judgments of membership, especially in relatively small categories, may be affected by actual frequencies. At any rate, speakers' subjective probability judgments might be affected by their typicality judgments, just as much as their typicality judgments might be affected by their probability judgments. Indeed, as noted, in making probability judgments, people tend to neglect sample sizes and prior odds, and to focus on the entities' similarity to the category (Tversky and Kahneman 1983).

2.2.5.4 Typicality and distributional patterns

Finally, typicality has distributional correlates. In a nutshell, the frequency of occurrence of a predicate (say, bird) with a word in a large corpus is similar to the occurrence frequency of its typical instances' names (robin, sparrow, etc.) with that word (Lynott and Ramscar 2001). How can these distributional frequencies in written language be explained? Again, they may be affected by the predicates' typicality-structure, just as much as typicality might be affected by them (Murphy 2002: 426-430).

2.2.6 The typicality ordering and learning

Some very robust findings, the order of learning effects, form evidence for tight relationships between typicality and learning. Most importantly, typical instances are acquired earlier than atypical ones, by children and adults (Rosch 1973; Anglin 1977; Murphy and Smith 1982; Mervis and Rosch 1981: 97-100).

2.2.6.1 Typicality and the order in which facts about classification are inferred

Developmentally, children tend to learn the typical members of the natural categories earlier. Children learn the good examples of, for instance, basic color categories, before learning the poor examples (Mervis and Rosch 1975). In addition, children are able to classify unfamiliar, yet typical animals like wombats and anteaters when they are not able to classify familiar but atypical animals (from the child's perspective) like butterflies or ants as being animal (Anglin 1977). Returning to our running example, this means that birdhood is normally determined first for bird types such as robins and pigeons, later on for chickens and geese, and last for ostriches and penguins. Similarly, non-birdhood is determined earlier for cows than for bats or butterflies. Thus, a normal acquisition order for the category bird is highly indicative of the typicality structure (figure 1). Why? A child, who possesses enough knowledge about the category dimensions, may be able to say that certain typical instances reach threshold. At the same time, this child may still be unable to say whether the atypical ones (which average less well in the dimensions) do (but not vice versa).

Figure 1: A normal acquisition order for the category bird is indicative of the typicality structure

[[bird]]c₀ … [[bird]]cᵢ … [[bird]]cₙ …
Similarly, in experiments in which subjects learn new categories, found that typicality correlated with acquisition order, measured in terms of number of training trials and error frequency. As typicality increases, the number of errors in the classification of items in a category reduces. Category learning reaches the criterion (a low enough error rate) earlier in trained typical instances. Children tend to learn that the good examples are members of, for instance, different categories of toys, before learning that the poor examples are members (Rosch 1973; Mervis and Pani 1980). The same effects were found in adults, in learning form concepts in cultures that do not posses them (Rosch 1973) and in adult learning of artificially invented categories such as dot patterns and stick figures (Rosch and Mervis 1975 experiments 5-6; Rosch, Simpson and Miller 1976; Mervis and Pani 1980).

In neural network simulations and in a study of aphasic patients, it was found that following training about the typicality dimensions, exposure to atypical items results in spontaneous recovery of categorization of untrained typical items, but not vice versa. Exposure to typical items does not result in recovery of categorization of untrained atypical items (Kiran and Thompson 2003). The membership of the typical instances was inferred from the membership of the atypical ones, but not vice versa.

In addition, in healthy adults, often typical instances which are seen for the first time are falsely thought to already be known (Reed 1988). For example, participants presumed to have identified criminals in a line-up, which in truth they had never seen, only because they obtained characteristic dimensions.

In these cases we see that, even when facts pertaining to the membership of atypical instances are directly learnt (or taught) before facts pertaining to the membership of more typical instances are learnt, knowledge of the category dimensions allows inferring the latter facts from the former facts, and not vice versa. Facts pertaining to the membership of typical instances do not license inferences regarding the membership of less typical instances. This is one sense in which we can say that the typical instances are acquired earlier.

2.2.6.2 Typicality and the order in which facts about classification are directly taught

Another sense in which we can say that the typical instances are acquired earlier is the following. Crucially, in the lack of knowledge (or direct training) about the category dimensions, the category membership of typical instances cannot be inferred based on knowledge about the category membership of less typical instances. In such circumstances, category acquisition is faster if initial exposure is to a typical category member, than if initial exposure is to an atypical member, or even to the whole denotation in a random order. Interestingly, the crucial factor is not the amount of examples but their typicality (Mervis & Pani 1980). Why? Because the typical instances form a better basis for generalization regarding what the category dimensions might be (Mervis & Pani 1980).

In chapter 5 I make this intuition more precise. I propose that in the lack of knowledge of the category dimensions, the early acquired entities are automatically treated as typical, and therefore their dimensions are treated as characteristic of the category (see the next section). The greater importance of the dimensions of the early acquired members in the selection of category dimensions (compared to dimensions which are frequent in the category in general), is further discussed in chapter 4 and on.

2.2.6.3 Typicality and inductive inferences about dimensions of categories and instances

The typicality of instances is known to affect our willingness to treat their properties as category dimensions (to extend our knowledge about the category dimensions by induction),
and as dimensions of other category instances (to extend our knowledge about the instances’ dimensions by induction). Rips (1975) and Osherson (1991) were the first to systematically study the conditions under which people conclude that an item possesses a certain property (say – having sesmoid bones or an ulnar artery), given a premise about another item possessing that property. For example, if robins are susceptible to a certain disease, how likely are we to infer that sparrows, ducks, ostriches or penguins are susceptible to this disease?

First, it was expected that the more typical the item in the premise would be, the stronger the inference would be. This expectation was firmly supported. For example, dimensions are more strongly inferred from robins to other birds than from penguins to other birds. Thus, induction is based on similarity (or typicality). Why? Typical entities are so regarded because they have many of the category dimensions. Atypical entities are so regarded because they violate many category dimensions. Thus, dimensions of typical entities are more likely category dimensions (or dimensions which are logically or causally related to the category dimensions) than dimensions of atypical entities are. Since dimensions of typical entities are more likely category dimensions, they are also more likely characterizing other category instances (or sub-types) than the dimensions of the atypical entities are. This explains the similarity effect.

Second, it was expected that dimensions would be more strongly inferred from (typical) items (e.g. robins) to the whole category (birds) than to atypical items (ostriches). This expectation was supported too, and it was called the inclusion fallacy. Why? Because atypical entities often violate the category dimensions (namely, the dimensions of the typical entities).

2.2.6.4 Frequency cues (coverage) play a smaller role in induction

You may expect that the more diverse the premise categories are (e.g. if they contain robins and ostriches rather than just robins and sparrows) and the more premises you add (the more bird types possess the dimension by premise), the stronger the inference to other instances will be. In fact, these two generalizations together were supported in western adults. They directly represent the effect of the degree of coverage of the category by the property. However, Lopez et al. (1992) found that kindergarten children did not rely on coverage. At the age of 8 (second grade) they only used coverage in certain situations (in inferences about all animals but not about an individual animal). In addition, Lopez et al. (1997) show that adults in different cultures do not use coverage. Thus, coverage-based induction is an acquired cultural practice, rather than a cognitive universal that everyone eventually develops (Murphy 2002: 252).

Similarly, you may also expect that when the classification of the premise item is uncertain, induction of dimensions will be based on the dimensions’ conditional probability in several potential categories (cf. the rational model by Anderson 1991). However, Murphy and Ross (1999) found that in such cases, induction of dimensions is only based on the most probable category for that item, not on multiple categories. In these experiments, subjects were presented with a whole set of objects (children drawings) of different geometrical shapes and colors, divided into categories by the child who was drawing them. The subjects were told that these were representative samples of the children’s drawings. Given a partial description of a new drawing (say – its color), the subjects had to estimate its category (whose drawing it was) and its other dimension (its form) as well as to estimate the probability of these estimations. The dimension probability estimations were affected by the dimension’s frequency in the most likely category, not by every possible category. Note that induction was based on frequency (within the most likely category) because subjects had no other choice. No other cues (learning order cues; cf. 2.2.6.2) were given to the subjects.
2.2.6.5 Typicality and memory

Finally, in addition to their early acquisition and their importance in triggering inductive inference, speakers also remember best the typical instances. They are most likely to be listed from memory, and their dimensions affect future remembrance of new entities and their dimensions (Heit 1997). For example, when speakers are initially exposed to joggers that wear expensive running shoes, they frequently falsely recollect joggers that do not wear expensive shoes as non-joggers or as joggers that do wear expensive shoes. In this case, new facts were corrected so as to match earlier ones.

When subjects are asked which stimuli they have seen previously, percentage of false recognition responses, and degree of confidence in seeing the stimuli, are both correlated with degree of typicality (Mervis & Rosch 1981).

2.2.7 The productive nature of the typicality effects

2.2.7.1 Linguistic evidence for the productive nature of the typicality effects

Typicality judgments are associated with very complex predicates. Numerous examples can be found in a simple Google search. For example, in (56a), we see a graded structure appear in a very complex noun phrase.

(56) a. … pretty much typical of a non-fan, non-entertainment, smart up-market British paper
b. You counter with an anecdotal tale about a non-typical non-developer. How does your counter-argument apply to a typical non-developer?
c. What were some exercises you would do on a typical non-running day? I read that they are mainly variations of pushups and situps...
d. There is one week where the format will be more typical of a non-seminar class
e. Her irritating non-performance is typical of a primarily young (read 'cheap') cast…
f. The music is typical of a non-CD game - that is to say, worthless. It's tinny and very electronic sounding.

We see that our ability to associate nouns with a gradable structure is highly productive. Speakers can generate infinitely many predicates, and, by and large, all the predicates that speakers generate seem to exhibit typicality effects. Thus, speakers must possess the means to generate a gradable interpretation for the infinitely many possible complex expressions in the language (for instance for predicates like male nurse or like red or white). These means are presumably either compositional, i.e. based on the meanings of the parts (e.g. red and white), or productive, i.e. based on some other generally available facts regarding the concept interpretation. Since our memory is finite, these means should amount to a finite set of rules for the composition of a denotation and a set of dimensions for the complex expression, from the denotations and sets of its parts, or from other facts pertaining to its interpretation, which are generally available in contexts.

Novel complex concepts are often called 'ad-hoc concepts', so as to emphasize the fact that their meaning is not retrieved from memory. Rather, it is computed on the fly (Barsalou 1983), and hence, it forms evidence for the assumption that a highly generative system plays a role in their construction.
Recently, focusing on basic lexical items, scholars argued that pragmatic effects, such as narrowing or widening of their denotations within a context of utterance, are also processes in which new ad-hoc concepts are created on the fly (Sperber and Wilson 1998; Carston 2002). As argued in the literature, the widening and narrowing of denotations and of quantification domains are frequently along dimensions (Kadmon and Landman 1993; von Fintel 1994; Lasersohn 1999), and, in fact, these dimensions can be identified with the typicality dimensions (Sassoon 2002). For example, generic sentences, like (57a), allow for exceptions (e.g. ducks or duck kinds which, for instance, lay grayish eggs). However, exceptions which are considered typical in the context of utterance are taken more seriously than exceptions which are considered atypical in that context. Generics with any, like (57b), express stronger generalizations, because their domain is widened so as to include also ducks or duck kinds which, in the context of utterance, are considered less typical, less normal or less relevant in some respect (Kadmon and Landman 1993).

(57)  
\begin{enumerate}
  \item A duck lays whitish eggs
  \item Any duck lays whitish eggs
\end{enumerate}

Finally, typicality scales play a role in the creation of scalar implicatures. For example, Bonnomi and Casalegno (1993) observe that only in examples like (58a) is ambiguous. On one reading exactly one person, the assistant headmaster, received me. On the other reading, the assistant headmaster was the most important person who received me. The implicature in this case is based on a status scale, but such implicatures can be derived based on typicality scales in just the same way (Sevi 2005). For example, on one reading of (58b), the speaker watched exactly one sport type, weightlifting. On the other reading, weightlifting was the most typical sport type watched. In (58c), the speaker watched no sport type less typical than swimming (but perhaps he did watch more typical sport types, if there are any).

(58)  
\begin{enumerate}
  \item A: Have you seen the headmaster?
    B: No, only the assistant headmaster received me.
  \item A: Have you managed to watch all the types of Olympic sport games?
    B: No, I only watched weightlifting.
  \item A: Have you managed to watch all the types of Olympic sport games?
    B: I wanted to, but I was very tired lately. I did not even watch the swimming contests.
\end{enumerate}

In sum, these findings suggest that, within each context, speakers productively use a set of rules and strategies to build a typicality ordering relation and dimension-set for both simple and complex concepts. These ordering relation and dimensions are tightly connected to the concepts' denotations and to the ways they can be restricted or stretched within contexts. Hence, they are firmly connected to the semantic interpretation of statements with quantifying expressions, hedges which mark sloppy or unusual interpretations, scalar particles like even and only, two-place relations like similar and identical, etc.

But what kind of rules and strategies do we use in building typicality-orderings and dimension-sets for complex concepts? Classical theories typically did not address this issue. When the issue was addressed, the various kinds of theories failed to be compositional (Osherson and Smith 1981; Kamp and Partee 1995). Scholars like Fodor (1998) assumed that all concepts are atoms (inherently non-compositional). In the last twenty years, though, an extensive research has offered evidence for connections between the typicality structures of complex concepts and of their constituents. The following section reviews the relevant findings and the views about concept combination that, based on these findings, have become
common in cognitive psychology. We will see that it is for the most part based on the research of concept combination, that the basic theories in linguistics and psychology are viewed as contradicting.

2.2.7.2 Dimensions of complex predicates: A composite-prototype representation

Recall that according to the intersective analysis of modified-nouns and conjunctions, modified-nouns and conjunctive predicates, \( P \land Q \), denote the intersection of their constituents' denotations (59).

\[
\forall w \in W: \quad [[P \land Q]]_w = [[P]]_w \cap [[Q]]_w
\]

From the psychological perspective, the truth-conditional theories are identified with the classical view of concepts as sets of necessary and sufficient conditions for categorization. Perhaps because of that, they are (or the intersection-rule is) identified with the wrong prediction that the dimension-set of a modified-noun is formed by the union of the constituents' dimension-sets (\( F(P \land Q) = F(P) \cup F(Q) \)).

Hampton (1987) analyzed the dimensions of modified-nouns and their constituents (dimensions that subjects listed and ranked for relative importance). The union-rule could account for about 80% of the dimensions which were produced for the modified-nouns. In particular, dimensions which were rated as necessary for one constituent were also rated as necessary to the modified-noun. Thus, by and large, the dimensions of modified nouns were predicted from the dimensions of their parts.

However, abundant exceptions to this general pattern occurred too. Accordingly, not all the dimensions of modified-nouns can be predicted from the dimensions of their parts. Some dimensions fail to be inherited from the parts to the whole, and others characterize the whole, but not the parts (emergent dimensions).

In Hampton (1987), modified-nouns had on average about 3 important dimensions less than the union-rule would predict. Dimensions which weighed little in one constituent could eventually not be inherited to the modified-noun, refuting the union-rule.

In general, failures of inheritance often result from greater dominance of a certain constituent in a modified noun (Hampton 1988, 1997; Chater and Myers 1990). For example, bird is the dominant constituent in both pets which are birds and birds which are pets. Entities' typicality ratings in these modified-nouns highly correlate with their ratings in bird (Hampton 1987; 1997). By and large, the dimensions of dominant constituents are regarded as more typical of the modified-noun than the dimensions of less dominant constituents.

Dominance effects seem to occur whenever the dimension set of the modifier cannot coherently combine with the dimension set of the noun. For example, if the non-definitional dimension manly is taken to be typical of male-nurse, manly individuals are regarded as more typical in male-nurse than less manly individuals equal in all other respects. Conversely, if feminine is taken to be typical of male-nurse, the manly individuals are regarded as less typical in male-nurse than the less manly ones. Consequently, at least one of these two dimensions has to be dropped in each context, in order to avoid an inconsistent interpretation. Hampton (1987) shows that the more compatible dimensions are with the dimension set, the more likely they are to remain in it. Hampton also found that dimensions common to both conjuncts receive a higher weight for the conjunction than do dimensions distinct to one conjunct (Hampton 1987: 64-66).

In general, Hampton (1987) concluded that only a special averaging method could predict the weight of each dimension, in each modified-noun, from its weights in the constituents. These findings were taken as counter-evidence to the formal theory, and positive evidence for
the creation of a composite prototype for modified-nouns, based on a special (non-Boolean) averaging method for the selection and weighing of the dimensions (Hampton 1997).

2.2.7.3 The emergence of new dimensions

The emergence of new dimensions in modified-nouns can be demonstrated by the fact that pet-birds are characterized by properties like lives in cages and can talk, which are neither typical of pets nor of birds alone. Similarly, small-spoons are typically made of metal and large-spoons are typically wooden. Boiled-eggs are hard whereas boiled-potatoes are soft. None of these dimensions characterizes any of the separate constituents. Moreover, it took subjects significantly longer to verify that, say, boiled celery is green than that boiled celery is soft, though soft, but not green, is an emergent dimension (Springer and Murphy 1992).

The emergent dimensions are usually viewed as refuting the idea of compositionality for dimension-sets, because they derive from experience with category members, rather than being logically entailed by Boolean composition rules for dimension-sets (Hampton 1997a).

2.2.7.4 The ordering relation of complex predicates is not completely determined by the ordering relations of the constituents

Hampton (1987; 1997) analyzed the typicality ratings of a list of entities in modified-nouns of the form ‘Ps which are Qs’ (like pets which are birds) and in their constituents (e.g. pets and birds). The following patterns emerged.

First, for any item d, it was possible to predict d’s typicality rating in a modified-noun, deg(d,P∧Q), from d’s ratings in the constituents, deg(d,P) and deg(d,Q), by an equation like (60a). W_P and W_Q represented the constituents’ weights, and W_P×Q represented the weight of the constituents’ interaction (the product deg(d,P) × deg(d,Q)). For example, the values for pets which are birds were: W_pets = .30, W_birds = .78, and W_pets×birds = .10.

Second, the typicality ratings in modified-nouns with negated constituents, deg(d,P∧¬Q), were predicted by adding a negative sign to the weight of the negated constituent (–W_Q). The interaction term was also negative, when significant (60b). For example, for pets which are not birds the weights were: W_p = .32, W_Q = −.75, and W_p×Q = −1.1. Why? Because the better an item is as an example of Q, the worse it is as an example of not-Q.

Third, given the logical connections between disjunctions, conjunctions and negations (P∨Q = ¬(¬P∧¬Q)), and the fact that negation affects the equation by changing the coefficient sign, Hampton predicted that the typicality ratings in disjunctions like hobbies or games, deg(d,P∨Q), would be given by adding a negative sign to the interaction term (–W_P×Q). Why? The value deg(d,P∨Q) ought to be identical to deg(d,¬(¬P∧¬Q)), which, in turn, should be given by an equation in which a negative sign is added to the weight of each negated-constituent (namely by the equation: (–W_P×Q)deg(d,P) + W_Qdeg(d,Q) + W_P×Q(deg(d,P) × deg(d,Q))). After the elimination of double negative-signs, this equation reduces to the one in (60c), with the negative interaction-weight. And indeed, using (60c), Hampton (1988) could predict the typicality ratings in disjunctions from the ratings in the disjuncts. In this case, the logical rules have triggered the discovery of the disjunctive pattern.

\[
\begin{align*}
\text{a. deg}(d, P \land Q) &= W_P \deg(d, P) + W_Q \deg(d, Q) + W_{P\land Q} \deg(d, P) \times \deg(d, Q), \\
\text{b. deg}(d, P \land \neg Q) &= W_P \deg(d, P) - W_Q \deg(d, Q) - W_{P\land Q} \deg(d, P) \times \deg(d, Q), \\
\text{c. deg}(d, P \lor Q) &= W_P \deg(d, P) + W_Q \deg(d, Q) - W_{P\lor Q} \deg(d, P) \times \deg(d, Q).
\end{align*}
\]
However, negated constituents sometimes had a decreased weight, because some dimensions were treated as characterizing both the predicate and its negation. For example, *animate* often characterizes both birds and entities that are not birds, and *bird-hood* characterizes both robins and non-robins. In general, the typicality ratings in modified-nouns were better fitted by the composite-prototype representation, where the weight of each dimension was adjusted by a special function (compared to the more coarse constituent-based equations in (60)).

### 2.2.7.5 Failures of intersective entailments (‘drop’ and ‘permutation’) in nouns

Finally, recall that (cf. 2.1.2.3) sentences with modified-nouns in predicate position like (61a) usually entail the sentences resulting from dropping some of the constituents (like (61b) and (61c)) or changing the constituent ordering (constituent-permutation, like in (60d); Landman (2000)). Such entailment-patterns form the basis for (they are directly predicted by) the intersective analysis of modified-nouns and conjunctions.

(61)  
a. Tweety is a four legged animal  
b. Tweety is four legged  
c. Tweety is an animal  
d. Tweety is an animal and is four legged

However, experimental evidence shows that the intersective entailment-patterns often fail in nouns. Scholars usually take this fact to refute the Boolean intersection-rule as well as other rules on which truth-conditional theories are based (Lakoff 1987; Hampton 1997a; Murphy 2002). Given the basic role that these rules play in linguistics, we should deal with these counter-examples.

First, experimental findings show that the denotations of modified-nouns may be more liberal than required by the constituents (this phenomenon is often called *overextension*). For example, not all the objects classed as *school-furniture* (desks, chairs, blackboards etc.) are also classed as *furniture* (Hampton 1988; Hampton 1997; Costello 2000). Very typical things in school are more likely to be classified as school-furniture than as furniture. Drop-entailments may fail even in modified-nouns of the form *Qs which are Ps*, where Q and P are both nouns. For example, (62a) may be considered true, and at the same time, (62b) may be considered false.

(62)  
a. Chess is a type of sports which are games  
b. Chess is a type of sport  
c. Chess is a type of games  
d. Chess is a type of games which are sports

Over-extension was also found in disjunctions, though to a lesser extent. Examples include cases, whereby participants are not sure to which one of the disjuncts (for example in *fruit or vegetables*) the item (say, *mushrooms, almonds*, etc.) belongs. They do know, however, that the item belongs to the denotation of at least one of the disjuncts (Hampton 1988a).

Second, permutation-entailments may fail, too. Frequently, entities’ typicality ratings in modified-nouns correlate more strongly with their ratings in the modifier than in the head noun (Hampton 1997). Thus, the interpretation of, for instance, *sports which are games* may not be identical to that of *games which are sports* (cf. (62a) versus (62d)). Interestingly, failures of permutation seem to disappear in disjunctions (Hampton 1998a).
Third, the characteristic effect found in the empirical research of disjunctions is under-extension. Under-extension happens when entities are classified as members in one or both of the disjuncts but not in the disjunction, or when entities are considered more typical (and more likely members) of the disjuncts than of the disjunction (e.g., Parsley for herbs or species; Hampton 1988a). This effect is taken to refute the union analysis of disjunctions, $P \lor Q$, namely the hypothesis that they denote the union of their constituents' denotations (63).

\[
(63) \quad \forall w \in W: \quad [\[P \lor Q\]]_w = [\[P\]]_w \cup [\[Q\]]_w
\]

In chapter 5 and on, I work out the predictions of the logical rules concerning the typicality ordering and dimension set of complex predicates. I show that the findings reviewed throughout 2.2.7 are not only compatible, but also partially motivated by the intersection rule and the other logical rules.

2.2.8 Hierarchical relations

2.2.8.1 Evidence for cognitive Taxonomies

The basic idea that people can use transitive reasoning patterns like "if all Ps are Qs and all Qs are Zs then all Ps are Zs", was established empirically (Murphy 2002: 200-210). Furthermore, certain findings attest to the psychological reality of lexical hierarchical relations (taxonomies). For example, in naming of objects, people tend to use categories like dog or bulldog which are part of a hierarchy more often than categories which cross hierarchies like drooling animal.

2.2.8.2 Hierarchical levels in cognitive taxonomies

Within the categories described as being connected by hierarchical relations (mostly taxonomic natural kind categories but also artifact categories), three cognitively different levels can be identified: The super-ordinate level (furniture, animal), the basic level (chair, bird) and the sub-ordinate level (kitchen table, sparrow), as demonstrated in table 2.

<table>
<thead>
<tr>
<th>Super ordinate categories:</th>
<th>Animals:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>breathes, eats,…</td>
<td></td>
</tr>
<tr>
<td>Basic level categories:</td>
<td>Bird:</td>
<td>Fish:</td>
</tr>
<tr>
<td></td>
<td>breathes, eats,…</td>
<td>breathes, eats,…</td>
</tr>
<tr>
<td></td>
<td>wings, feathers,…</td>
<td>has fins, gills, can swim…</td>
</tr>
<tr>
<td>Subordinate categories:</td>
<td>Canary:</td>
<td>Salmon:</td>
</tr>
<tr>
<td></td>
<td>breathes, eats,…</td>
<td>breathes, eats,…</td>
</tr>
<tr>
<td></td>
<td>wings, feathers…</td>
<td>has fins, gills, can swim…</td>
</tr>
<tr>
<td></td>
<td>yellow, small…</td>
<td>pink, edible, swims upstream…</td>
</tr>
</tbody>
</table>

(Names of) basic level concepts are most commonly used. Accordingly, they are learnt earlier and categorized faster, they have shorter and more familiar names, they are more tightly associated with a common shape (e.g. a table contour; a car contour) and a common type of body movement (say –sitting for chairs), etc. (Murphy 2002). Basic level categories form the best compromise between being informative (being associated with many dimensions) and being distinctive (being different from other categories in the same level). Subordinate
categories are characterized by only few additional dimensions, and super categories are characterized by significantly less dimensions. Hence, for the purposes in most contexts of use, the latter are usually not regarded as informative enough but the former are regarded as too informative. The basic level categories are usually as informative as required, in terms of the number of inductive inferences you can draw based on their use. Furthermore, basic level categories are more distinctive with respect to other categories in that level than sub-ordinate categories. For instance, chairs are different from tables more than kitchen tables are different from office tables (Rosch et al. 1976).

2.2.8.3 The hierarchical relations as intransitive relations ("Ps are typically Qs")

Unlike Collins and Quillian (1969), Hampton (1982) argues that the taxonomic relations (e.g. the relation that holds between animal and bird) do not necessarily represent class inclusion. Put simply, they should not be described by a relation like "all Ps are Qs", but by a relation like "Ps are typically Qs". For example, Hampton (1982) has found that subjects who agree that chairs are a type of furniture and that car seats are a type of chair are sometimes unwilling to allow that car seats are a type of furniture. These examples are parallel to the drop-entailment failures, except that they were found in categories and their sub-categories (rather than in modified nouns and their constituents). For example, experiments show that subjects may regard (64a) and (64b) as true, and, at the same time, regard (64c) as false.

(64)  a. The big Ben is a clock
       b. Clocks are types of furniture
       c. The big Ben is a furniture

2.2.8.4 Are the hierarchical relations stored or computed?

Hierarchical relations may be either stored or computed from the concepts' dimensions (Collins and Quillian 1969). The following effects are predicted by the hypothesis that the hierarchy is computed from the concepts' dimensions. First, the category size effect (i.e. the fact that verification of sentences such as a canary is a bird is quicker than the verification of a sentence such as a canary is an animal), would be explained by the similarity between the dimension-sets of the exemplar (canary) and the smaller super-ordinate category (bird), compared to the larger super-ordinate category (animal). Second, the online typicality effect (the fact that verification of sentences such as a canary is a bird is quicker than of sentences such as an ostrich is a bird), would be explained as well, because, again, typical exemplars share more dimensions with the super-ordinate category than atypical ones.

Then again, stored hierarchical relations help to achieve a more economical representation. Dimensions are placed at the highest level possible, and only there. The hierarchy may also comprise both fixed relations and computed relations.

2.3 Intermediate conclusions and a last, but not least piece of data

In this chapter, basic facts about predicates and their interpretation (concepts), which are often in the focus of investigation in semantics and in psychology, were put side by side. The juxtaposition of the psychological and linguistic facts revealed the following problems. On the one hand, a variety of semantic facts show that nouns differ from adjectives in that they are usually incompatible with degree structures. On the other hand, the psychological facts
show that, conceptually, nouns are gradable (most importantly, judgments of denotation membership in nouns are graded), and the fact that nouns are susceptible to modification by some degree operators shows that this gradable structure has to be part of the semantic interpretation of nouns. Similarly, conceptually, nouns are clearly associated with ordering dimensions. But grammatical operations cannot access these dimensions. Finally, substantiated psychological findings show that, quite often, the intersective inferences fail. Yet, from the semantic perspective, they seem to be basic and persuasive.

Adapting the view that nouns are gradable will help bridging the gap between linguistics and psychology, and to account for the fact that nouns are compatible with certain degree modifiers. But the main difficulty with incorporating a gradable analysis of nouns (and noun phrases) into the semantic theory is that important distinctions might become blurred. First, if nominal concepts denote degree functions, it is not clear why they are incompatible with (within-predicate) comparatives, equatives and other degree modifiers. Second, if nominal concepts are multi-dimensional, it is not clear why it is impossible to quantify over their dimensions. A larger problem is looming behind these questions, namely the problem of giving an adequate account of the semantic distinction between nouns and adjectives. Given this perspective on the problems with which we are concerned, I would like to conclude this chapter by drawing attention to another important distinction between nouns and adjectives.

We have described corroborated findings suggesting that nominal concepts (nouns and noun phrases) map entities to their mean degree on the nominal dimensions. Nevertheless, precisely in this, nouns differ from multi-dimensional adjectives (like healthy, normal, similar, typical, etc.) Usually researchers give little or no attention to this important fact. In 7.5.4, I present novel empirical findings based on a corpus study of uses of exception phrases (which are only licensed in the presence of expressions whose semantics involves universal-quantification). I have found that such phrases are often used with adjectives, either in their positive form (as in healthy except for blood pressure) or when negated (as in not sick except for blood pressure), while never occurring with basic nouns like bird or apple (as in # bird except for size / flying). These findings support the hypothesis that when a predicate is classified as adjectival, its dimensions are combined using quantifiers, not averaging operations. For example, in adjectives like healthy and typical of a bird, the denotation consists of entities that fall under all the dimensions (which are, e.g., healthy in every respect). In adjectives like sick and atypical, the denotation consists of entities that fall under some dimension (which are, e.g., sick in at least one respect). In that, adjectival predicates sharply differ from nominal ones. In chapters 5-9 of this dissertation, I propose a theory in which nouns are gradable. The linguistic contrasts between nouns and adjectives are explained based on a distinction in the type of degree functions that are associated with nominal versus adjectival concept names.

Throughout the dissertation, I will continue to use the term typicality to refer to gradability in nominal concepts (as it is usually used), and not to gradability in the adjective typical. As I point out in chapter 4, some phenomena that are conventionally regarded as "typicality effects", are actually related to the adjective typical, not to gradability in nouns. In chapter 5 and on, I discuss the differences and the (tight) connections between the gradable structures of nouns like bird and the gradable structures of adjectival phrases like typical of a bird.

With all these facts and observations in mind, we will now turn to previous accounts that exist for them – to contemporary theories about vagueness, gradability, and typicality, in semantics and psychology.
Part II
Existing Theories
3 VAGUENESS, GRADABILITY, AND TYPICALITY: AN OVERVIEW OF LINGUISTIC THEORIES

Here is what I am going to do in this chapter:

Section 3.1 briefly reviews models for the representation of vagueness which are standardly used in linguistics.

Section 3.2 discusses linguistic theories of gradability. The discussion is organized by subject: Section 3.2.1 focuses on dominant accounts of the connection between gradability and vagueness. Section 3.2.2 focuses on the controversy concerning the nature of the degrees (numerical or not). Section 3.2.3 focuses on dominant accounts of the polarity effects. Section 3.2.4 focuses on dominant accounts of the phenomenon of incommensurability (impossibility of between predicate comparisons). Section 3.2.5 focuses on dominant accounts of the compositional derivations of the semantics of (within predicate) comparisons.

Now, in order to account for the first set of facts (especially the fact that nouns are infelicitous in the comparative), linguistic theories usually assume that nouns like *bird* and *apple* are neither vague nor gradable. However, by doing that, these theories fail to provide any explanation for the second set of facts (2.1). The variety of substantial empirical findings in the field of cognitive psychology, which provide evidence for the counter assumption (that nouns are inherently gradable), question the adequacy of these theories. The linguistic phenomena presented in 2.2 provide further evidence against them. In section 3.3, I will therefore examine in some detail the well known linguistic analysis of Kamp and Partee (1995), which forms an attempt to account for the typicality effects, while maintaining the assumption that nouns are not gradable in the usual linguistic sense. We will see that this theory is highly problematic and cannot be maintained.

3.1 The representation of vagueness

By and large, in formal grammars, the denotation of a predicate P in world w and time h is treated as a set of objects, \([[P]]_{w,h}\). The denotation of the negated predicate, \([[-P]]_{w,h}\), is the result of application of the Boolean complement operation to the predicate denotation. For example, the denotation of *not a bird* is the complement of the denotation of *bird*, within the contextually given domain of objects, D (as stated in (1)). Thus, the standard semantic model is a model of total information. Such a model determines for each individual and each property in a situation, context or world, whether it has that property or not. There is no third possibility – no gap containing individuals that one does not know if they have that property or not.

\[
(1) \quad [[-P]]_{w,h} = D - [[P]]_{w,h}
\]

However, an important refinement of the notion of intension was made about forty years ago, in order to account for problems of vagueness and partial information. Dynamic discourse exchange models and vagueness models were developed. The representation of partial information was found fruitful in the analysis of many semantic problems (see for example in Stalnaker 1975; van Fraassen 1969; Kamp 1975; Fine 1975; Veltman 1984; Groenendijk & Stokhof 1984; Landman 1991 and others). Stalnaker (1975) shows that many aspects of a state of partial information can be modeled with the set of worlds which are compatible with the information in c (those worlds which might still be found to be the actual world). Others (van Fraassen 1969; Kamp 1975; Fine 1975; Veltman 1984; Landman 1991 etc.) claimed that partiality enters into the basic semantics, in that the basic logic of predicates (or categories) is assumed to be three-valued. This means that semantic interpretation is
relative to information states (or contexts), in which predicate denotations are only partially known. We do not associate a predicate (or a category) in a context (information state) with one denotation (the set of objects to which the predicate applies), but with a triple consisting of a positive denotation, 

\[ [[P]]_c^+ \]

the set of individuals that in \( c \) are positively known to fall under \( P \), a negative denotation, 

\[ [[P]]_c^- \]

the set of the individuals that in \( c \) are positively known not to fall under \( P \), and a gap, 

\[ [[P]]_c^? \]

the remaining individuals. Vagueness models, \( M^* \), contain a set \( C \) of partial contexts and a relation, \( \leq \), which represents information growth – namely, the order in which entities are categorized under the predicates through contexts and their extensions, as demonstrated in Figure 2.

![Diagram](image.png)

**Figure 2:** The contexts structure in a standard vagueness model \( M_c \)

In the minimal context under \( \leq \) (the context of zero information), \( c_0 \), all predicate denotations are empty. In the maximal contexts under \( \leq \) (total contexts \( t \); the supervaluations in van Fraassen 1969), every entity is either in the negative or in the positive denotation of each predicate. Each intermediate context is extended (or followed) by a set of other contexts in which more entities are added to the denotations. A context \( c_1 \) is extended by another context \( c_2 \), \( c_1 \leq c_2 \), iff the positive and negative denotations of each predicate in \( c_1 \) are subsets of (and therefore "extended by") the positive and negative denotations of that predicate in \( c_2 \).

For example, if in a context \( c \) the positive denotation of `tall`, 

\[ [[tall]]_c^+ \]

consists of only very tall items, and the negative denotation of `tall`, 

\[ [[tall]]_c^- \]

consists of only very short items, then in \( c \), we do not yet know if anything else, which is neither very tall nor very short, is `tall` or not. Similarly, we may have a context \( c \), where the denotation of `chair` consists of only one item, the prototypical chair, \( p_{chair} \), and the denotation of `non-chair` consists only of items which are very clearly not chairs, including apples, tables, and so on, and also, say, the prototypical sofa, \( p_{sofa} \), but not things like an armchair, a stool, a chair with less than 4 legs, etc. About the latter items, we do not yet know in \( c \) if they are a `chair` or not. But each partial context \( c \) is extended by a set of total extensions, \( T_c \) (where \( T_c = \{ t \in T : c \leq t \} \)). These represent all the states of total (complete) information compatible with the partial information in \( c \) (all the ways in which things may yet turn out to be, given the information in \( c \)). In each of the contexts \( t \) in \( T_c \), the complete sets of `tall` and `non-tall` things, `chairs` and `non-chairs`, etc. are specified. Each item is either in the denotation of `tall` or in the denotation of `non-tall`, and each item is either in the denotation of `chair` or in the denotation of `non-chair`. Finally, a consistency constraint requires that positive and negative denotations never overlap.

In each context \( c \) in an information structure, some statements are directly known to be true. A larger set of statements directly follows from the statements which are known to be true. van Fraassen 1969 classified these statements under the notion `super-true`. A statement is `super-true` iff it is true in every total context extending \( c \), i.e. it is indirectly known in \( c \) (it is entailed by the information in \( c \)). On the predicate level, for each predicate \( P \), one can distinguish between things which are directly known to be \( P \) in \( c \), and things whose \( P \)-hood indirectly follows from the entire information in \( c \): things that must be \( P \) (cf. Veltman 1984). Accordingly, we will use the notation 

\[ [[P]]_c \]

for the set of things which may have not been directly classified under \( P \) in \( c \), but end up falling under \( P \) in every total context extending \( c \).
The notation $[\neg P]$ will stand for the set of things which may have not been directly classified under not-P in c, but end up falling under P in no total context extending c.

Formally, a vagueness model $M^*$ for a set of entities $D$ (van Fraassen 1969; Kamp 1975; Fine 1975; Veltman 1984; Landman 1991, etc.) is a tuple $M^* = \langle C, \leq, c_0, T \rangle$, such that:

(2) $C$ is a set of partial contexts:
In each $c$ in $C$, an $n$-place predicate $P$ is associated with partial positive and negative denotations (sets of $n$-tuples), $\langle [[P]]^+_c, [[P]]^-_c \rangle$.

(3) $\leq$ is a partial order on $C$:

a. $c_0$ is the minimal element in $C$ under $\leq$:
\[ \forall P \in \text{PRED}: [[P]]^+_c = [[P]]^-_c = \emptyset \]
Denotations are empty in $c_0$.

b. $T$ is the set of maximal elements under $\leq$:
\[ \forall t \in T, \forall P \in \text{PRED}: [[P]]^+_t \cup [[P]]^-_t = D^n \]
Denotations are maximal in each $t$ in $T$.

c. **Consistency**:
\[ \forall t \in T, \forall P \in \text{PRED}: [[P]]^+_t \cap [[P]]^-_t = \emptyset \]
Positive and negative denotations are disjoint.

d. **Monotonicity**: $\forall c_1, c_2 \in C$, s.t. $c_1 \leq c_2$:
\[ \forall P \in \text{PRED}: [[P]]^+_t \subseteq [[P]]^+_c \cup [[P]]^-_c \]
A context $c_1$ is extended by another context $c_2$ iff all predicate denotations in $c_1$ are subsets of the corresponding denotations in $c_2$.

e. **Totality**:
\[ \forall c \in C, \exists t \in T: c \leq t \]
Every $c$ has some maximal extension $t$.

f. **"Super-denotations"**: $\forall t \in T, \forall c \in C$:
\[ [[P]]_t = \cap \{ [[P]]^+_t \mid t \in T, c \leq t \} \]
\[ [[\neg P]]_t = \cap \{ [[P]]^-_t \mid t \in T, c \leq t \} \]
P-hood of an entity $d$ can be inferred in $c$ iff $d$ belongs in $[[P]]^+_t$ in any total extension $t$ of $c$.

3.2 The analysis of Gradability

3.2.1 The connections between vagueness and Gradability

3.2.1.1 The vagueness-based approach to gradability and comparison

A well-established tradition illustrates that vagueness models are useful for the representation of the distinction between gradable and non-gradable predicates. Researchers observed that vague predicates (namely, by and large, adjectives) are gradable. Given this generalization, gradability was analyzed as vagueness-dependent (Lewis 1970; Lewis 1979; Kamp 1975; Fine 1975; McConnell Ginnet 1973; Seuren 1973; Klein 1982; Landman 1991 etc.)

This approach to the analysis of gradability and comparison can be characterized by the following three principles:

(4) a. Each gradable predicate $P$ is directly associated with a partial set of entities.

b. Degrees are delineations (possible boundary specifications) for vague predicates.

c. The interpretation of the comparative form $\leq P$ is derived from the interpretation of the positive form $P$. An entity is more $P$ than other entities iff it falls under $P$ relative to more delineations. That is, iff it falls under $P$ relative to some possible boundary specifications, which exclude the other entity from $P$’s denotation.
Let me explain these principles in more detail.

For the analysis of gradability and comparison, most researches use simplified vagueness models which contain but one partial context $c$ (the ground model) and a set $T_{c}$ of the total contexts $t$ extending $c$, as demonstrated in Figure 3. The intermediate steps between $c$ and each $t$ are thought to be unimportant (Kamp 1975; Kamp and Partee 1995). The total contexts are thought to represent different ways to fix the cutoff points between the tall and non-tall entities (Lewis 1979). In some of them only very tall entities are regarded as tall enough to be considered tall; in others, also entities that are just fairly tall, are considered tall, etc. The cutoff point in a context $t$ is usually called the standard value for being tall in $t$, standard$tall(t)$. In each context $t$, we consider $tall$ those entities that reach the standard value for being tall in $t$.

\[
\begin{array}{|c|}
\hline
\text{Standard value for being tall in } t_{n} & 2.00 \text{ meters} \\
\text{Standard value for being tall in } t_{m} & 1.95 \text{ meters} \\
\text{Standard value for being tall in } t_{j} & 1.90 \text{ meters} \\
\text{Standard value for being tall in } t_{u} & 1.85 \text{ meters} \\
\text{...} & ... \\
\hline
\end{array}
\]

Figure 3: The context structure in a simplified vagueness model $M_{c}$

If in $c$ we do not yet know what the standard is, we can only consider $tall$ those entities which are tall in every total context above $c$. Using van Fraassen's (1969) notion of super-truth (truth in every total context), we can state that $Sam$ is tall in $c$ (or put more explicitly, $Sam$ must be tall in $c$, where $must$ is interpreted epistemically; Veltman 1984) iff $Sam$ is tall in every total context above $c$ (that is, $Sam$'s height reaches the standard, be it what it may).

(5) Supertruth: $[[Sam \text{ is tall}]]_{c} = 1$ iff $\forall t \in T$, $t \geq c: [[Sam \text{ is tall}]]^{t}_{c} = 1$

Regarding comparatives, the gist of theories like Kamp (1975) and Fine (1975) is that a comparative statement like $Dan$ is taller than $Sam$ is considered true in a context $c$ iff $Dan$ is tall relative to more standards, that is: $Dan$ is tall in more total contexts above $c$, compared to $Sam$. If $Sam$ reaches a certain standard of tallness, $Dan$ certainly reaches this standard, but not vice versa. $Dan$'s height reaches certain standards which $Sam$'s height does not reach. Thus, in some total context $t$ $[[Sam \text{ is Tall}]]^{t}_{i} = 0$ and $[[Dan \text{ is tall}]]^{t}_{i} = 1$ but in no total context $[[Sam \text{ is Tall}]]^{t}_{i} = 1$ and $[[Dan \text{ is tall}]]^{t}_{i} = 0$.

(6) a. $[[Dan \text{ is (at least) as tall as } Sam]]_{c}^{+} = 1$ iff:

\[
\{ t \in T_{c} \mid [[Sam \text{ is Tall}]]^{t}_{c} = 1 \} \subseteq \{ t \in T_{c} \mid [[Dan \text{ is tall}]]^{t}_{c} = 1 \}
\]

b. $[[a \text{ is as } P \text{ as } b]]_{c}^{+} = 1$ iff $\{ t \in T_{c} \mid [[P(a)]]^{t}_{c} = 1 \} \subseteq \{ t \in T_{c} \mid [[P(b)]]^{t}_{c} = 1 \}$

(7) a. $[[Dan \text{ is taller than } Sam]]_{c}^{+} = 1$ iff

\[
\{ t \in T_{c} \mid [[Sam \text{ is Tall}]]^{t}_{c} = 1 \} \subset \{ t \in T_{c} \mid [[Dan \text{ is tall}]]^{t}_{c} = 1 \}
\]

b. $[[a \text{ is more } P \text{ than } b]]_{c}^{+} = 1$ iff $\exists t \in T_{c} : [[P(a)]]^{t}_{c} = 0$ and $[[P(b)]]^{t}_{c} = 1$ and $\forall t \in T_{c} : [[P(a)]]^{t}_{c} = 1$ and $[[P(b)]]^{t}_{c} = 0$.

Imagine a context $c$, where $Dan$ is 1 meter tall and $Sam$ is .95 meters tall in $c$. Both are rather short, but we can still confidently say that $Dan$ is taller in $c$. According to Kamp (1975), we see some total context $t$ (a total extension of $c$), in which the standard is loose enough to
render Dan, but not Sam, tall. Klein (1980: 12) criticizes Kamp's approach by arguing that it does not explain how the mechanisms for making vague predicates more precise can lead us to a context in which the standard is altered that much. In Klein's (1980) implementation of the vagueness-based analysis of gradability, the comparison class replaces the notion of a standard value (or the use of degrees). The comparison class, $X_{\text{tall}}$, is a set of entities functioning as a local domain for the interpretation of tall and not-tall in each context of use. The comparison class is related to the topic of conversation (the set of things that the participants in a conversation happen to be talking about). Klein (1980) shows that the comparison class is a means of reducing the vagueness in the meaning of gradable adjectives like tall. Like delineations (contexts), comparison classes are associated with partial interpretation functions for the gradable adjectives in the language. Crucially, we can reduce any comparative statement like Dan is taller than Sam to one in which the local domain, $X_{\text{tall}}$, would consist of no entity but Dan and Sam. According to Klein, in such a domain, by virtue of our natural conceptual capacity to make classification decisions, we will classify Dan as tall and Sam as short. Hence, for Klein (1980), the given comparative statement is true iff there is a comparison class (such as the set $\{[\{\text{Dan}\}]_c, [\{\text{Sam}\}]_c\}$), relative to which Dan is tall is true but Sam is tall is false.

(8) $[[\text{Dan is taller than Sam}]]^{+}_{c,X} = 1$ iff $\exists X' \subseteq X: [[\text{Dan is tall}]]^{+}_{c,X'} = 1$ and $[[\text{Sam is tall}]]^{+}_{c,X'} = 0$

Similarly, Klein (1980) argues that an entity is considered very tall in a context c iff it is considered tall even when the positive denotation of tall forms the comparison class. Thus, very boosts the standard for membership (so to speak) by restricting the local domain to be the positive denotation (to tall individuals).

Cross linguistically, the morphological form of comparative predicates like taller is more complex than that of the positive form tall (for a detailed cross linguistic review of comparison morphologies see Klein 1991). The vagueness-based approach aims at explaining this phenomenon by taking the meaning of the positive form to be more basic. The principles in (6), (7) and (9) are implementations of this view. We see in them that the meaning of the comparative depends on the meaning of the positive form (e.g. the extensions of tall in different total contexts). The meaning of the positive form is given by our natural ability to classify entities as, e.g., tall and non-tall. Klein's (1980) preference of comparison classes over degrees (or standards) further emphasizes this point. He proposes that, generally, an important means of vagueness reduction consists of treating the denotation gap as a new local domain (comparison class). This allows determining the status of some gap members under tall (intuitively, those which are the tallest in the new domain become denotation members and those which are the least tall become non-members). This process can be repeated by treating the rest of the entities (the reduced gap) as a new local domain, up to the point that the gap is eliminated.

Finally, vagueness-based theories postulate additional principles. It is not enough to state that a comparative statement like Dan is taller than Sam is true when Dan is tall relative to more standards (or comparison classes) than Sam is. The connections between the comparative relation and the denotation are tighter. Intuitively, any entity must be considered tall, if some other entity which is equally or less tall than it is considered tall. In addition, any entity must be considered as not tall, if some other entity which is equally or more tall than it

\footnote{Klein (1980) discusses yet another implementation of the vagueness-based approach, grounded on McConnell Ginnet's (1973) analysis. This implementation makes use of natural functions like the ones denoted by degree modifiers (very, fairly, barely, etc.) The idea is that Dan is taller than Sam in a context c iff there is a natural function f (say, the denotation of very very), such that Dan is f-tall but Sam is not f-tall.}
is considered not tall. Therefore, vagueness-based analyses of gradability postulate separately that total contexts must conform to constraints that capture these intuitions. For example, Klein's (1991) principle grad (p. 684) is given in (9).

\[
(9) \quad \forall t \in T, \forall P \in \text{PRED}, \forall d_1, d_2 \in D:\n\]
\[
a. \quad \text{If } d_1 \in [[P]]^+_t \text{ and } <d_2, d_1> \in [[\geq P]]^+_t, \text{ then also } d_2 \in [[P]]^+_t. \]
\[
\quad \text{If } d_1 \text{ is } P \text{ and it is equally or less } P \text{ than } d_2 \text{ then also } d_2 \text{ is } P. \]
\[
b. \quad \text{If } d_2 \in [[P]]^+_t \text{ and } <d_2, d_1> \in [[\geq P]]^+_t, \text{ then also } d_1 \in [[P]]^+_t. \]
\[
\quad \text{If } d_2 \text{ is not } P \text{ and it is equally or more } P \text{ than } d_1 \text{ then also } d_1 \text{ is not } P. \]

3.2.1.2 Problems with the vagueness-based approach to gradability

Problem 1: Comparison classes cannot replace the use of (standard) degrees

The problem with Klein's (1980) attempt to replace degrees with comparison classes is that Klein has to disallow cases in which the negative or the positive denotations are empty (Kennedy 2002; Landman 2005). For instance, imagine again that Dan is 1 meter tall and Sam is 0.95 meters tall in c. Klein has to assume that, in evaluating the truth of a statement like Dan is taller than Sam relative to the simplest comparison class (namely, the one containing Sam and Dan), we (always) map Dan to the positive denotation and Sam to negative denotation of tall (otherwise, the statement will wrongly come out false in c). But intuitively, in the given example, both Dan and Sam may be mapped to the negative denotation. Thus, Klein's analysis fails to represent cases in which everybody that actually exists is tall, not-tall, or, for instance, healthy in some respect. For the same reasons, Klein's analysis fails to represent cases in which everybody that is tall is also very tall (Landman 2005).

Kamp (1975) has to assume that some dubious contexts exist, where the standards are loose enough to render (very short entities like) Dan tall. But Klein (1980) has to assume that we always rely on such dubious contexts (Landman 2005). Contrary to these assumptions, lacking knowledge about a standard (that is particularly loose), we will not decide that Dan is tall. Consequently, the notion of a comparison class and the notion of a standard degree are usually taken to be supplementary (Klein 1991; Kennedy 2002).

Problem 2: Are denotations more basic than comparisons?

The vagueness-based approach presumes to have solved the puzzle created by the morphological complexity of the comparative, compared to the positive form of predicates (across languages), because according to this approach the meaning of, e.g. more P, is composed of the meaning of P together with the meaning of more. But the basic idea of this approach (namely, that the denotation is determined by reaching a certain threshold) only emphasizes the converse basic intuition. We judge that Dan is tall is true in a given context by virtue of the fact that Dan's degree of height reaches P's contextually given standard (\[[[\text{Sam is tall}]]^+_t = 1 \iff [[[\text{Sam}]]]_t \in [[[\text{tall}]]]_t \iff \text{deg}([[\text{Sam}]], \text{tall}, t) \geq \text{standard}(\text{tall}, t)).\ The conceptual capacity underlying this judgment seems to be the ability to order entities by their height. If so, our capacity to make ordering judgments (or to map entities to degrees) is more basic, not our ability to make classification decisions (Kennedy 2002).

If this observation is correct, then the vagueness-based analysis, insightful as it is, does not explain why in language after language, the comparative form is derived from the positive form, rather than vice versa. The reasons for the universality of this compositional derivation are still poorly understood (see also the discussion in Landman 2005).

Furthermore, principles like (10a-b) (which take the ordering or the mapping to degrees to be conceptually primitive), immediately capture the generalizations which are stipulated in
(6)-(7) and (9): If Sam's height is smaller than Dan's height, then there must exist some standard which Dan reaches but Sam does not ((6)-(7)). For every total context (and the standard in it), it must be the case that if Sam reaches the standard so does Dan (9a), and if Dan does not reach the standard, neither does Sam (9b). Thus, crucially, the degrees (or entity ordering) seem to be the basis upon which speakers construct the information structure the way they do.

\[(10) \quad \forall c \in C: \]
\[\text{a. } [[\text{Dan is (at least) as tall as Sam}]]_c = 1 \iff \forall t \in T, t \geq c: \text{deg}([[\text{Dan}]], \text{tall}, t) \geq \text{deg}([[\text{Sam}]], \text{tall}, t)\]
\[\text{b. } [[\text{Sam is tall}]]_c = 1 \iff \forall t \in T, t \geq c: \text{deg}([[\text{Sam}]], \text{tall}, t) \geq \text{standard}(\text{tall}, t)\]

Having said this, we can now ask whether it is the ordering that is the conceptual primitive (and the degrees are derived from the ordering; cf. Cresswell 1976) or it is the mapping to degrees that is conceptually basic (the grammar links predicates with full fledged numerical degrees; cf. Bartsch and Vennemann 1972). This question is addressed in 3.2.2.2.

**Problem 3: The vagueness-based analysis applies to gap members only**

The most serious weakness of a standard-based analysis like the one in Kamp (1975), namely, of a principle like (11a), is that it applies to gap members only (Klein 1991). All the entities already known to be tall in c, are tall in all the total contexts extending c. So they are all tall in the same set of standards (total contexts). Hence, they are wrongly predicted to all be equally tall (11b). But intuitively, two tall individuals can stand in the relation taller than to each other.

\[(11) \quad \text{a. } [[a \text{ is as } P \text{ as } b]]_c^+ = 1 \iff \{t \in T_c \mid [[P(a)]]_t = 1\} \subseteq \{t \in T_c \mid [[P(b)]]_t = 1\}\]
\[\text{b. } \text{Wrong Prediction: } \forall d_1, d_2 \in [[\text{tall}]]_c^+: d_1, d_2 \in [[\text{equally tall}]]_c^+ \implies \forall d_1, d_2 \in [[\text{tall}]]_c^-: d_1, d_2 \in [[\text{equally tall}]]_c^+\]

**Problem 4: Vagueness in (and context dependency of) the ordering relation**

Intuitively, the relations that hold between two entities may be unknown (if, say, their heights are unknown). This is particularly true for comparatives such as more normal bird or more stereotypical tall person, whose interpretations (the set of dimensions, their relative importance and accordingly the ordering of the entities) is highly context-dependent.

Principles like (11a), though, cannot represent the fact that the truth value of a comparative statement can fail to be known, because they state that the truth conditions of a comparative statement in a context c depend on the whole set of total contexts in the vagueness model based on c, M_c. Thus, the truth value of comparison statements does not vary across different total contexts within M_c. Hence, their truth or falsity is always known.

**Problem 5: Is gradability associated with vagueness?**

Another problem for the vagueness-based approach is that some gradable predicates do not seem to be vague. Gradable predicates like tall are often called relative because their standard is highly context-dependent (12a). But this is not the case with other predicates, namely, the so called absolute predicates, whose standard is fixed by their semantics (Rotstein and Winter 2005; Kennedy and Mcnally 2005). Absolute predicates are called partial when they simply require the argument to possess some non-zero degree in the ordering dimension that they...
introduce (so, the most minimal degree is already enough for the predicate to hold of the argument). For example, visible and open are partial predicates because an entity falls under them iff it has a non-zero level of visibility or open-ness (12b). Absolute predicates are called total when they require the argument to possess the maximal degree in the relevant dimension. For example, full is a total predicate because entities fall under it iff they are maximally full (Rotstein and Winter 2004; Kennedy and McNally 2004). In sum, we have the three types of gradable predicates (12c).

(12) a. Relative predicates have a context-dependent standard for membership (tall, cool, clever, big, small, heavy, light, bald, old).
   b. Partial predicates have a minimum-standard (awake, visible, open, bent, wet, known, famous, hilly, alive, eaten, written, able to cope with, available, accurate, needed, wanted rest, hated, envied, loved, admired woman, worried, kissed, punched, met).
   c. Total predicates have a maximum standard (full, flat, closed, straight, dry, dead, unknown, inaccurate, unable, unavailable, unneeded, unwanted).

By virtue of their fixed standards, usually absolute predicates exhibit characteristic entailment patterns (Rotstein and Winter 2004; Kennedy and McNally 2004). First, typically, in partial (minimum standard) predicates, any non-zero degree in P entails P-hood, but in relative predicates many non-zero degrees may be below the contextual standard. Thus, the interpretation of (13a), but not of (13b), is intuitively judged to be a contradiction. Second, the negation of a total predicate entails the assertion of its (partial) antonym, but in relative predicates entities may fall under neither P nor P's antonym. For instance, not closed entails open (14a), but not short does not entail tall (14b). Fourth, mid-point modifiers like half or partially entail P-hood in partial predicates and non-P-hood in total predicates (15a-b). They entail membership under neither P nor not-P in relative predicates (15c). Fifth, in minimum standard predicates x is more P than y entails x is P (16a). In maximum standard predicates x is more P than y entails y is not P (16b). Comparative phrases with a relative predicate P entail neither that x is P nor that y is not P (16c), etc.

(13) a. # The door is not open, but it is still ajar          [contradiction]
   b. Sam is not tall but his height is normal for his age     [No contradiction]

(14) a. The door is not closed ⇒ The door is open.
   b. Sam is not short ⇔ Sam is tall.

(15) a. The door is half open ⇒ The door is open.
   b. The door is half closed ⇒ The door is not closed.
   c. The tree is half tall ⇒ The tree is (not) tall.

(16) a. The door is more open than the window ⇒ The door is open.
   b. The door is more closed than the window ⇒ The window is not closed.
   c. Rod A is longer than Rod B ⇒ Rod A is long.
   ⇒ Rod B is not long.

Finally, relative predicates license for arguments that help to fix a standard by constraining the local domain (17a). Yet, absolute predicates, whose standard is fixed semantically, usually do not license for arguments (17b-c) (Kennedy 2002). A for argument seems to be acceptable in total predicates (as in empty for a popular film theater) as a means of reducing the standard only if (otherwise) the domain necessarily ends up empty (∀t ≥ c: [[empty]] ⋂ [[popular film theater]] t = ∅).
Thus, we have good reasons to think that the standard of absolute predicates is fixed semantically. Yet, absolute predicates are gradable. For example, they are perfectly felicitous in within predicate comparisons (e.g. we can naturally say about entities that they are fuller, more open, less visible, etc.) This speaks against the association of gradability with vagueness.

3.2.1.3 Intermediate conclusions and a bit of a preview of my proposal

Intuitively, there are tight connections between vagueness and gradability. However, we saw that the vagueness-based approach fails to correctly describe these connections. In chapter 5 and onwards, I argue that the source of the problems with this approach is that gradability is not coupled with vagueness per se, but with the order in which vagueness is removed (the order in which the membership of entities in the denotation is learnt or inferred) through contexts and their extensions. (In order to formalize this intuition, (and directly following Landman 1991), I will add back into the model the intermediate contexts; that is, I will use a standard (full) vagueness model, like the one illustrated in Figure 2 in 3.1.) Chapter 8 shows that this proposal solves the problems discussed in this chapter, and is supported by robust empirical findings ("the order of learning effects").

As for the morphological complexity of comparative predicates, in chapter 5 and onwards, I propose that the reasons for this phenomenon can only be appreciated following a detailed examination of the semantics of nouns and multi-dimensional adjectives, which traditionally were not the primary focus of gradability theories. The point is that, intuitively, different types of predicates differ with respect to whether in effect categorization is based on reaching a threshold degree in the degree function they are linked with, or not. When you look at the denotation of many multi-dimensional adjectives (healthy, normal, typical of a bird etc.), you see that it is indeed not construed by a threshold degree. Consider for example healthy, in a context in which health is measured by the dimensions blood pressure, pulse and lung functions. Imagine that Dan is within the norm in only two of these dimensions, but in these two dimensions Dan has the ideal value (in the third he is only almost within the norm). Imagine also, that Sam is within the norm in all these dimensions, but she is very close to the norm limits (within the normative ranges of these dimensions, her levels are maximally far from the ideal levels). In other words, we can imagine a scenario in which (18a-c) are all true (note that we can average on three degrees with three different units, if the units are normalized).

(18)

a. Dan's average in blood pressure, pulse and lung functions is higher than Sam's
b. Sam is healthy wrt to blood pressure, pulse and lung functions
c. It is not the case that Dan is healthy wrt to blood pressure, pulse and lung functions

d. Sam is healthy
e. It is not the case that Dan is healthy

Intuitively, in this scenario, Sam is healthy, but Dan is not, because Sam, but not Dan, reaches the norm in all the contextually relevant respects ((18b) entails (18d) and (18c) entails (18e)).
Now, the question is whether Dan is healthier than Sam or Sam is healthier than Dan in such a scenario. Intuitively, the latter statement is correct (18f). But this shows that it is not the case that we directly compare Sam's average degree in the healthy dimensions to Dan's average degree in the healthy dimensions. Had we done that, we would have judged Dan to be healthier than Sam ((18a) would have entailed that Dan is healthier than Sam).

f. It is not the case that Dan is healthier than Sam

What do we do, then? In chapter 5 and onwards I argue that in one-dimensional adjectives like tall, the denotation is fixed by a threshold rule, and in multi-dimensional adjectives like healthy and sick, the denotation is fixed by dimension conjunction and dimension disjunction, respectively (namely, based on the standards of the dimensions). Thus, the use of the positive form of the adjective does not presuppose that the members reach the adjective's own standard (that the comparative relation is predetermined). On the other hand, the use of more does presuppose that the denotations of its predicative argument are predetermined (as assumed by the vagueness approach and demonstrated in (18)). Consequently, the meaning of the positive predicate is more basic than that of the comparative (notwithstanding the fact that the degrees are a conceptual primitive). In chapter 5 and onwards, I present my proposals regarding the classification and ordering criteria in multi-dimensional adjectives. I demonstrate that these proposals help predict a variety of facts, such as the licensing of with respect to modifiers, quantification over respects, and except phrases.

3.2.2 The nature of the degrees: The ordinal-scale versus interval-scale controversy

3.2.2.1 Measurement levels

Do natural language predicates map entities to numerical degrees? Before addressing this issue, it will be useful to consider the following four-level classification of scalar properties or degree functions (assignments of numbers to objects, along a dimension). It goes back to Stevens (1946; 1975) and is widely used in statistics (Babbie 2004).

The first level in this classification is called nominal. The only significance that nominal degree functions have is in the fact that entities are assigned same or different values. If the values are numerals, the choice of numerals is irrelevant, and the only comparisons that can be made between variable values are equality and inequality. There are no "less than" or "greater than" relations among the values, nor operations such as addition or subtraction. An example is the set of eye colors (brown, blue, green, etc.), the set of truth values {0,1}, the marital status of a person, or the model of a car.

The second level is called ordinal. At this level, the numbers assigned to objects represent their rank order (1st, 2nd, 3rd etc.). Comparisons of "greater than" and "less than" can be made, in addition to equality and inequality. However, operations such as addition and subtraction are still meaningless. Examples include the results of a horse race, which state only which horses arrived first, second, third, etc. but no time intervals.

The third level is called interval, where, in addition to the features of an ordinal level, equal differences between values represent equivalent intervals. Thus, operations such as addition and subtraction are meaningful. But the zero point on the scale is arbitrary, and negative values can be used. Thus, ratios between numbers on the scale are not meaningful, and operations such as multiplication and division cannot be carried out directly (only ratios of differences between pairs of values can be expressed; one difference can be twice the other, etc.) Examples are the year date in many calendars and temperature on a Celsius or Fahrenheit
scale. The fact that the zero Celsius degree is mapped to the water freezing point is arbitrary. This point does not correspond to non-existence of temperature. Accordingly, it is meaningless to say that 20°C is twice as hot as 10°C (it is not the case that 20°C represents a double amount of heat).

The forth level is called ratio. Ratio functions have all the features of interval functions, as well as meaningful ratios between values. Operations such as multiplication and division are therefore meaningful. The zero value on a ratio scale is non-arbitrary. Most physical quantities, such as mass, length or energy are measured on ratio scales; so is temperature measured in Kelvin, that is, relative to absolute zero. Other examples include age, length of residence in a given place, etc.

The linguistic theories of gradability and comparison can be divided into two main approaches, which I will call the ordinal scale analysis and the interval scale analysis. The ordinal scale approach entertains the assumption that grammar does not link gradable predicates with full fledged numerical degrees, but only with ordinal scales (sets of degrees which represent the ordering between each two entities but not the precise distance between them). The interval scale approach entertains the opposite assumption, namely that gradable predicates map arguments to numerical degrees with a plus (or difference) operation defined on them. In the following, I describe the two approaches and their problems in more details.

3.2.2.2 An ordinal scale analysis of gradability and comparison

The ordinal approach takes the ordering relation between entities (the denotation of comparative predicates) to be conceptually primitive, and the entities’ degrees to be derived from them. Quoting Sapir 1944:

It is very important to realize that psychologically all comparatives are primary in relation to their corresponding absolutes (“positives”) (Sapir 1944: 95).

In accordance, this approach to the analysis of gradability and comparison can be characterized by the following two principles:

\begin{align}
\forall c \in C:
\quad & a. \text{ The ordering of entities relative to } P, \text{ which forms the denotation of the derived comparative is a semantic primitive } ([\text{at least as } P])_c \subseteq D \times D. \\
& b. \text{ Degrees are equivalence classes under this ordering relation: } \\
& \forall d_1 \in D: \deg(d_1, P, c) = \{d_2 \in D \mid <d_1,d_2>_c \in [=[P]]_c\}
\end{align}

Let me explain these principles in more detail.

According to principle (19a), the ontology underlying language interpretation includes, among other things, a directly given partial ordering, \(\leq_P\), on the entity domain, along each gradable predicate \(P\) (Sapir 1944; Cresswell 1976). For example, if \(D\) is the individual domain, the linguistic symbol \(\text{at least as tall}\) is directly assigned a set of entity pairs as a denotation, \([\text{at least as tall}])_c \subseteq D \times D, \text{ in each context } c. \text{ The symbols taller and equally tall denote two subsets of this relation (20).}

\begin{align}
\forall c \in C:
\quad & a. \quad [\text{equally tall}]_c = \{<d_1,d_2>_c \in D \times D \mid <d_1,d_2>_c \in [\text{at least as tall}])_c, \quad <d_2,d_1>_c \in [=[P]]_c\}.
\quad & b. \quad [\text{taller}]_c = \{<d_1,d_2>_c \in D \times D \mid <d_1,d_2>_c \in [\text{at least as tall}])_c, \quad <d_2,d_1>_c \not\in [=[P]]_c\}.
\end{align}
According to principle (19b), we can use \( \preceq P \) in order to partition the domain into equivalence classes, that is, sets of individuals which are considered equal under \( \preceq P \) (Cresswell 1976).\(^2\) Take, for example, the relation *equally tall*. It is reflexive (every entity is exactly as tall as itself), symmetric (it is easy to see given (20b), that if, for instance, the pair \( <[[Dan]]_c,[[Sam]]_c> \) falls under *equally tall*, then so does the pair \( <[[Sam]]_c,[[Dan]]_c> \) and transitive (if, for instance, Dan and Sam are equally tall, and Sam and Sue are equally tall, then so are also Dan and Sue). Thus, *equally tall* denotes an equivalence relation. It partitions the domain into sets of entities which are equally tall (*equivalence classes*).

A degree of height on this approach is an equivalence class under *equally tall* (Cresswell 1976) – a set of objects which are all *equally tall*. Consider the multi-dimensional predicate *healthy* when ordered with respect to the dimension *blood pressure* (*healthy wrt bp*). In this view, the set of health degrees, \( S_{\text{healthy wrt bp}} \), is not a set of blood pressure degrees, but a set of equivalence classes under \( \preceq \text{healthy wrt bp} \). For any entity \( d_1, \text{deg}(d_1,\text{healthy wrt bp},c) = \{d_2 \in D: <d_1,d_2> \in [[=\text{healthy wrt bp}]]_c \} \). So the degrees in \( S_{\text{healthy wrt bp}} \) are indirectly ordered by \( \preceq \text{healthy wrt bp} \) as (21) states. They are not full fledged numerical degrees.

\[
\forall c \in C, \forall d_1,d_2 \in D: \text{deg}(d_1,P,c) = \text{deg}(d_2,P,c) \iff <d_1,d_2> \in [[=P]]_c \\
\text{deg}(d_1,P,c) < \text{deg}(d_2,P,c) \iff <d_1,d_2> \in [[<P]]_c
\]

The ordered equivalence classes can be associated with any sequence of numbers which correctly represents the ordering between them. E.g. we can define \( \text{deg}_{\text{healthy wrt bp}} \) to be a function from \( D \) to the set of rational numbers \( \mathbb{R} \), which satisfies constraint (21), i.e. any two entities \( d_1 \) and \( d_2 \) have to be mapped onto the same number (\( \text{deg}(d_1,\text{healthy wrt bp},c) = \text{deg}(d_2,\text{healthy wrt bp},c) \)) iff they are *equally healthy wrt bp* (\( <d_1,d_2> \in [[=\text{healthy wrt bp}]]_c \)) and \( d_1 \) has to be mapped onto a smaller number than \( d_2 \) iff \( d_2 \) is *healthier wrt bp* (\( <d_1,d_2> \in [[<\text{healthy wrt bp}]]_c \)). However, this change will make little difference, since this analysis assumes that there is no established interval size between each two degrees. The difference between each two consecutive degrees in \( S_P \) is meaningless. The plus or difference operations are therefore not defined on \( S_P \). These operations are only defined when there is an agreed upon unit which represents the interval between any two degrees.

The vagueness-based approach as presented in, for instance, Klein (1980), treats degrees as ordinal. Yet, vagueness-based implementations are compatible with an interval scale view (cf. Klein 1991; Kamp and Partee 1995). The use of an ordinal scale (a set of degrees which represent the ordering between each two entities but not the precise distance between them), fit best gradable predicates like *happy, sad or bald* which do not seem to be associated with numerical values. Indeed, Moltmann (2006) proposes an analysis without full fledged numerical degrees precisely for those predicates, but not for predicates like *tall or long*. The latter can be modified by numerical degree phrases, as in *two meters tall* or *twice as tall*, which seem to presuppose the existence of an interval scale, as shown next.

3.2.2.3 Problems for the ordinal scale approach

**Problem 1: Numerical degree modifiers**

von Stechow (1984a) argues that it is not clear how the meaning of statements with numerical degree modifiers, such as those in (22), can be captured within the vagueness approach.

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\(^2\) In that, principle (19b) parallels the analysis of moments of times as equivalence classes of events relative to some ordering on these events.
A suggestive solution is found in Klein 1991 (p: 684). If the unit of measurement is centimeters, then (22a) can be said to be true iff there are exactly 100 standards that Dan reaches. (22b) can be said to be true iff there are exactly six standards that Dan reaches but Sam does not reach. Namely, if Sam is 1 meter tall, and Dan 106cm tall, the six standards would be \{101,102, 103,104,105,106\}. Similarly, (22c) would be true iff Dan reaches twice as many standards as Sam. This proposal brings us closer to the view that gradable adjectives denote interval scale properties (Russell 1905; Bartsch and Vennemann 1972; Hellan 1981; von Stechow 1984; Kennedy 1999). This view is discussed in 3.2.2.5.

(22) a. Dan is 1 meter tall
b. Dan is 6 centimeters taller than Sam.
c. Dan is twice as tall as Sam is

**Problem 2: Multi-dimensional predicates**

The ordinal approach falls short in describing the way the degrees in different dimensions are combined to form the meaning of multi-dimensional adjectives, like *intelligent* or *healthy*, and their derived comparative.

Generally, it is assumed that if \( P \) is multi-dimensional, first a contextually relevant ordering dimension \( F \) needs to be specified for \( P \), and only then a partial ordering along \( F \), \( \leq_F \), can be associated with \( P \). However, when two or more dimensions are involved in the ordering of entities along \( P \) (say, \( F_1 \) and \( F_2 \)), it is not clear how the ordering along \( P \), \( \leq_{P[F_1,F_2]} \), is composed from (or related to) the ordering in \( F_1 \), \( \leq_{F_1} \), and the ordering in \( F_2 \), \( \leq_{F_2} \). In particular, if one entity is more \( F_1 \) and another is more \( F_2 \), which one is more \( P \)? The answer to this question requires notions like 'distance between' or 'sum of' degrees: Intuitively, the entity which is more \( P \) is such that, roughly, the sum of its (weighted) degrees in \( F_1 \) and \( F_2 \) is bigger (for further discussion see chapters 4, 7 and 8).

**Problem 3: Between-predicate comparisons**

The vagueness-based approach as is fails to give a correct analysis of between-predicate comparisons (Kennedy 1999). For instance, (23a) is predicted to be true iff *the table is long* is true in more total contexts than *the sofa is wide* is true in. (23b) is predicted to be true iff *Dan is tall* is true in more total contexts than it is false in. The problem is that nothing in the analysis predicts the infelicity of (23b-e), that is, the incommensurability of certain predicate pairs, and in particular, pairs of a predicate and its antonym; nor are the differences between positive and negative predicates with respect to modification by numerical degree phrases accounted for (23d-e).

(23) a. The table is longer than the sofa is wide.
b. # Dan is taller than he is not tall / is short.
c. # Dan is taller than he is happy.
d. # Dan is 1 meter short.
e. Dan is 1 meter tall.
f. This is more a bird than a horse

While comparing different predicates (as in (23a) and (23f)), a unified scale has to be reconstructed (or the relative positions of entities on each scale need to be determined). This seems to require the use of interval scales, where the distance between entities' degrees is
well defined and hence the notion of relative position on a scale is well defined (for further discussion see chapters 7 and 9).

**Problem 4: The morphological complexity of the comparative wrt the positive form**

A serious problem, which has finally led to the rejection of Creswell's (1976) ordinal approach, concerns compositionality (Kamp 1975; Klein 1980). According to the ordinal analysis, the meaning of the positive form of a predicate (say, *tall*) is derived from the meaning of the comparative form (e.g. *taller*). Thus, this analysis fails to predict the fact that, cross linguistically, the morphological complexity of comparative relations is greater than that of predicates in the positive form. That is, the meaning of *taller* needs to be composed from the (basic) meaning of *tall* and the meaning of the comparative morpheme *more*.

### 3.2.2.4 An interval scale analysis of gradability and comparison

This approach to the analysis of gradability and comparison (Russell 1905; Wunderlich 1970; Bartsch and Vennemann 1972; Cresswell 1976; Hellan 1981; Klein 1982; von Stechow 1984; Kamp and Partee 1995; Kennedy 1999; Schwarzschild and Wilkinson 2002; Rotstein and Winter 2005; Landman 2005) can be characterized by the following two principles:

1. **The interpretation of gradable predicates involves mapping of entities into full fledged numerical degrees, closed under addition.**
2. **The comparative morpheme *more* denotes the plus (or difference) operation.** Roughly, an entity $d_1$ is more $P$ than some other entity $d_2$ iff the difference between $d_1$'s (maximal) degree and $d_2$'s (maximal) degree in $P$ is bigger than zero.

Let me describe these principles in more detail.

According to principle (24a), the interpretation of gradable predicates like *tall* is assumed to be associated with a scale, namely with several elements, roughly as described in (25) (Kennedy 1999: 188; Rotstein and Winter 2005; Landman 2005).

1. **A dimension** $F_{tall}$ (height)
2. **A set of units of measurements** $U_{tall}$ (centimeter, meter, etc.)
3. **A dense set of points** (degrees), $S_{tall}$. Degrees are often formalized as tuples, $<n,u,F>$, containing a real number $n$, a unit $u$, and a dimension $F$ (so at least for predicates with established units, $S_P = \{<n,u,F_P>: n \in \mathbb{R}, u \in U_P}\$).
4. **A linear ordering** on $S_{tall}$, $\leq_{tall}$, which states for each two points which one represents the larger degree under tall (for example, $\leq_{short}$ is inverse wrt $\leq_{tall}$).
5. **A set of measure functions**, $\text{deg}_{tall,u}$, for each $u \in U_{tall}$, such that $\text{deg}_{tall,u}$ maps each entity $d$ in a context $c$ to a degree in $S_{tall}$: $\text{deg}_{tall,u}(d,c) = <n,u,\text{height}>$, for some $n \in \mathbb{R}$.

Dimensions like length, height or happiness are usually taken to be primitive qualities or attributes that permit grading (Sapir 1944; Bartsch and Vennemann 1972; Cresswell 1976; Bierwisch 1989; Kennedy 1999). Dimensions are stipulated also within the ordinal approach. In order to better appreciate the notion of a measure function, or an interval-scale property (as opposed to the notion of an ordinal function or an ordinal scale property), let us consider the length-measuring function. There is an object, a rod, whose length is agreed to be 1 meter. Thus, the meter serves as a measurement unit, based on which we can define a scale with
meaningful intervals. Let the symbol '$\geq$' and '+' stand both for relations between numbers and for relations between degrees, as specified in (26a). Let the symbol a-b stand for the concatenation (the placing degrees to end) of two rods a and b. A measure function for the dimension length and the unit meter, $deg_{\text{long, meter}}$ (or an interval scale property $\text{long in meters}$) maps entities (in a context $c$) to degrees, subject to the constraint in (26b). For each two entities $d_1$ and $d_2$, $deg_{\text{long, meter}}(d_1,d_2,c) = deg_{\text{long, meter}}(d_1,c) + deg_{\text{long, meter}}(d_2,c)$. Thus, we can now meaningfully speak about the distance between entities' degrees in long, and indeed, according to principle (24b), that is precisely what we do when we use comparative statements, as demonstrated in (26c-d) (the unit variable is often suppressed in the representation of the interpretation of sentences without unit names like (26d)).

(26) $\forall P \in \text{PRED}, \forall u \in U_P,$
    a. $\forall n,m \in R: <n,u,F_P> \geq <m,u,F_P>$ iff $n \geq m$ and
      $<n,u,F_P> + <m,u,F_P> = <n+m,u,F_P>$

    b. $\forall c \in C, \forall d_1,d_2 \in D:$ $deg_{P,u}(d_1 \text{°} d_2,c) = deg_{P,u}(d_1,c) + deg_{P,u}(d_2,c)$

    c. $[[\text{Rod A is 1 meter longer than rod B}]]_c = 1$ iff
      $deg_{\text{long, meter}}(A,c) = deg_{\text{long, meter}}(B,c) + <1,\text{meter, length}>$

    d. $[[\text{Rod A is longer than rod B}]]_c = 1$ iff $\exists n > 0, p \in S_{\text{long}}, \exists u \in U_p$:
      $deg_{\text{long, u}}(A,c) = deg_{\text{long, u}}(B,c) + p$

3.2.2.5 Problems with the interval scale approach

**Problem 1:** Most predicate interpretations appear to have characteristics of ordinal scales

The main problem for the interval-scale approach comes from the obvious fact that we do not associate beauty, happiness, and most other things with established means of measuring (units, numbers, etc.) This is reflected in not using numerical degree phrases such as two meters with most predicates (happy, beautiful, intelligent, etc.) or nominalizations (happiness, beauty, intelligence, etc.) In some languages, numerical degree modifiers are allowed with a restricted set of positive predicates (like the English predicate tall as in Dan is two meters tall). In other languages, numerical degree modifiers are allowed with a restricted set of nominalizations (as in (Hebrew:) gova shney meter 'height of two meters'), or in the comparative form (as in two meters taller). In addition, the set of positive predicates which allow this modification varies considerably between languages (Moltmann 2006). So, when we decide to posit an abstract set of numerical degrees as part of the interpretation of predicates or nominalizations then our theory faces several immediate problems.

First, it is not intuitive (as Kamp and Partee 1995, for instance, explicitly admit).

Second, given a certain scale (number set), there is much indeterminacy in the mapping of individuals to numbers (Kamp and Partee 1995). This is certainly true of predicates like happy. Given the set of real numbers between 0 to 1, why would a certain person have a degree 0.25 rather than say 0.242 in happy? Kamp and Partee (1995) present a general suggestion for a solution to this problem in terms of vagueness with regard to the correct degree function in each context. In this setting, a context is associated with a set of degree functions (the ones that are still candidates for being the correct one), such that we may only know in a certain context that, e.g., the degree of a penguin in bird ranges between 0.25 and

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3 Different implementations of this approach differ as to the way they represent and derive this interpretation for comparison statements, as discussed in 3.2.5. The notions dimension and degree are further discussed in section 3.2.3 that focuses on between predicate comparisons and in section 3.2.4 that focuses on polarity.
0.242. However, they contend that this idea is still not quite intuitive and they do not develop it into a full-fledged analysis.

Third, in most of the predicates there is much indeterminacy regarding the number set. Which set of numbers represent the degrees of predicates like happy or beautiful? Moltmann (2006) maintains that, given this indeterminacy, the assumption that numerical degrees play a role in the semantics of predicates and comparatives, creates a meaning intention problem. How do speakers know the meaning of their utterances when they are using expressions which (presumably) denote degrees, such as (27a)?

Finally, Moltmann (2006) observes that adverbial modifiers of gradable adjectives, like the ones in (27b), do not denote properties of numbers. For example, numbers which represent beauty degrees can be neither strange nor visible, but particular instantiations of the property beautiful in individuals may be both strange and visible. According to Moltmann (2006), this speaks against the idea that the gradable adjective beautiful maps individuals to numerical degrees, or denotes a relation between individuals and such degrees. Moltmann (2006) proposes to replace degrees with the notion trope, which stands for the type of entities which form the meaning of nominal forms such as happiness (the particular instantiations of the property happy in particular individuals).

(27) a. Dan is as happy as Sue is. Bill isn’t that happy.
    b. Strangely beautiful; visibly happy; fatally weak; deliberately silent

Problem 2: Problems with the linear ordering $\leq_P$ on the degrees in $S_P$

In the implementations of the interval scale approach, the scale of a predicate is a set of degrees ordered along some predicate specific ordering relation. This is taken to be the case because it is usually assumed that one set of degrees $S_P$ may be ordered by several different ordering relations per dimension. For instance, predicates and their antonyms (say, tall and short) are usually viewed as sharing the dimension and scale, but their ordering is represented by reversed ordering relations on this scale. If P is associated with $>_P$, $P^\text{ant}$ is associated with $\leq_P$ (cf. Rullmann 1995). This view is problematic in several respects.

First, the need to stipulate a predicate specific ordering relation in the interpretation of predicates weakens the idea underlying these theories, namely, that the mapping to degrees forms the basis for the predicate’s ordering relation (the entity ordering), and not vice versa.\(^4\)

Further, the theories in the interval-scale approach do not explicate how the ordering of the degrees is fixed in different types of predicates based on their dimensions. In antonym pairs, the connections between the ordering of $P$’s dimension and the ordering of $P^\text{ant}$ are relatively clear. But in some cases, such as predicates of the form $P$ wrt $Q$ (say, healthy wrt blood pressure [bp] or typical of a bird), $Q$ is intuitively taken to be the ordering dimension (e.g. predicates like healthy wrt bp naturally appear to be measured by blood pressure degrees: $S_{\text{healthy wrt bp}} = S_{\text{bp}}$), but the connections between the ordering of $Q$ (bp) and the ordering of $P$ (healthy) is not trivial. Neither a larger degree nor a smaller degree in blood pressure necessarily represents a higher degree in healthy wrt bp. Furthermore, multi-dimensional adjectives $P$ (like healthy) can be paraphrased as "$P$ wrt $Q_1 \ldots Q_n$", but the theories fail to state the connections between the degrees in $Q_1, \ldots, Q_n$ and in $P$. (Naturally, understanding the graded structure of predicates like healthy wrt bp would be a crucial step towards an improved analysis of predicates like healthy wrt bp, pulse, and lung functions, and

\(^4\)The discussion in 3.2.4 and chapter 5 and onwards suggests that the stipulation of a predicate specific ordering relation can be spared (in chapters 5-9 I propose a theory where the degrees in all the predicates are ordered by the natural ordering on the real numbers, $>$).
bare multi-dimensional predicates like *healthy*, or *typical of a bird*). In sum, we need to look more deeply into the systematic ways in which different predicate types order entities along their dimensions (the ways the dimensions' degrees are mapped into the predicates' degrees).

Let us briefly look at the issues involved in this task. Consider again the predicate *healthy wrt bp* (assuming that it has a unit u and the dimension bp).

On the one hand, intuitively, some range of bp degrees, say, any degree between \(<n, u, bp>\) and \(<m, u, bp>\) in \(S_{bp}\) (such that \(n \leq m\)) is considered within the norm. Any bp degree above \(<n, u, bp>\) or under \(<m, u, bp>\) is ranked higher under \(\leq_{healthy \, wrt \, bp}\), than any degree under \(<n, u, bp>\) or above \(<m, u, bp>\). In addition, smaller deviations from these norm bounds are ranked higher under \(\leq_{healthy \, wrt \, bp}\) (for instance, \(<m+1, u, bp>\) is ranked higher than \(<m+2, u, bp>\), and \(<n-1, u, bp>\) is ranked higher than \(<n-2, u, bp>\), and within the norm bounds, smaller deviations from the middle of the normal range are ranked higher under \(\leq_{healthy \, wrt \, bp}\) (for instance, \(<(n+m)/2, u, bp>\) is the maximum degree under \(\leq_{healthy \, wrt \, bp}\) (28).

\[
\begin{align*}
(n+m)/2, u, bp & \leq_{healthy \, wrt \, bp} (n+m)/2 + 1, u, bp \\
=_{healthy \, wrt \, bp} & (n+m)/2 - 1, u, bp
\end{align*}
\]

In a context where health is measured by bp and pulse, and the axes in Figure 4 are taken to stand for degrees in bp and pulse, the points falling within the dotted square represent denotation members (entities which are *healthy wrt bp and pulse*).

On the other hand, if we take it that comparative morphemes denote the difference operation on numbers (the numbers in entities' degrees; cf. (24b)), then we must assume that the degrees of *healthy wrt bp* are ordered by the natural ordering on the real numbers \(>\) or \(<\). Thus, the actual dimension that fixes the ordering relation of *healthy wrt bp* is not bp (\(\leq_{healthy \, wrt \, bp} \neq \leq_{bp}\) and \(S_{healthy \, wrt \, bp} \neq S_{bp}\)), but a reconstruction of the dimension bp based on speakers' knowledge about the relations between health and blood pressure. Strictly speaking, saying that *healthy* (wrt bp) is ordered by blood pressure is wrong. This is an imprecise (but short) way to say that it is ordered by a dimension which is reconstructed from this measure in the way just described. Similarly, nouns like *bird* are often described as being ordered by dimensions like *size* or *small*. This is an imprecise (but short) way to say that *size* is the means of measuring and the actual dimension is a reconstruction from the size measures (this reconstruction may be roughly denoted by predicates like *robin's size*). In figure 5, the axes are taken to stand for the degrees in the dimensions which are reconstructed from blood pressure and pulse (*healthy wrt bp* and *healthy wrt pulse*). The dotted square which represents the denotation falls between the degree n’ or m’ (the degrees into which the norm bounds n and m are mapped) and the maximum under \(\leq_{healthy \, wrt \, bp}\), \((n+m)/2\)' (the degree into which the optimal bp degree \((n+m)/2\) is mapped).
The theories in the interval-scale approach must assume that we reconstruct the dimensions of predicates in some way, but they do not specify ways to do that.

If we adopt the representation illustrated in Figure 5, this has several implications.

First, it entails that predicates like \textit{healthy wrt bp} must have a maximum degree, \((n+m)/2)\', and their other degrees must represent extents of deviations from \((n+m)/2\). Several bp degrees may be mapped to the maximum.

Second, we need to be able to represent the fact that the ordering between entities with different bp degrees may be partially unknown. Often, in multi-dimensional adjectives or nouns, only the maximum (the prototype) is known, and the distance of entities from it varies depending on the weights assigned to the different dimensions.

Third, this representation presupposes that the dimensions of multi-dimensional predicates are independent. For instance, it follows from it that being closer to the norms of \textit{blood pressure} always improves one's health, regardless of one's status in other physiological parameters. But empirical findings show that dimensions are not always independent in that sense (e.g. it is possible that a high \textit{bp} degree will improve one's health when one has a high degree in \textit{pulse} and at the same time this \textit{bp} degree will have the opposite effect when one has a low degree in \textit{pulse}) This problem can be solved by treating any set of dependent dimensions either as a unique complex dimension or as non-dimensions (concerning this issue, see the discussion of the exemplar theory in chapter 4).

\subsection*{3.2.2.6 Conclusions and implications}

Any analysis which takes up the interval scale approach needs to explain why the meaning of most of the gradable predicates appears to have characteristics of ordinal scales. They do not have an established unit such that distances between degrees can be expressed based on it, they do not admit numerical degree modification, and there is much indeterminacy wrt their scale (number set) and wrt the number to which they map each entity.

In addition, the theory needs to be supplemented by a precise formulation of the systematic ways in which different predicate types are ordered along their dimensions (the ways the dimensions' degrees are mapped into the predicates' degrees), so that the implications of these formulations will be worked out and be empirically supported or refuted.

In chapter 5 and onwards, I propose a formulation for the systematic ways in which multi-dimensional adjectives, predicates of the form \textit{P wrt Q}, negative predicates, and nouns are ordered along their dimensions. My proposal does not associate the degree-set of predicates with a predicate specific ordering relation. I propose that relatively simple operations (which I call transfer, reversal and distance operations) produce degree functions (and scales) for all the predicates, whose degrees are naturally ordered by the standard "equal or bigger than" relation of the real numbers. Thus, the interpretation of a predicate becomes more economical.

\subsection*{3.2.3 Between-predicate comparisons}

\subsubsection*{3.2.3.1 Incommensurability}

Certain statements (sometimes called \textit{sub-deletion comparatives}) involve comparisons of degrees in two different predicates. In order to abstract away from theoretical considerations, I called these statements \textit{between-predicate comparisons} (29), as opposed to statements that involve comparisons of two degrees in one predicate (30), which I called \textit{within-predicate comparisons}. Some pairs of predicates cannot occur in between-predicate comparisons (31).
The table is longer than the sofa is wide
The table is longer than the sofa is
a. # The table is longer / more long than the sofa is beautiful
b. # Dan is taller / more tall than Ram is intelligent
c. # John is taller / more tall than he is not
d. # John is taller / more tall than he is short

A dominant account of the incommensurability phenomenon uses dimensions as a means of distinguishing between scales (Kennedy 1999; 2001). Principle (32) states that two predicates P and Q are commensurable only if they share not only the number-set, but also the dimension.

A sub-deletion comparative construction (a between-predicate comparison)
NP1 is more P than NP2 is Q is well-formed only if the compared adjectives P and Q have the same number-set and dimension (Kennedy 1999: 59).

According to Kennedy (1999 p. 100), long and wide order entities along the same dimension, because width and length are both measures of linear extents (and hence they may be measured by the same units). They differ in that they order the entities according to different perpendicular aspects, and hence they are linked with different measure functions (which may map entities to different degrees).

In Kennedy (2001) the generalization is described in terms of sorts of degrees. Compared adjectives are required to define ordering relations between the same sorts of degrees. In order to account for cross-polar anomalies (incommensurability between the degrees of predicates and of their negations or antonyms; cf. (31c-d)), negative and positive predicates are analyzed as denoting different sorts of degrees (cf. 3.2.4).

3.2.3.2 Problems with the incommensurability account

Problem 1: Counterexamples

The main difficulty in assuming that comparisons of entities’ degrees in predicates with different dimensions are impossible is that nominal predicates freely occur in such comparisons.

a. Bling Bling says "tweet" (I'm convinced he's more a bird than a cat).
b. … giving me three bits of furniture which she didn't want anymore (a coat rack, chair, and stool thing which is really more a table than anything else
c. The "wall" was rolling backward until it came to a horizontal position, now being more a table than a wall
d. Chevy is more a car than a truck
e. The ostrich is more of a bird than the platypus is a mammal
f. A bat is no more a bird than a whale is a fish

If we can compare entities’ degrees along these predicates then we can make comparisons which involve several different dimensions. Since linguists usually consider nouns not to be gradable, these data were overlooked in the past.
Problem 2: The notion of a 'dimension' is not well defined

In order to correctly predict that degrees of different predicates are not necessarily comparable, the theory relies heavily on the notion of a dimension. However, the notion of a dimension is hardly well defined.

Moltmann (2006) observes that this notion does not reduce to a set of degrees under a certain ordering relation, because (predicates with) different dimensions (e.g. length, width and height) have the same set of degree and the same ordering on this set.

Kennedy (1999) takes the measure functions of the predicates long, wide and high to be different, not their dimensions. But, intuitively, we do feel that length, width and height are different (as Kennedy puts it, they order the entities according to different perpendicular aspects). And if long and wide share the same dimension, then a dimension cannot even reduce to a predicate meaning (since obviously, different predicates are assumed to share a dimension) or to the set of constraints on the mapping of entities to degrees (since the measure functions of long and wide may map entities to different degrees). If so, then what is a dimension? The same definitional vagueness applies to Landman's (2005) notion of a dimensional measure.

The notion of a dimension $F_P$ of a predicate $P$ (like tall or happy) can be clarified by assuming, following Moltmann (2006), that $F_P$, like the corresponding nominal forms of predicates $P$ (e.g. height, happiness), denotes the set of the particular instantiations of $P$ in the individuals in the domain. But then wide and long will no longer share a dimension (because, my width and my length, for instance, are not identical), and their ability to co-occur in comparative statements will not be explained.

Apparently, however, two predicates are equivalent iff they share the same set of instantiations (iff their nominalizations are equivalent). Thus, defining a dimension by using the predicate's nominalization only pushes the need for an account one level up. We still need to explain how the set of instantiations (or anything else in the interpretation of the predicate) permits grading. Moltmann (2006) assumes that two entities $d_1$ and $d_2$ stand in the relation more $P$ than iff $d_1$ possesses a larger quantity of $P$-hood (of the thing that forms the denotation of the predicate's nominalization) than $d_2$. This is intuitively correct, but rather vague. How do we count (or measure amounts of) instantiations of health? In addition, as we have already seen, this cannot be the whole story. For instance, healthy wrt bp does not order entities by quantity of blood pressure (so to speak). So we still need to say how "quantities of health wrt bp" are determined based on "quantities of bp" (for further discussion of this quantity metaphor see chapter 5).

3.2.3.3 A preview of my analysis of dimensions and between-predicate comparisons

In chapter 5, I argue that a dimension can profitably be viewed as just another (gradable) predicate in the language, whose degree function (and hence entity ordering) is connected to the degree function (and hence ordering) of the predicate which it is a dimension of. The dimensional degree function fixes or constrains (in the case of multi-dimensional adjectives like healthy wrt blood pressure and pulse) the degree function of the predicate. For example, the predicate normal blood pressure may function as a dimension for the predicate healthy and the predicate flying may function as a dimension for the predicate typical of a bird. This may happen because the mapping of entities to degrees in these dimensions may affect the mapping of entities to degrees in the predicates. Ultimately, the theory should state precisely how the dimensions affect the ordering in the predicate. This issue is at the core of the psychological research and it will be discussed extensively in chapters 4 and onwards.
At any rate, again, *wide* and *long* no longer share a dimension and their ability to co-occur in comparative statements is not be explained. We see that in both of the attempts to make the notion 'dimension' precise, the incommensurability account in (32) is lost. Thus, we seem to be in need of an improved account. In chapters 5, 7 and 9, I propose such an improved account. The crucial observation in chapter 7 is that degrees in different predicates (even in predicates with different units) can be compared iff they are normalized. I discuss several conditions under which the values of a degree function can be normalized (for example, one method for normalization requires that the scale will have a minimum and a maximum), and I show that they correctly predict the facts. In addition, comparisons that are based on a shared unit (as in *The table is longer than the sofa is wide*) are analyzed in a different way in chapter 9.

3.2.4 Polarity

3.2.4.1 A boundedness account of the polarity effects

Winter (2001: 7) suggests that, for every predicate pair P and Q such that Q is P's antonym, P is *positive* and Q is *negative* iff for each individual which is P to the extent of n units, things may be more than *n units Per*, but nothing can be *more than n units Qer*. This is demonstrated in (15b) with the predicates *tall* and *short*.

(34)  *Tall* is positive and *short* is negative, because for each individual d which is *n meters tall*, there are many entities which are *more than n meters taller than d*, but no entity is *more than n meters shorter than d*

3.2.4.2 Problems with the boundedness account and preview of my proposal

The test illustrated in (34) is problematic as it is. Consider for example the pair *hot* (or *warm*) and *cold*. *Hot* or *warm* are positive and *cold* is negative. Yet, for each individual d whose heat is ranked as 20 degrees Celsius, there are (many entities which are more than 20 degrees warmer than d, but also) many entities which are more than 20 degrees colder than d.

Note also that numerical degree modifiers are not freely licensed with both *warm* and *cold* (Kennedy 2001). In chapter 5 and onwards, I claim that the connection between the failure of the test in (34) and the infelicity of numerical degree modifiers with the positive predicate *warm* is not accidental. Both happen when the positive predicate denotes an interval scale property (a transferred zero scale), as opposed to a ratio scale property. Negative antonyms are typically linked with interval scales (even when the positive predicates are mapped to ratio scales), so they are typically characterized both by the property that is attributed to them by principle (34), and as failing to license numerical degree modifiers. Positive predicates, however, may also denote interval scales and then they also exhibit these properties.

3.2.4.3 An 'extent' account of the polarity effects

Before presenting the 'extent' account of the polarity effects, we need to present the basic tenets of the relational analysis of gradable predicates, and the quantificational analysis of the comparative morphemes. On the relational-quantificational analysis, gradable predicates are sortally different from non-gradable ones (Russell 1905; Cresswell 1976; Hellan 1981; Hoeksema 1983; von Stechow 1984; Heim 1985; Moltmann 1992; Rullmann 1995). While a non-gradable predicate denotes a set of entities (or the characteristic function of such a set), a gradable predicate P denotes a relation between entities and degrees. This is formally stated in
(35a), where $\delta$ is a variable over degrees, and $\text{deg}(x, P)$ is an object language expression, "$x$'s degree in $P$"; such that in any context $c$, $\text{deg}(x, P)$ denotes the degree of $x$'s referent in $P$, $\text{deg}(x, P, c) \in S_P$\footnote{I use 'deg' both in the object language expression 'deg(x,tall)' and in the meta-language expression 'deg(d,tall,c)'; but the first argument in the former is a term in the object language while in the second it is a term's referent (an object in D).}. For example, the predicate $\text{tall}$ is equivalent to the relation "$\lambda\delta\lambda x. \text{deg}(x, \text{tall}) \geq \delta$" (that relation between entities $x$ and degrees $\delta$, such that $x$'s degree exceeds $\delta$).

\[(35)\]

For any gradable predicate $P$ and context $c$:

\begin{enumerate}
  \item $P \iff \lambda\delta\lambda x. \text{deg}(x, P) \geq \delta$
  \item $[[P]]^+_{c}(\text{standard}(P, c))(d) = 1$ iff $\text{deg}(d, P, c) \geq \text{standard}(P, c)$
  \item $[[P]]^-_{c}(\text{standard}(P, c))(d) = 1$ iff $\text{deg}(d, P, c) < \text{Standard}(P, c)$
\end{enumerate}

This approach assumes that within a context $c$, one degree in $S_{\text{tall}}$ forms a categorization criterion, $\text{standard}(\text{tall}, c) \in S_{\text{tall}}$, such that entities are taken to fall under the one-place predicate 'tall' iff their (maximal) height exceeds this degree. Formally, it is $[\text{standard}(P, c)]^+_{c}$ that stands for our one-place predicate 'tall' (see (35b)). For example, statements like Sam is tall are considered true in a context $c$ iff Sam's height, $\text{deg}([[\text{Sam}]]_{c}, \text{tall}, c)$, reaches the standard for tallness in $c$. Syntactically, in relational-quantificational theories, a degree term like $1 \text{ meter}$ is assumed to fill the specifier position of adjectival phrases (Bersnan 1973). Thus, as demonstrated in (36), $\text{Sam is 1 meter tall}$ is assumed to be true, roughly, iff the measure function associated with the interpretation of the adjective $\text{tall}$, $\text{deg}_{\text{tall,meter}}$, maps Sam in $c$ to a degree which exceeds the degree denoted by the degree term $1 \text{ meter}$.

\[(36)\]

\[
[[ \text{[Sam is [ [1 meter]DEGP [tall]AP]VP]IP} ]_{c} = 1 \quad \text{iff} \quad \text{deg}([[\text{Sam}]]_{c}, \text{tall}, c) \geq <1, \text{meter}, \text{height} >
\]

This approach differs also from Measure function theories (Wunderlich 1970; Bartsch and Vennemann 1972; Kamp and Partee 1995; Kennedy 1999; Rotstein and Winter 2005), in which a gradable adjective like $\text{tall}$ is viewed as denoting the degree function itself, that is, a function from entities to their maximal degree under $\text{tall}$ (37a).

\[(37)\]

\begin{enumerate}
  \item $[[\text{tall}]]_{\text{AP}} \iff \lambda x. \text{deg}(x, \text{tall})$
\end{enumerate}

Still, in theories like Kennedy (1999), the adjective phrase meaning $[[\text{tall}]]_{\text{AP}}$ is assumed to be modified by a constituent which is analyzed as a degree head DEG (p. 121). This constituent takes a gradable adjective meaning (AP) and returns a relation between degrees $\delta$ and objects $x$, such that $x$'s degree in the adjective is greater than $\delta$. The result is a relation between entities and degrees (37b). Thus, the final truth conditions of statements like Sam is tall, Sam is taller than Dan, or Sam is one meter tall are essentially the same as those assumed by the relational approach.

This approach is opposed to the approach of Kamp (1975) and Klein (1980), who treat one-place gradable adjectives as denoting ordinary properties (non-relational predicates $\lambda x. P(x)$). They move the degree parameters (the standard degree, delineation, comparison class, etc.) from the logical form into the contextual coordinate (as in: $[[P]]_{c,\text{standard}(P, c), X_{c}}$...).

Syntactically, in relational-quantificational theories, a degree term like $1 \text{ meter}$ is assumed to fill the specifier position of adjectival phrases (Bersnan 1973). Thus, as demonstrated in (36), $\text{Sam is 1 meter tall}$ is assumed to be true, roughly, iff the measure function associated with the interpretation of the adjective $\text{tall}$, $\text{deg}_{\text{tall,meter}}$, maps Sam in $c$ to a degree which exceeds the degree denoted by the degree term $1 \text{ meter}$. Still, in theories like Kennedy (1999), the adjective phrase meaning $[[\text{tall}]]_{\text{AP}}$ is assumed to be modified by a constituent which is analyzed as a degree head DEG (p. 121). This constituent takes a gradable adjective meaning (AP) and returns a relation between degrees $\delta$ and objects $x$, such that $x$'s degree in the adjective is greater than $\delta$. The result is a relation between entities and degrees (37b). Thus, the final truth conditions of statements like Sam is tall, Sam is taller than Dan, or Sam is one meter tall are essentially the same as those assumed by the relational approach.
Finally, in the relational approach, the clause in the scope of \textit{than} in comparison statements is viewed as denoting a set of degrees. For instance, in \textit{Dan is taller than Sam is}, the subordinate clause \textit{Sam is} is viewed as denoting the set of heights that can be truthfully applied to Sam (38a).

\begin{equation}
\text{a. } \left[ \left[ \left[ \text{Sam is tall} \right]_{CP} \right]_{c} = \{ p \in S_{\text{tall}} \mid \left[ \left[ \text{Sam is } \delta \text{ meters tall} \right]_{c,g(\delta)} = 1 \} \right. \\
= \{ <0, \text{meter}, \text{tall}>, \text{deg}([\left[ \text{Sam} \right]_{c}, \text{tall}, c]) \} \right.
\end{equation}

Thus, it is usually assumed that the morpheme \textit{than} introduces an operation on the degree-set denoted by the \textit{than}-clause. According to von Stechow (1984), for example, the $\sigma$ operation takes this degree set as an argument and returns its \textbf{maximal} degree, $\text{deg}([\left[ \text{Sam} \right]_{c}, \text{tall}, c])$, as demonstrated in (38b).

\begin{equation}
\text{b. } \left[ \left[ \left[ \text{than Sam is tall} \right]_{PP} \right]_{c} = \sigma(\left[ <0, \text{meter}, \text{tall}>, \text{deg}([\left[ \text{Sam} \right]_{c}, \text{tall}, c]) \right] ) = \text{deg}([\left[ \text{Sam} \right]_{c}, \text{tall}, c])
\end{equation}

Furthermore, it is often assumed that the comparative morpheme \textit{more} introduces an existential quantifier which binds a degree variable $\delta$. For example, the comparative phrase \textit{taller than Sam is} denotes the property that one has iff one is $\delta$ tall, where $\delta$ is a degree which is greater than the maximal degree in the \textit{than} clause. \textit{Dan is taller than Sam is} is true, then, iff there is a degree $\delta$, which is greater than Sam's (maximal) degree and Dan is $\delta$ tall, i.e. iff there is some degree $p$ such that Dan's degree is at least as great as $p$ plus Sam's degree.

\begin{equation}
\text{c. } \left[ \left[ \text{Dan is taller than Sam is} \right]_{c} = 1 \text{ iff:} \\
\exists p \in S_{P}, \text{ s.t. } p > \text{deg}([\left[ \text{Sam} \right]_{c}, \text{tall}, c]): \text{deg}([\left[ \text{Dan} \right]_{c}, \text{tall}, c]) \geq p \text{ iff} \\
\exists p \in S_{P}, \text{ s.t. } p > 0: \text{deg}([\left[ \text{Dan} \right]_{c}, \text{tall}, c]) \geq \text{deg}([\left[ \text{Sam} \right]_{c}, \text{tall}, c]) + p
\end{equation}

Other theories in the relational approach differ in the type of operator they assume to be introduced by the \textit{than} morpheme, as shown in (39a-d).

\begin{equation}
\text{a. } \text{deg}([\left[ \text{Dan} \right]_{c}, \text{tall}, c]) \text{ exceeds some height in CP} \quad \text{(Hellan 1981)} \\
\text{b. } \text{deg}([\left[ \text{Dan} \right]_{c}, \text{tall}, c]) \text{ exceeds the height in CP} \quad \text{(Russell 1905)} \\
\text{c. } \text{deg}([\left[ \text{Dan} \right]_{c}, \text{tall}, c]) \text{ exceeds the maximal height in CP} \quad \text{(von Stechow 1984)} \\
\text{d. } \text{deg}([\left[ \text{Dan} \right]_{c}, \text{tall}, c]) \text{ exceeds every height in CP} \quad \text{(Cresswell 1976; Klein 1982)}
\end{equation}

In addition, in some theories, the comparative morpheme \textit{more} functions as a degree determiner. It denotes a relation between the two sets of degrees denoted by its arguments. \textit{Dan is taller than Sam is} is true iff every degree of length that can be truthfully applied to Sam, can be truthfully applied to Dan too, but there is a degree of length that can be truthfully applied to Dan, but not to Sam. These truth conditions are equivalent to those in (38c-d).

\begin{equation}
\text{e. } \{ p \in S_{\text{tall}} \mid [\left[ \text{Sam is } \delta \text{ meters tall} \right]_{c,g(\delta)} = 1 \} \subset \\
\{ p \in S_{\text{tall}} \mid [\left[ \text{Dan is } \delta \text{ meters tall} \right]_{c,g(\delta)} = 1 \} \quad \text{(Cresswell 1976; Moltmann 1992)}
\end{equation}
Having said this, we can now return to the extent account of the polarity effects.
We said earlier that degrees are usually taken to stand for points on a scale. We have to
further clarify this point. It is often assumed in the gradability literature (cf. Seuren 1978;
Seuren 1984; von Stechow 1984; Kennedy 1999, 2001) that measure functions \( f \) map entities
d to degrees \( f(d) \), but adjectives \( P \) map entities \( d \) to extents, namely to sets of degrees, the set of
degrees smaller or equal to \( f(d) \). This set is characterized by the function \( [\lambda \delta. \ deg(x,P) \geq \delta]_{c,g(x,d)} \). Examples like (40) are usually taken to show that many different degrees may be
truthfully assigned to each individual. Consider for example the individual Dan and the
predicate \( \text{tall} \). We may truthfully assign to Dan any degree above zero up to his maximal
height. This set of degrees is characterized by the function \( [\lambda \delta. \ deg(Dan,\text{tall}) \geq \delta] \). The comparative as tall as Dan seems to behave as a definite description, denoting a unique
degree, the maximal degree in this set, \( [\text{the-max}(\lambda \delta. \ deg(Dan,\text{tall}) \geq \delta)] \).

\[
(40) \quad \text{Sam is as tall as Dan is, and, in fact, taller}
\]

In accordance with that, for each predicate \( P \), \( P \)'s degrees are often taken to be intervals
on a scale (\( \text{extents} \)), i.e. non-empty convex subsets of \( S_P \). Positive predicates \( P \) (like \( \text{tall} \))
are viewed as mapping entities \( d \) into positive extents – intervals which cover the set of degrees that can be truthfully applied to them, \( \{p \in S_P \mid \deg(d,P,c) \geq p\} \) (in any \( c \), the points from \( \deg(d,P,c) \) downward, as stated in (41b)). Negative predicates \( P \) (like \( \text{short} \)) are viewed as mapping entities \( d \) into negative extents – intervals which cover the set of degrees that cannot be truthfully applied to them, \( \{p \in S_P \mid \deg(d,P,c) \leq p\} \) (the points from \( \deg(d,P,c) \) upward, as stated in (41c)).

\[
(41) \quad \text{a. A P-extent, } I, \text{ is a subset of } S_P (I \subseteq S_P), \text{ which is convex } (\forall p_1, p_2 \in I, \forall p_3 \in S_P, p_1 \leq p_2 \leq p_3 : p_3 \in I).
\]

\[
(41) \quad \text{b. Positive predicates map entities into positive extents, } I^+ \subseteq S_P, \text{ namely into convex subsets of } S_P \text{ which are ranging from some point } p_2 \text{ downward (initial intervals): } I^+ = \{I \subseteq S_P \mid \exists p_2 \in S_P: I = \{p_1 \in S_P \mid p_1 \leq p_2\}\}
\]

\[
(41) \quad \text{c. Negative predicates map entities into negative extents, } I^- \subseteq S_P, \text{ namely into convex subsets of } S_P \text{ which are ranging from some point } p_1 \text{ upward (complements of initial intervals): } I^- = \{I \subseteq S_P \mid \exists p_1 \in S_P: I = \{p_2 \in S \mid p_1 \leq p_2\}\}.
\]

In von Stechow's (1984) proposal, the presence of negation in comparative clauses is felt to be anomalous (as in Dan is taller than Sam is not), because the than clause receives the interpretation in (42). The anomaly is explained by the fact that there are infinitely many heights which Sam does not have. The set \( \{p \in S_{\text{tall}} \mid [\text{[Sam is } \delta \text{ meters tall]}]_{c,g(\delta)} = 0\} \) has no maximum and, as a result, the sentence fails to denote a proposition. The infelicity of negative antonyms like \( \text{short} \) in the scope of than in between-predicate comparisons (as in Dan is taller than Sam is short) is accounted for in the same way.

\[
(42) \quad [\text{[than Sam is not]}]_{c} = \sigma (\{p \in S_{\text{tall}} \mid [\text{[Sam is } \delta \text{ meters tall]}]_{c,g(\delta)} = 0\})
\]

\[
= \sigma (\deg([\text{Sam}])_{c,\text{meter},\text{tall},c},<\infty,\text{meter, height }>) \)
\]

Kennedy (1999) considers this anomaly to be part of the general problem in comparing
degrees on different scales. Positive and negative predicates (whether negations or antonyms)
are linked with different sets of degrees (extent-scales), \( I^+_P \) and \( I^-_P \) (cf. principle (41)). Consequently, given a principle like (32), they are linked with non-commensurable extents.
Moreover, the incompatibility of negative predicates with numerical degree modifiers (#Dan is two meters short) is accounted for in the following way. We saw that, on this approach, a comparative relation like more P reduces to the subset relation for extents (cf (39e)). Two entities d₁ and d₂ stand in the relation denoted by as P as iff the extent to which d₁ is P, I₁, is as big as the extent to which d₂ is P, I₂, that is I₂ ⊆ I₁ (Kennedy 2001). An entity d falls under the predicate two meters tall iff d is taller than two meters, i.e. iff the extent to which d is tall in meters (the set of degrees below d's maximal height, {p∈Sₜₐₗ: p ≤ deg(d,meter,tall,c)}) is at least as big as the extent (the set of degrees below) 2 meters ({{p∈Sₜₐₗ | p ≤ <2,meter,height>}}) (43a). For instance, if Dan is 2.10 then Dan falls under the predicate two meters tall because the set of degrees representing the extent 2 meters (the interval [<0,meter,height>,<2,meter,height>]) is a subset of the set of degrees representing Dan's positive extent along the dimension height (the interval [<0,meter,height>,<2.1,meter,height>]).

(43) a. [[Dan is two meters tall]]ₑ = 1 iff
   {p∈Sₜₐₗ | p ≤ <2,meter,height>} ⊆ {p∈Sₜₐₗ | p ≤ deg([[Dan]]ₑ,meter,tall,c)}

b. *[[Dan is two meters short]]ₑ = 1 iff
   {p∈Sₜₐₗ | p ≤ <2,meter,height>} ⊆ {p∈Sₜₐₗ | deg([[Dan]]ₑ,meter,short,c) ≤ p}

No entity d can be two meters short, because this requires that the negative extent of d along the dimension height (the set of degrees above d's maximal height, {p∈Sₜₐₗ: deg(d,meter,tall,c) ≤ p}, would be a subset of another set of degrees below 2 meters (43b). Even if Dan is very short (say, 1 meter tall), the extent 2 meters (the interval [<0,meter,height>,<2,meter,height>]) is not a subset of the set of degrees representing Dan's negative extent along the dimension height (the interval [<1,meter,height>,<∞,meter,height>]). Hence, the statement can never be judged as true (Kennedy 2001). The idea is that a proper initial segment of a scale (a positive extent) can be a subset of another initial interval. However, a proper initial segment of a scale (a positive extent) can never be a subset of a proper complement of an initial interval (a negative extent).

Finally, according to the extent approach, clausal comparatives like Dan is taller than Sam is license negative polarity items (NPIs) like ever (cf. (44)) by virtue of the than clause being a downward entailing context. A linguistic operation O is downward entailing (and hence licenses negative polarity items) iff for every two sets X and Y, such that X⊆Y, O(X) is entailed by O(Y). The extent analysis predicts that clausal complements of comparatives would license NPIs, because for every two degree-sets X and Y, such that X⊆Y, it follows that σ(Y) is at least as big as σ(X). Hence, whenever a degree (e.g. Dan's degree) is bigger than σ(Y), it is also bigger than σ(X) (and whenever Y is a subset of Dan's extent, so is Y).

(44) a. We bought more wine than we could ever drink
b. Sue is taller than any boy is

3.2.4.4 Problems with the 'extent' account and preview of my proposal

Problem 1: Wrong predictions regarding the licensing of numerical degree modifiers

This main problem with the analysis pertaining to the licensing of numerical degree modifiers is that it suffers from wrong predictions.
The problem with the proposal of von Stechow (1984) is that in his formulation of the account it is crucial for this analysis that the predicate's number-set \( S_P \) will have no maximum. However, the idea that scales have no maximum is problematic, given that Rotstein and Winter (2005) and Kennedy and McNally (2005) show that some predicates do have an upper bound (for instance, \textit{full} and \textit{empty}).

An additional problem pertaining to Kennedy's (2001) proposal as well is that the sentences in (43) are predicted to have the same truth conditions as the sentences in (45). This is ok for (45a), but it poses a problem with (45b), which is a felicitous sentence. In fact, it is predicted that \textit{nobody is shorter than two meters} will always be true, which is obviously wrong (Landman 2005).

\begin{align*}
(45) & \quad a. \quad [\text{Dan is taller than two meters}]_c = 1 \text{ iff } \\
& \quad \{p \in S_P \mid p \leq 2, \text{meter, height} > \} \subset \\
& \quad \{p \in S_P \mid \delta \leq \text{deg}([\text{Dan}], \text{meter, tall}, c)\} \\
& \quad b. \quad [\text{Dan is shorter than two meters}]_c = 1 \text{ iff } \\
& \quad \{p \in S_P \mid p \leq 2, \text{meter, height} > \} \subset \\
& \quad \{p \in S_P \mid \text{deg}([\text{Dan}], \text{meter, short}, c) \leq p\}
\end{align*}

For the sentence in (45b) it is desirable to have \textit{two meters} denote a negative extent as well (the set of points above 2, \( \{p \in S_P \mid <2, \text{meter, height} \leq p\} \)), as shown in (46). For instance, if Dan is 1 meter tall, the negative extent \textit{2 meters} (the interval \( <2, \text{meter, height}, <\infty, \text{meter, height} > \)) is a subset of Dan's negative extent along the dimension height (the interval \( <1, \text{meter, height}, <\infty, \text{meter, height} > \)). Maybe it is for this reason that the statement in (45b) is judged true. But there is nothing in the analysis that predicts that the degree modifier \textit{two meters} should be interpreted as a negative extent in (45b), but, crucially, not in (43b).

\begin{align*}
(46) & \quad [\text{Dan is shorter than two meters}]_c = 1 \text{ iff } \\
& \quad \{p \in S_{\text{tall}} \mid <2, \text{meter, height} \leq p\} \subset \\
& \quad \{p \in S_P \mid \text{deg}([\text{Dan}], \text{meter, short}, c) \leq p\}
\end{align*}

In chapters 5, 7 and 9, I discuss two types of degree functions, \textit{ratio} functions and \textit{interval} functions. I propose that negative predicates are linked with interval functions and I show that this proposal correctly predicts the fact that numerical degree modifiers (such as \textit{two meters}) are inappropriate with them, though they can be freely licensed with their comparative forms.

\textit{Problem 2: Problems with the account of cross polar anomalies}

Kennedy's (2001) account of cross-polar anomalies (like \# \textit{Dan is taller than Sam is short}) crucially depends on his account of the general incommensurability phenomenon (cf. principle (32)). However, it was shown in 3.2.3.2 that this account is problematic, in that it makes incorrect predictions and it is not well defined. Thus, it cannot be maintained as it is.

\textit{Problem 3: Problems with the account of the felicity of NPIs in clausal comparatives}

Schwarzschild and Wilkinson (2002) show that \textit{than} clauses are not downward entailing, and, hence, should not be analyzed as such, despite the fact that NPIs are licensed in \textit{than}-clauses. For example, given the assumption that Mary is a girl, \textit{x teased every girl} entails that \textit{x teased Mary}. The direction of entailment is supposed to be reversed if these two expressions are
embedded within a downward entailment context. Indeed, (47a) and (47b) entail (47c) (Every boy who teased every girl also teased Mary; hence, by (47a), he was sent to the headmaster). This entailment pattern shows that every is indeed downward entailing on its first argument. However, (48a) and (48b) do not entail (48c). Thus, the than-clause in a clausal comparative does not seem to form a downward entailment context. The occurrence of negative polarity items in these contexts calls for an alternative explanation (for further discussion see Landman 2005).

(47)  
a. Every boy who teased Mary was sent to the headmaster  
b. Mary is a girl  
c. Every boy who teased every girl was sent to the headmaster

(48)  
a. John is more famous than Mary is  
b. Mary is a girl  
c. John is more famous than every girl is

Problem 4: The account of adjectives as mapping entities to extents

The extent approach encounters another basic problem. The fact that many degrees can truthfully apply to each entity forms the basis of the extent analysis. However this fact seems to stem from the fact that numerals in general allow for 'at least', 'at most' and 'exactly' readings. Numerals in degree constuctions are no exceptions (Schwarzschild and Wilkinson 2002; Landman 2005). The main data from the review in chapter 2 are repeated below (for explanations see 2.1.7.3).

(49)  
a. Parents of 2 children get in free of charge (at least 2)  
b. I can take 4 persons in this car (at most 4)  
c. How many children do you have? 2. (exactly 2)

(50)  
a. If you are as tall as Dan, you can reach the basket (at least as tall as Dan)  
b. If your car is two centimeters taller than Dan's car, it will not pass under that bridge (at least 2 more)  
c. If your car is as tall as Dan's, it will pass under that bridge (at most as tall as Dan's)  
d. If your car is two centimeters taller than Dan's car, it will have no problem passing under that bridge (at most 2 more)  
e. How tall is Dan? He is 2 centimeters taller than Peter (exactly 2 more)

Thus, as in Kadmon's (1987) treatment of the semantics of numerals, the semantics of degree modifiers and of the equative and comparative morphemes should be one that allows at least, at most and exactly interpretations. The pragmatic theory should ultimately determine when and how these relational meanings show up. But if this is the case, then we need not assume that gradable adjectives map entities to extents. This assumption is more complicated than that of the 'one degree' approach and it is counterintuitive. In fact it is highly intuitive to think of entities as possessing a unique degree per predicate (their so called 'maximal' degree).

Problem 5: The interaction of the comparative morpheme with quantifiers

On this approach, an in situ interpretation of quantified noun phrases gives the wrong truth conditions (Schwarzschild and Wilkinson 2002). Take for example the context in (51).
Individual heights in meters:

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>Tony</th>
<th>Uri</th>
<th>Veronica</th>
<th>Sam</th>
<th>John</th>
<th>Dan, Kim, Lee, Mary, Nelly</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.55</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.60</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.65</td>
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<tr>
<td>1.70</td>
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</tr>
<tr>
<td>1.75</td>
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<td></td>
</tr>
<tr>
<td>1.80</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The than CP meaning in (52a) is understood to be the set of heights that can be truthfully applied to everyone in the context, namely \( \{ p \mid p \in (<0,\text{meter},\text{height}>,<1.50,\text{meter}, \text{height}>) \} \). The maximal number in this set is 1.50 (52b). Thus, it is wrongly predicted that Dan is taller than everyone else is would be considered true in this model, because Dan's height is 1.70 meters tall and this degree is greater than some / the / the maximal / every degree that everyone else's height reaches (e.g. 1.50 meters tal). Further mistaken predictions are generated for various other quantifiers. For example (53a-b) are wrongly predicted to be true.

\[
\begin{align*}
(52) & \quad a. \quad [\text{everyone else is (tall)}]_c = \{ p \in S_{\text{all}} \mid \forall d: \deg(d, \text{tall}, c) \geq p \} \\
& \quad = (<0,\text{meter}, \text{height}>,<1.50,\text{meter}, \text{height}>) \\
& b. \quad [\text{than everyone else is (tall)}]_c = \sigma(\{ p \in S_{\text{all}} \mid \forall d: \deg(d, \text{tall}, c) \geq p \}) \\
& \quad = <1.50,\text{meter}, \text{height}> \\
& c. \quad [\text{Dan is taller than everyone else is}]_c = 1 \iff \\
& \quad \exists p > \sigma( (<0,\text{meter}, \text{tall}>,<1.50,\text{meter}, \text{tall}>) ): \deg([\text{Dan}]_c, \text{tall}, c) \geq p \iff \\
& \quad \exists p > <1.50, \text{meter}, \text{height}> : \deg([\text{Dan}]_c, \text{tall}, c) \geq p \iff \\
& \quad \deg([\text{Dan}]_c, \text{tall}, c) > <1.50,\text{meter}, \text{height}> \\
(53) & \quad a. \quad \text{Hellene is taller than exactly 5 of the others are} \\
& b. \quad \text{Hellene is taller than only one of the others is}
\end{align*}
\]

Consequently, the relational-quantificational and measure function theories must assume that these quantifiers are obligatorily raised outside the than clause. That is, statements with quantifiers like every and most in a than clause, like (54a), are interpreted along the lines of (54c), not in (54b) (unless everybody is equally tall). Similarly, if statements with negation in a than clause, like (55a), have any interpretation at all, they are interpreted along the lines in (55c), and not as in (55b).

\[
\begin{align*}
(54) & \quad a. \quad \text{Dan is taller than every boy} \\
& b. \quad \neq \text{Dan is taller than the maximal degree n such that that every boy is n tall} \\
& c. \quad = \text{For every boy, Dan is taller than his maximal height n} \\
(55) & \quad \# \quad \text{Dan is taller than Sam is not} \\
& b. \quad \neq \text{Dan is taller than the maximal degree n such that Sam is not n tall} \\
& c. \quad = \text{It is not the case that Dan is taller than Sam's maximal height n}
\end{align*}
\]

But the fact that the than clause is an extraction island is in discrepancy with the assumption that the quantifier (or connective) may (or in fact must) take wide scope, as (55c) seems to suggest. Both overt and covert Wh-movement is forbidden from a clause under than (56a-b).

\[
\begin{align*}
(56) & \quad a. \quad * \quad [\text{which bird}, \text{are you tall}_{i,pp} \text{ than } [\text{cp } t_t \text{ t was}]]? \\
& b. \quad * \quad \text{She asked who was richer than who else was}
\end{align*}
\]

Thus, it is likely that quantifier raising is forbidden too. In certain cases, quantifier raising would violate syntactic islands (56c-d).
c. Kim is richer than Dan was and than most of his children will ever be.
d. [most of his children], Kim is richer than Dan was and than t, will ever be.

In addition, usually, quantifier raising is not obligatory. Narrow scope readings exist along with wide scope readings. Quantifier rising is even forbidden for certain quantifying expressions, like both and usually, which again is inconsistent with the assumption that in comparatives raising is obligatory. In conclusion, there are good reasons to assume that these quantifiers are interpreted in situ (Larson 1988).

Finally, Schwarzschild and Wilkinson (2002) show that in certain cases, the non-in-situ (wide scope) interpretation is incorrect too. Statements with intensional operators in the than clause, like (57a), are interpreted along the lines of (57c), not (57b). For example, (57a) is intuitively true in a context where Max predicted that most of his students will get between 80 and 90 on the exam and Dan got 96. But the truth conditions in (57b) predict that (57a) will be judged false, as Max made no predictions about the marks of any particular individuals.

\[(57)\]
\[\begin{align*}
a. \text{Dan did better than Max predicted most of his students would do.} \\
b. \neq \text{Most of Max's students are } x \text{ such that Dan did better than the maximal degree } n \text{ such that Max predicted that } x \text{ will do } n \text{ well} \\
c. = \text{Dan did better than the maximal degree } n \text{ such that Max predicted that most of his students will do } n \text{ well}
\end{align*}\]

### 3.2.4.5 A dimensional account of the polarity effects

The dimensional account of the polarity effect is part of the supremum theory (Landman 2005). Basically, this is a measure function type of analysis. According to the supremum theory, degrees are triples \(<n, u, F>\), where \(n\) is a real number, \(u\) is a unit (like meter) and \(F\) is a dimensional measure like height. For a measure \(F\) and unit \(u\), \(S_{u,F}\) is the set of degrees \(\{<n, u, F> | n \in \mathbb{R}\}\).

Each gradable adjective \(P\) is associated with a dimensional tuple \(\text{DIM}_P\) as stated in (58a). For example, \(\text{DIM}_{\text{tall}}\) consists of the dimensional measure \(\text{height}\); an ordering relation for degrees, \(\triangleright_{\text{tall}}\); a supremum operation (which for any subset \(X\) of \(S_{\text{tall}}\) gives the minimum or lower bound of \(X\), \(\cup_{\text{tall}}(X)\) (58c)); a difference relation for degrees, \(\ll_{\text{tall}}\); a measure function \(\text{deg}_{\text{tall}}\) (which maps entities in \(D\) into heights in \(S_{\text{tall}}\)), and a standard for \(\text{tall}\), \(\text{Standard}_{\text{tall}}\), and for the antonym \(\text{short}\), \(\text{Standard}_{\text{tall-ant}}\).

\[(58)\]
\[\begin{align*}
a. \text{DIM}_P &= <F_P, \triangleright_P, \cup_{\text{p}}, \ll_{\text{p}}, \neg_P, \text{deg}_P, \text{Standard}_P, \text{Standard}_{\text{pant}}^> \\
b. \triangleright_p &= \{<a, b> | a = <n, u, F_P> ; b = <m, u, F_P> ; n > m\}. \\
c. \forall X \subseteq S_P, \cup_{\text{p}}(X), \text{ the supremum (minimum or lower bound) of } X \text{ under } \triangleright_p, \\
\text{is the unique element of } S_P \text{ s.t. } \forall x \in X: x \geq_p \cup_{\text{p}}(X) \text{ and } \forall s \in S_P \text{ if } \forall x \in X: x \geq_p s \text{, then } \cup_{\text{p}}(X) \geq s. \\
d. \neg_p(a, b) &= <n-m, u, F_P> \text{ for any } a = <n, u, F_P> \text{ and } b = <m, u, F_P>
\end{align*}\]

The negative antonym of \(P\) is viewed as denoting the converse dimension, \((\text{DIM}_P)^c\). Consider for example, the dimension of short, \((\text{DIM}_{\text{tall}})^c\) (59a). It consists of the measure \(\text{height}\) and degree function \(\text{deg}_{\text{tall}}^c\), which (as in Rullmann 1995) are assumed to be the same as the dimensional measure and degree function of the positive counterpart, e.g. \(\text{height}\) and \(\text{deg}_{\text{tall}}\). The other parts of the dimensional tuple of the negative predicate are assumed to be converse to those of the positive predicate. For example, the ordering relation of \(\text{short}\), \(\triangleright_{\text{short}}\),
is assumed to be the converse of the ordering relation of tall, namely \( \geq_{\text{tall}} \). The difference operation of short, \(-\text{short}\) is assumed to be the converse of that of tall, namely \(-_{\text{tall}}(a,b) = \text{short}(b,a)\), for any a and b. The supremum relation of short is assumed to be the converse of the supremum relation of tall, namely the infimum relation of tall \((\cup_{\text{tall}}^c(X)\) is the maximum or upper bound of \(X\) under \(\geq_{\text{tall}}\) \((59d)\)). Finally, the converse standard of tall is the standard of short and vice versa.

\[
\begin{align*}
(59) \quad & \text{For } P_{\text{ant}}, P's \text{ antonym:} \\
& a. \text{ DIM}_{\text{Pant}} = \langle F_P^c, \geq_P^c, \cup_P^c, -P^c, \text{deg}^c_P, \text{Standard}^c_P, \text{Standard}^c_{\text{Pant}} \rangle \\
& b. \quad F_P^c = F_P; \quad \text{deg}^c_P = \text{deg}_P \\
& c. \quad \geq_P^c = \prec_P^c = \{ \langle b, a \rangle \mid a = \langle n, u, F_P \rangle ; b = \langle m, u, F_P \rangle ; n > m \}. \\
& d. \quad \cup_P^c = \cap_P, \text{ where } \forall X \subseteq S_P, \cap_P(X), \text{ the infimum (maximum or upper bound)} \text{ of } X, \text{ is the unique element of } S_P \text{ s.t. } \forall x \in X: x \leq_P \cap_P(X) \text{ and } \forall s \in S_P \text{ if } \forall x \in X: x \leq_P s \text{, then } \cap_P(X) \leq s. \\
& e. \quad -P(a,b)^c = -P(b,a) \text{ for any } a = \langle n, u, F_P \rangle \text{ and } b = \langle m, u, F_P \rangle \\
& f. \quad \text{Standard}^c_P = \text{Standard}^c_{\text{Pant}}; \quad \text{Standard}^c_{\text{Pant}} = \text{Standard}^c_P
\end{align*}
\]

On this theory, the basic type of number names is that of proper names (three uniquely denotes the number 3). Combined with a certain relation R (at least \(\geq\), at most \(\leq\), or exactly \(=\)), they shift to a predicate type, and denote either numbers (say, \(\lambda\text{n.n} \geq 3\)) or objects (plural individuals \(\lambda x. |x| \geq 3\)). Similarly, the basic type of degree terms like 3 meters tall is that of a proper name, but they can also shift to a predicate type, and denote either a degree set (say, \(\lambda\text{n.} \leq 3. \text{meters, tall}\) or an on entity set \(\lambda x. \text{deg}(x, \text{meter, tall}) = \langle 3, \text{meters, tall}\rangle\), the set of entities which the measure function maps to the degree three meters tall). Similarly, a positive predicate like tall can denote both a degree property \((\lambda\text{tall}^c \geq_{\text{tall}} \text{Standard}_{\text{tall}})\) and an entity property \((\lambda x. \text{deg}(x, \text{tall}) >_{\text{tall}} \text{Standard}_{\text{tall}})\), relations like taller than can apply to both degree pairs \((\lambda\text{tall}^c, \lambda\text{tall}^c, \geq_{\text{tall}} \text{Standard}_{\text{tall}}, \text{Standard}_{\text{tall}})\) and entity pairs \((\lambda y. \lambda x. \text{deg}(x, \text{tall}) >_{\text{tall}} \text{deg}(y, \text{tall}))\) and relations like exactly 3 meters taller than can apply to both degree pairs \((\lambda\text{tall}^c, \lambda\text{tall}^c, \geq_{\text{tall}} \text{Standard}_{\text{tall}}, \text{Standard}_{\text{tall}})\) and entity pairs \((\lambda y. \lambda x. \text{deg}(x, \text{tall}) >_{\text{tall}} \text{deg}(y, \text{tall}) = \langle 3, \text{meters, tall}\rangle)\).

The infelicity of negative predicates with numerical degree modifiers is accounted for by assuming that, in languages like English, predicates like tall are ambiguous between their adjectival interpretations (the entity and degree interpretations that were described above) and a dimensional measure interpretation. In the latter, tall has a different syntactic category MEU (not ADJ) \((60a)\). This interpretation occurs in statements with numerical degree modifiers like two meters tall \((60b-d)\).

\[
\begin{align*}
(60) \quad & a. \quad \text{[[ two meters [tall] MEU ]]c} = F_{\text{tall}} = \text{height} \\
& b. \quad \text{[[ two meters is tall ]]}c = 1 \text{ iff } \langle 2, \text{meter, height} \rangle \geq \text{Standard}_{\text{tall}} \\
& c. \quad \text{[[ two meters [tall] MEU [deg] ]]} = \text{[[} \lambda\text{x. deg}(x, \text{meter, F}_{\text{tall}}) = \langle 2, \text{meter, F}_{\text{tall}} \rangle \text{ ]]}c \\
& d. \quad \text{[[ two meters [tall] MEU [deg] ]]} = \text{[[} \lambda\text{x. deg}(x, \text{meter, F}_{\text{tall}}) = \langle 2, \text{meter, F}_{\text{tall}} \rangle \text{ ]]}c \\
& e. \quad \text{[[ two meters [short] MEU [deg] ]]} = \text{[[} \lambda\text{x. deg}(x, \text{meter, F}_{\text{short}}) = \langle 2, \text{meter, F}_{\text{short}} \rangle \text{ ]]}c = \bot
\end{align*}
\]

Other predicates, including negative predicates like short, are simply assumed not to have the dimensional measure interpretation \((60e)\).

In contrast, in within-predicate comparison statements (like Sam is taller than Dan), predicates have their adjectival interpretation, not their dimensional measure interpretation.
Since all the predicates are associated with the adjectival interpretation (positive and negative alike), statements like two meters shorter are felicitous.

But then again, in between-predicate comparison statements (as The sofa is wider than the table is long), the predicate in the than-clause has a dimensional interpretation. Thus, cross polar comparisons with negative predicates in the than-clause (as in Sam is more tall than she is short) are not felicitous.

Within this analysis, the comparative morpheme more than is assumed to be ambiguous, too. When combined with a predicate (a dimensional tuple) like tall and short, the comparative morpheme meaning depends on the dimension’s polarity. For example, when combined with tall, more tall than denotes the subtraction relation in DIM_{tall} (\lambda \delta_2 \cdot \lambda \delta_1 \cdot \delta_1 <_{\text{tall}} \delta_2), and when combined with short, more short than denotes the subtraction relation in DIM_{short} (namely, the converse relation, \lambda \delta_2 \cdot \lambda \delta_1 \cdot \delta_2 \leftarrow_{\text{all}} \delta_1). Finally, when the comparative morpheme more is combined with a degree term rather than with a predicate (as in more than n meters) it is taken to denote the subtraction relation \lambda \delta_2 \cdot \lambda \delta_1 \cdot \delta_1 \rightarrow \delta_2.

The negative morpheme less than reverses the order of the arguments, so it denotes the converse relations, namely, when combined with tall, less tall than denotes the relation in DIM_{short} (\lambda \delta_2 \cdot \lambda \delta_1 \cdot \delta_2 \leftarrow_{\text{all}} \delta_1) and when combined with short, less short than denotes the relation in DIM_{tall} (\lambda \delta_2 \cdot \lambda \delta_1 \cdot \delta_2 \leftarrow_{\text{all}} \delta_1), etc.

As for the different possible pragmatic enrichments of the interpretation of comparatives, Landman (2005) represents them by assuming that a phonologically null phrase exc or inc enters the semantics. The covert constituent exc has the meaning more than 0 (\lambda P \cdot \lambda \delta \cdot \delta > 0_{\text{up}, \text{FP}}>) and the covert constituent inc has the meaning at least zero (\lambda P \cdot \delta \cdot \delta \geq 0_{\text{up}, \text{FP}}>). The meanings of the comparative morpheme and the covert constituent combine via functional composition. This yields a two place relation (> or \geq). When exc is composed with more it yields the more P than relation (61a). When exc is composed with less P than it yields the less than relation (61b).

(61)  
a. exc \cdot more than \iff \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\lambda \delta \cdot \delta >_P 0_{\text{up}, \text{FP}}) \cdot (\delta_1 < \delta_2) = \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\delta_1 > \delta_2) = \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\delta_1 > \delta_2 + 0_{\text{up}, \text{FP}}) = \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\delta_1 > \delta_2) = \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\delta_1 < \delta_2) = (\text{more than}) 
b. exc \cdot less than \iff \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\delta_1 \leq \delta_2) = \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\delta_1 < \delta_2) = (\text{less than})

When inc is composed with more it yields the at least as P as relation (61c) and when inc is composed with less it yields the at most as P as relation (61d).

(62)  
c. inc \cdot more than \iff \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\delta_1 \geq \delta_2) = \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\delta_1 \leq \delta_2) = (\text{at least as}) 
d. inc \cdot less than \iff \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\delta_1 \leq \delta_2) = (\text{at most as})

An overt numerical degree phrase like n meters can combine with more in just the same way as inc or exc do (62).

(62)  
a. n meters \cdot more than \iff \lambda P \cdot \lambda \delta_2 \cdot \lambda \delta_1 \cdot (\lambda \delta \cdot \delta >_P n_{\text{meters}, \text{FP}}) = (\delta_1 < \delta_2) =

\footnote{Note that the argument order of any expression \varphi can always flip flop by applying a simple type shift: \lambda x_1 \cdot \lambda x_2 \cdot P(x_2)(x_1).}

\footnote{The interpretation of than in phrasal comparatives does not contribute anything to the interpretation of the sentence (than denotes a function whose output is identical to its input). This is not the case in clausal comparatives. The more complex interpretation of than in clausal comparatives is given when these are discussed.}
\[
\lambda P.\lambda \delta_2.\lambda \delta_1.(\delta_1 - \delta_2) >_p \langle n, \text{meters}, F_p \rangle = \\
\lambda \delta_2.\lambda \delta_1.(\delta_1 - \delta_2) >_p \langle \delta_2 + <n, \text{meters}, F_p \rangle \\
(= \text{n meters more than})
\]

b. \text{n meters - less than} \iff \lambda P.\lambda \delta_2.\lambda \delta_1.(\delta_1 <_p \delta_2 - \langle n, \text{meters}, F_p \rangle ) \\
(= \text{n meters less than})

If a relation like the one in (61a) is fed with (i.e., takes as an argument) a degree term like \text{n meters} it yields a predicate (63).

\[
(63) \quad [\text{exc - more than}] \text{n meters} \iff \lambda P.\lambda \delta_2.\lambda \delta_1.(\delta_1 >_p \delta_2)(\text{n meters}) = \\
\lambda \delta_2.\lambda \delta_1.(\delta_1 >_p \langle n, \text{meters}, F_p \rangle ) \\
(= \text{more than n meters})
\]

When this predicate combines with \text{more} (via functional application, again) it yields the two place relation \text{more than n meters more} (64). And so on and so fourth.

\[
(64) \quad a. \quad [\text{exc - more than}] \text{n meters} \cdot \text{more} \iff \\
\lambda P.\lambda \delta_2.\lambda \delta_1.(\lambda \delta. \delta >_p \langle n, \text{meters}, F_p \rangle)(\delta_1 - \delta_2) = \\
\lambda P.\lambda \delta_2.\lambda \delta_1.(\delta_1 - \delta_2) >_p \langle n, \text{meters}, F_p \rangle = \\
\lambda P.\lambda \delta_2.\lambda \delta_1.\langle \delta_2 - \langle n, \text{meters}, F_p \rangle \rangle \\
(= \text{more than n meters more})
\]

b. \quad b. \quad [\text{exc - more than}] \text{n meters} \cdot \text{less} \iff \\
\lambda P.\lambda \delta_2.\lambda \delta_1.(\lambda \delta. \delta >_p \langle n, \text{meters}, F_p \rangle)(\delta_1 - \delta_2) = \\
\lambda P.\lambda \delta_2.\lambda \delta_1.\langle \delta_1 - \delta_2 \rangle >_p \langle n, \text{meters}, F_p \rangle = \\
\lambda P.\lambda \delta_2.\lambda \delta_1.\langle \delta_2 - \langle n, \text{meters}, F_p \rangle \rangle \\
(= \text{more than n meters more})
\]

c. \quad c. \quad [\text{exc - less than}] \text{n meters} \cdot \text{more} \iff \\
\lambda P.\lambda \delta_2.\lambda \delta_1.(\lambda \delta. \delta >_p \langle n, \text{meters}, F_p \rangle)(\delta_1 - \delta_2) = \\
\lambda P.\lambda \delta_2.\lambda \delta_1.\langle \delta_2 - \delta_1 \rangle >_p \langle n, \text{meters}, F_p \rangle = \\
\lambda P.\lambda \delta_2.\lambda \delta_1.\langle \delta_1 + \langle n, \text{meters}, F_p \rangle \rangle \\
(= \text{more than n meters more})
\]

d. \quad d. \quad [\text{exc - less than}] \text{n meters} \cdot \text{less} \iff \\
\lambda P.\lambda \delta_2.\lambda \delta_1.(\lambda \delta. \delta >_p \langle n, \text{meters}, F_p \rangle)(\delta_1 - \delta_2) = \\
\lambda P.\lambda \delta_2.\lambda \delta_1.\langle \delta - \delta_1 \rangle >_p \langle n, \text{meters}, F_p \rangle = \\
\lambda P.\lambda \delta_2.\lambda \delta_1.\langle \delta_2 + \langle n, \text{meters}, F_p \rangle \rangle \\
(= \text{more than n meters more})
\]

Finally, the supremum theory accounts for the felicity of NPIs in clausal comparatives (as in \text{Dan is richer than Sam could ever dream to be}) by assigning them a more complex structure (compared to phrasal comparatives like \text{Dan is richer than Sam}). The \text{than} clause is not assumed to be downward entailing, but it is assumed to contain a supremum operation, and it is assumed that NPIs can be licensed in the scope of this operation (for further discussion see Landman 2005).

Landman (2005) develops the idea that the interpretation of clausal comparatives is mediated by a supremum operation based on observations and insights of Schwarzschild and Wilkinson (2002). Schwarzschild and Wilkinson (2002) propose a theory that links gradable predicates with ordered intervals, but this theory is significantly different from the extent theory. They argue that the interpretation of the \text{than} clause in, for instance, \text{Dan is taller than every boy is} involves an interval which covers the heights of all the boys. The interpretation of the \text{than} clause in \text{Dan did better than Max predicted most of his students would do} involves an interval which covers the marks which Max predicted most boys will have (\{80...90\}). Therefore, they propose to treat a gradable predicate as denoting an interval (the interval that
covers all the heights of all its potential members) and the comparative operator as denoting a relation between intervals. We will now discuss this analysis and then return to the interpretation of comparison statements in the supremum theory.

3.2.4.6 The interval theory (Schwarzschild and Wilkinson 2002)

Let \( I_s \) be the set of intervals of the scale \( S \). Two intervals may stand either in the part of \((\subseteq)\) relation (for instance, the interval \([1.50,1.60]\) is part of the interval \([1.50,1.70]\)) or in the less than \((<)\) relation (the latter case occurs if they are separated by some non-empty interval; for instance, the interval \([1.50,1.60]\) is less than the interval \([1.70,1.75]\)). The part-of and less than relations are partial (partially overlapping intervals stand neither in the part of nor in the less than relation), and they exclude each other (no two intervals stand in both the less than and part of relations).

A set of degrees (an 'interval'), \( I(d,P) \), is assigned to each entity or entity-set \( d \) and predicate \( P \) (65a). For example, if Dan is 1.55m tall and Sam is 1.65m tall, then \( I(Dan,tall) \) is \([1.55,1.55]\) and \( I(Dan and Sam,tall) = [1.55,1.65]) \). An interval \( I \) covers \( d \) relative to \( P \), \( (I(d,P) \subseteq I) \), iff the set of \( d \)'s degrees in \( P \), \( I(d,P) \), is contained in \( I \). E.g. the interval \([1.50,1.60]\) covers Dan relative to \( tall \). An interval \( I \) covers \( P \) iff \( I(P) \subseteq I \) (the interval assigned to \([P]\)), \( I(P) \), is contained in \( I \).

(65)  a. \( \text{deg}(d,P) = I(d,P) \subseteq S_P \)

The system is persistent, as stated in (65b). In addition, any two intervals which cover \( d \) relative to \( P \) must have a common part (the set of \( d \)'s degrees in \( P \), \( I(d,P) \)), as stated in (65c).

b. Persistence:
\[
\forall d,P, \forall I \in I_s \text{ s.t. } I(d,P) \subseteq I: \quad \forall I' \in I_s: \quad I \subseteq I' \rightarrow I(d,P) \subseteq I'
\]
(If \( I \) covers \( d \) relative to \( P \), all the intervals \( I' \) covering \( I \) do so too)

c. Overlap:
\[
\forall P,d, \forall I,J \in I_s \text{ s.t. } (I(d,P) \subseteq I, I(d,P) \subseteq J): \exists K \in I \text{ s.t. } K \subseteq I, K \subseteq J, K \neq 0.
\]
(Any two intervals which cover \( d \) relative to \( P \) have a common part)

A difference operation between intervals, \( I-K \), is defined in (65d). \( I-K \) is the interval below \( I \) and above \( K \), if this interval exists. Otherwise \( I-K \) equals zero. For example, \([1.70,1.75] - [1.50,1.60] = (1.70,1.60) \), and \([1.50,1.60] - [1.70,1.75] = I_0 \).

d. Differentials:
\[
\forall I,K \in I_s, \text{ s.t. } K < I: \quad \forall J \in I_s, K < J < I \leftrightarrow J \subseteq [I-K]
\]
(I–\( K \) is the interval below \( I \) and above \( K \) if this interval exists; otherwise, \( I–K = 0 = I_0 \)).

Finally, a maximalization operation \( \sigma \) is defined for intervals. When it applies to a predicate \( P \), it takes the set of intervals that cover \( P \), \( \lambda I.I(P) \subseteq I \), and returns the largest interval all of whose non-empty subsets cover \( P \).

e. Maximalization:
\[
\sigma(P) \text{ is the largest interval all of whose non-empty subsets cover } P.
\]
A positive predicate like *tall* denotes an interval, $I(\text{tall})$, on its scale (66a). *Sam is tall* is true iff this interval covers Sam (66b).

(66)  
\[ a. \quad \llbracket \text{tall} \rrbracket_c = I(\text{tall}) \subseteq S_{\text{tall}} \]
\[ b. \quad \llbracket \text{Sam is tall} \rrbracket_c = 1 \text{ iff } I(\text{Sam}, \text{tall}) \subseteq I(\text{tall}) \]

_Sam is somewhat tall_ or _Sam has some length_ is true iff Sam is mapped to a non-zero interval (Sam has some length; (66c)). _Sam has no height_ is true iff Sam is mapped to zero (Sam has no length; (66d)). Numerical degree modifiers like _two meters_ are assumed to denote differentials (66e).

(66)  
\[ c. \quad \llbracket \text{Sam is somewhat tall} \rrbracket_c = \llbracket \text{Some} \rrbracket_c(I(\text{Sam}, \text{tall})) = 1 \text{ iff } I(\text{Sam}, \text{tall}) \neq I_0 \] (Sam's degree is not zero).
\[ d. \quad \llbracket \text{Sam has no height} \rrbracket_c = \llbracket \text{No} \rrbracket_c(I(\text{Sam}, \text{tall})) = 1 \text{ iff } I(\text{Sam}, \text{tall}) = I_0 \] (Sam's degree is zero).
\[ e. \quad \llbracket \text{Sam is 2 meters tall} \rrbracket_c = 1 \text{ iff } \llbracket \text{2 meters} \rrbracket_c(I(\text{Sam}, \text{tall}) - I_0) \] (Sam's degree is at least 2 meters above $I_0$).

Finally, let $\delta$ be a degree (interval) variable. An interval $I$ falls under the predicate $\lambda \delta \text{ Sam is } \delta \text{ tall}$ iff $I(\text{Sam}, \text{tall}) \subseteq I$ (66f).

(66)  
\[ f. \quad \llbracket \lambda \delta. \text{ Sam is } \delta \text{ tall} \rrbracket_c = \{ I \in I_{\text{Sam}} \mid I(\text{Sam}, \text{tall}) \subseteq I \}. \]

Comparatives are assumed to denote relations between intervals. A comparative of the form _x is diff more P than y_ is separated into the following ingredients: A comparative morpheme (_more_), a numerical degree predicate (_diff_), a matrix clause (_x is $\delta$ P_) and an embedded clause (_y is $\delta$ P_). The clauses are thought to contain a degree (interval-type) trace. This trace is bound by a lambda operator, so these clauses function as predicates of intervals. When the differential is not realized, _some_ is filled in the _diff_ slot. The standard principle of the interval scale approach (24b) extends to this approach, in that a comparative relation holds of its arguments iff there is a positive difference between their degrees in P. However, the crucial insight of Schwarzschild and Wilkinson (2002) is that the comparative relation is interpreted inside the _than_ clause, not outside of it!

Thus, the comparative _x is diff more P than y is P_ is true iff $[\lambda \delta. \lambda.X. [\lambda y. t' P](\sigma(\lambda.Y. \text{diff}(X-Y))))$, that is, iff the matrix clause ($\lambda \delta. \lambda.X. [\lambda y. t' P]$) is satisfied by the maximal interval $X$ (each non empty part of which is) such that the subordinate-clause ($\lambda \delta. \lambda.Y. \text{diff}(X-Y)$) is satisfied by the maximal interval $Y$ that (each non empty part of which is) is separated from $X$ by the differential _diff_ (iff $I(x, P) \subseteq \sigma(\lambda.X. I(y, P) \subseteq \sigma(\lambda.Y. \text{diff}(X-Y))$); this condition is satisfied iff the interval that is lying below $I(x, P)$ and above $I(y, P)$ is of the size denoted by _diff_ (iff $\llbracket \text{diff} \rrbracket_c(I(x, P) - I(y, P))$).

Consider for example, the following context.

(67)  
<table>
<thead>
<tr>
<th>1.50</th>
<th>1.55</th>
<th>1.60</th>
<th>1.65</th>
<th>1.70</th>
<th>1.75</th>
<th>1.80</th>
<th>1.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sam</td>
<td>Ruth</td>
<td>John</td>
<td></td>
<td>Kim, Lee, Mary, Nelly</td>
<td>Dan,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uri</td>
<td></td>
<td>Sam</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Veronica</td>
<td>Sam</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Dan is 1 meter taller than Sam is tall is true iff Dan's degree in tall (the interval [1.75,1.75]) is contained in the maximal interval X (each non empty part of which is) such that Sam's degree in tall (the interval [1.50,1.50]) is contained in the maximal interval each non empty part of which is separated from X by 1 meter (68). This condition is satisfied iff (I(Dan,tall) − I(Sam,tall)) ∈ [[1 meter]]c (iff the interval below I(Dan,tall) and above I(Sam,tall) is larger than 1 meter). This is not the case in the given context because I(Dan,tall) − I(Sam,tall) = ([1.75,1.75] − [1.50,1.50]) = (1.50, 1.75) ∉ [[(at least) 1 meter]]c. Dan is 1 centimeter taller than Sam is tall is true in the given context because (1.50,1.75) ∈ [[(at least) 1 centimeter]]c.

Landman (2005) observes that this analysis of the comparative uses only the bounds of the intervals that cover the arguments. Hence, it can be formulated in terms of points together with a supremum and infimum operations, rather than in terms of intervals. Landman (2005) formulates such a theory (the supremum theory) and shows that it is equivalent to the interval theory. Thus, it correctly captures the meaning of statements with quantifiers in the than clause. This theory assigns quantified noun phrases the standard semantics (it treats them as generalized quantifiers). Moreover, it more easily extends to less comparatives. The nature of the difference operation in Schwarzschild and Wilkinson (2002) makes it harder to extend the analysis to less comparatives, in the absence of negative intervals. The next section presents a simplified version of this supremum theory, which nonetheless preserves the important ideas of Landman (2005).

3.2.4.7 The interpretation of comparative statements in the supremum theory

Let us call the relation derived from the sequence of numerical degree predicates and comparative morphemes in a comparison sentence (e.g. more than two meters more P than, i.e. the relation "\( \lambda \delta_2 \lambda \delta_1. \delta_1 >_{\text{tall}} \delta_2 +_{\text{tall}} <_{2\text{,meters,\text{height}}} \)" the \( \alpha \)-relation. Based on this relation Landman (2005) defines two notions.

First, the \( \alpha_0 \) relation is defined to be the corresponding relation where the differential is reset to zero (e.g. more than zero meters more P than, i.e. the relation "\( \lambda \delta_2 \lambda \delta_1. \delta_1 >_{\text{tall}} \delta_2 \)" (this is simply the relation \( >_{\text{tall}} \) in the dimensional tuple of tall; so are all the \( \alpha_0 \) relations).

Second, the supremum of \( \alpha \), \( \cup_\alpha \), is defined to be the supremum in the dimensional tuple of P, \( \cup_{\text{p}} \), iff the number of occurrences of negative morphemes (say, less) in \( \alpha \) is even. Let it be the converse operation (the infimum, \( \cap_{\text{p}} \)), otherwise. For instance, the supremum of more
than zero meters more tall is \( \cup_{\text{tall}} \), because there is an even number of occurrences of negative elements in this relation (zero). The supremum of less than zero meters less short is \( \cup_{\text{short}} \) (namely, \( \cap_{\text{tall}} \)), because there is an even number of occurrences of negative elements in this relation (two). The supremum of more than zero meters less short is \( \cap_{\text{short}} \) (namely, \( \cup_{\text{tall}} \)), because there is an odd number of occurrences of negative elements in this relation (two).

The interpretation of a (comparative) predicate has the form "be \( \alpha \) than \( \phi \)", i.e. it is built from an \( \alpha \) relation (i.e. from everything in the matrix clause up to predicate preceding the morpheme than, except for the noun phrase that forms the subject), and the interpretation of the phrase or clause \( \phi \) (the scope of than), which in clausal comparatives contains a supremum operation.

Let us begin with the simpler case of phrasal comparative predicates (predicates of the form is \( \alpha \) \( P \) than \( [\phi]_{NP} \)). As in Schwarzschild and Wilkinson (2002), the \( \alpha \) relation is interpreted inside the scope of than, so the interpretation is assumed to be \( \lambda x.\phi(\lambda y.\alpha(\text{deg}(x,P),\text{deg}(y,P))) \). Consider for example the comparative predicate is taller than every boy. In this example, the predicative argument \( P \) is tall, the \( \alpha \) relation is more \( P \) than (i.e. taller), \( \phi \) is the noun phrase every boy (\( \lambda Q.\text{every}(\text{boy},Q) \)), so the resulting interpretation is as given in (70a).

\[
\text{(70)} \quad \text{Phrasal comparative predicates:}
\begin{align*}
\text{a.} & \quad \text{is taller than every boy} \iff \\
& \quad \lambda x. (\lambda Q.\text{every}(\text{boy},Q))(\lambda y. (\text{taller-than}(\text{deg}(x,\text{tall}), \text{deg}(y,\text{tall})))) = \\
& \quad \lambda x. (\lambda Q.\text{every}(\text{boy},Q))(\lambda y. (\text{deg}(x,\text{tall}) > \text{tall} \text{deg}(y,\text{tall}))) = \\
& \quad \lambda x. \text{every}(\text{boy}, \lambda y. \text{deg}(x,\text{tall}) > \text{tall} \text{deg}(y,\text{tall}))) \\
\text{b.} & \quad \text{Dan is taller than every boy} \iff \\
& \quad \text{every}(\text{boy}, \lambda y. \text{deg}(\text{Dan},\text{tall}) > \text{tall} \text{deg}(y,\text{tall})) \\
\text{c.} & \quad [[\text{Dan is taller than every boy }]]_{c} = 1 \text{ iff } \\
& \quad \forall b \in [[\text{boy}]]_{c} : \text{deg}([[\text{Dan}]]_{c}, \text{tall}, c) > \text{tall} \text{deg}(b, \text{tall}, c)
\end{align*}
\]

The interpretation of the more complex clausal comparative predicates (predicates of the form "is \( \alpha \) than \( \beta \)\( CP \)) is roughly \( \lambda x.\phi_0(\text{deg}(x,P), \cup_0(\lambda \delta.\phi_\delta(\lambda y. \alpha(\delta, \text{deg}(y,P)))))) \). Consider for example the comparative predicate is taller than every boy is. The predicative argument \( P \) is tall, the \( \alpha \) relation is more \( P \) than (i.e. taller), \( \phi \) is the clause every boy is (\( \lambda Q.\text{every}(\text{boy},Q) \)). We apply \( \alpha \) within the than clause, but we also apply \( \phi_0 \) (i.e. the relation taller than) outside, to the supremum of the degree-set resulting by applying taller than to every boy. We get the set of objects whose heights are above the supremum (minimum or lower bound) of the set consisting of all the degrees bigger than all the boys' heights, as illustrated in (71a).

\[
\text{(71)} \quad \text{Clausal comparative predicates:}
\begin{align*}
\text{a.} & \quad \text{is taller than every boy} \iff \\
& \quad \lambda x. (\text{taller-than}(\text{deg}(x,\text{tall})), \\
& \quad \cup_\text{tall}(\lambda \delta. (\lambda Q.\text{every}(\text{boy}-\text{is}-\text{Q}))(\lambda y. (\text{taller-than}(\delta, \text{deg}(y,\text{tall})))))) = \\
& \quad \lambda x. \text{deg}(x,\text{tall}) > \text{tall} \cup_\text{tall}(\lambda \delta. \text{every}(\text{boy}, \lambda y. \delta > \text{tall} \text{deg}(y,\text{tall}))) \\
\text{b.} & \quad \text{Dan is taller than every boy} \iff \\
& \quad \text{deg}(\text{Dan},\text{tall}) > \text{tall} \cup_\text{tall}(\lambda \delta. \text{every}(\text{boy}, \lambda y. \delta > \text{tall} \text{deg}(y,\text{tall}))) \\
\text{c.} & \quad [[\text{Dan is taller than } [[\text{every boy is }]]_{CP }]]_{c} = 1 \text{ iff } \\
& \quad [[\text{deg}(\text{Dan},\text{tall}) > \text{tall} \cup_\text{tall}(\lambda \delta. \text{every}(\text{boy}, \lambda y. \delta > \text{tall} \text{deg}(y,\text{tall}))))]_{c} = 1 \text{ iff }
\end{align*}
\]
Dan's height is above the supremum (minimum or lower bound) of the set of degrees bigger than all the boy's height iff
\[ \forall b \in [[\text{boy}]]_c : \deg([[\text{Dan}]]_c, \text{tall}, c) > \text{tall} \deg(b, \text{tall}, c) \]
Dan's height is above every boy's height

For instance, if the tallest boy is two meters tall, the supremum (minimum or lower bound) of the set of degrees bigger than all the boys' heights is 2 (the lower bound of the set of degrees bigger than 2, \( \{ \delta \in \text{S}_{\text{tall}} : \forall b \in [[\text{boy}]]_c : \delta > \text{tall} \deg(b, \text{tall}, c) \} \)), is 2. So Dan is required to be taller than two meters.

(72) and (73) are examples of a clausal comparative predicate with a negative predicative argument, short, and with a complex \( \alpha \) relation, more than two meters more, respectively.

### (72) Clausal comparative predicates with a negative predicative argument:

a. \([[[\text{Dan} \text{ is shorter than every boy is }]]]_c = 1 \text{ iff} \]
de\( \deg([[\text{Dan}]]_c, \text{short}, c) > \text{short} \cup \text{short} (\{ \delta \in \text{S}_{\text{short}} : \forall b \in [[\text{boy}]]_c : \delta > \text{short} \deg(b, \text{short}, c) \}) \)
Dan's height is below the infimum (maximum or upper bound) of the set of degrees smaller than every boy's height iff
\[ \forall b \in [[\text{boy}]]_c : \deg([[\text{Dan}]]_c, \text{tall}, c) < \text{tall} \deg(b, \text{tall}, c) \]
Dan's height is below every boy's height

For instance, if the shortest boy is one meter tall, the infimum (maximum or upper bound) of the set of degrees smaller than all the boys' heights is 1 (the lower bound of the set of degrees bigger than 2, \( \{ \delta \in \text{S}_{\text{tall}} : \delta < \text{tall} < 1, \text{meter, height} \} \)), is 1. So Dan is required to be shorter than one meter.

### (73) Clausal comparative predicates with a complex \( \alpha \) relation:

a. \([[[\text{Dan} \text{ is more than two meters taller than every boy is }]]]_c = 1 \text{ iff} \]
de\( \deg([[\text{Dan}]]_c, \text{tall}, c) > \text{tall} \cup \text{tall} (\{ \delta \in \text{S}_{\text{tall}} : \forall b \in [[\text{boy}]]_c : \delta > \text{tall} \deg(b, \text{tall}, c) + <2, \text{meters, height}> \}) \)
Dan's height is above the supremum (minimum or lower bound) of the set of degrees bigger than every boy's height plus two meters iff
\[ \forall b \in [[\text{boy}]]_c : \deg([[\text{Dan}]]_c, \text{tall}, c) > \text{tall} \deg(b, \text{tall}, c) + <2, \text{meters, height}> \]
Dan's height is above every boy's height plus two meters

### 3.2.4.8 Problems with the dimensional account and preview of my proposal

**Problem 1: The dimensional account is highly stipulative**

The dimensional account for the polarity effects is fully compositional and it produces the right predictions. However, it is highly stipulative. For example, the dimensional measure height is assumed to be part of the interpretation (the dimensional tuple) of both the predicate tall and the predicate short, and it has to be stipulated that, nonetheless, tall has a dimension-interpretation (it can denote its dimension height), but short does not. Thus, the theory may be descriptively adequate, but it is hardly explanatory. In addition, nothing in the theory tells us what a dimension might be.
In chapter 5, I propose an answer for the latter question, according to which a dimension is no more than a predicate in the language, whose degree function constrains the degree function of the predicate it is a dimension of. On this analysis, positive and negative antonyms do not share the dimension. However, I propose how the degree function of the negative predicate is built from the degree function of the positive predicate. I show that the properties of the resulting function correctly predict the polarity effects. Thus, this proposal is more detailed (it tells you what a dimension is), and more explanatory (it does not rely on a stipulation of a lack of a dimensional interpretation in negative predicates).

**Problem 2: The ordering relation and difference operation in the dimensional tuple can be dispensed with**

The analysis in Rullmann (1995) and Landman (2005) assumes that the degree functions of positive predicates and their negative antonyms are identical. As a consequence, a different ordering relation has to be associated with the scale of each predicate, which is, >, for positive predicates and its converse, <, for negative ones. In such a theory, the comparative morpheme *more* needs to be ambiguous between its use with positive and negative predicates (as in *more tall* and *more short*), as its interpretation has to depend on the predicate's polarity (it is either the difference operation or its converse). In addition, when used with numerical degree terms (as in *more than two meters*), it denotes the difference operation.

In chapters 5-9, I propose the *reversed theory*, according to which the degree function of negative predicates is inversely related to that of their positive antonyms. I show that, for this purpose, it is not necessary to assume that degrees of negative predicates are negative extents (cf. von Stechow 1984; Kennedy 1999; Winter 2001). Rather, the interpretation of, e.g., *short* involves two stages. First, the degree function of its positive antonym *tall* is accessed. Second, it is replaced by a function which is inversely related to it. In the reversed theory, we need not associate predicates with (predicate specific) ordering relations at all. They are all naturally ordered by the bigger relation for the reals, >. Thus, the interpretation of a predicate becomes more economical. Consequently, the comparative morphemes (e.g. *more* and *less*) are not ambiguous. They simply denote the difference operation ($\lambda \delta_2 \lambda \delta_1, \delta_1 - \delta_2$) and its converse ($\lambda \delta_2 \lambda \delta_1, \delta_2 - \delta_1$), respectively, as they should. This simplifies the theory also in that no difference relation needs to be associated with the dimensional interpretation of a predicate (because even when combined with a predicate, *more* and *less* do not denote the dimensional difference operations). Thus, the interpretation of a predicate becomes even more economical.

**Problem 3: The supremum operation in the dimensional tuple can be dispensed with**

It is highly reasonable to assume that the interpretation of *than* is dependent on the interpretation of the comparative relation that occurs earlier in the sentence (the occurrences of *more* and *less* in relations like *more than two meters less tall than* etc.), given that *than* and *more* or *less* usually co-occur (intuitively, they are strongly connected). However, it is less intuitive to think that the interpretation of *than* is dependent on the polarity of the predicate in the comparative statement. In chapters 5-9, I present a simplified theory in which the interpretation of predicates is more economical in that it does not include a supremum operation. The supremum operation in the interpretation of clausal comparatives is fixed solely based on the number of occurrences of negative morphemes in the $\alpha$ relation (excluding the predicate). By virtue of the fact that negative predicates denote reversed functions, this theory makes the same predictions as the original supremum theory. To sum up, by combining it with a new account for the polarity effect, the supremum theory becomes simpler and more explanatory.
3.3 Typicality in linguistic theories

We see that, generally, semantic theories assume that in gradable predicates (unlike non-gradable ones), entities possess the properties to different extents (or degrees). Entities are judged to be instances of gradable predicates iff the extent to which they satisfy the relevant gradable property (the ordering dimension) is within the norm, that is, iff they reach the standard for membership under that predicate, standard(P,c) (75a). The qualities or attributes that permit grading are denoted by comparative relations like 'taller' or "less tall" (75b) (von Stechow 1984; Kennedy 1999).

(74) a. A standard-based membership criterion:

\[
[[P]]_c = \{ d \in D \mid \deg(d,P,c) \geq \text{standard}(P,c) \}
\]

b. A degree-based ordering criterion:

\[
[[\text{more } P]]_c = \{ <d_1,d_2> \in D^2 \mid \deg(d_1,P,c) \geq \deg(d_2,P,c) \}
\]

Semantic theories assume that the interpretation of non-gradable predicates, including nouns, does not involve any mapping of individuals to degrees along ordering dimensions. This accounts for the incompatibility of nouns with the operations denoted by comparative relations in within-predicate comparisons (like "more P than"), superlatives, and other degree modifiers. Nor are nouns associated with a comparison class, according to these theories. This ought to explain the fact that failures of intersective inferences are not easily observed in nouns, compared to adjectives (2.1).

However, the facts in 2.2 show that nouns behave very much like our semantics for gradable predicates would expect (they map entities to degrees, they are linked with ordering dimensions, they exhibit non-intersective effects, etc.) The typicality (graded structure) effects in nouns are robust and pervasive. Citing Murphy (2002):

As a general observation, one can say that whenever a task requires someone to relate an item to a concept, the item's typicality influences performance (Murphy 2002: 24).

Furthermore, nouns combine with more-of comparatives, and with degree modifiers like pretty much, and they are licensed in between-predicate comparisons (comparisons of the form more P than Q) more freely than adjectives are (2.1.5.6). Thus, by assuming that nouns are non-gradable, linguists pay a heavy price in terms of the dissociation between the semantics they assume for nouns and many other things that we know about them. We need to give up the assumption that nouns are non-gradable. Their infelicity in, for instance, within-predicate comparisons, must have reasons other than lack of gradable meaning.

Kamp and Partee's (1995) influential analysis, the supermodel theory, is an exception in the semantic landscape, in that it assigns nouns a gradable structure. Given its central status in semantics, I will dedicate this section to showing that nonetheless, it fails to correctly predict the typicality effects.

3.3.1 Background: Multi-valued semantics

Kamp and Partee's main innovation within the analysis of typicality, is in the use of a logic with three truth values and the technique of Supervaluations (cf. 3.1), as opposed to the standard use of a logic with multiple truth values (such as fuzzy logics) in the truth conditional analysis of typicality in artificial intelligence, cognitive psychology, and linguistics (Zadeh 1965; Lakoff 1973; Osherson and Smith 1981; Lakoff 1987). A critical review of fuzzy semantics and a
variety of problems in modeling natural language using fuzzy semantics can be found in Osherson and Smith (1981) and Kamp and Partee (1995).

### 3.3.1.1 Fuzzy models

In classical logics, a proposition may take as a truth value either 0 or 1. In fuzzy logics, a proposition may take as a truth value any number in the real interval \([0,1]\). For example, such a model can assume the following facts:

(75) 
\begin{align*}
  a. & \text{ The truth value of the proposition } a \text{ robin is a bird } = 1 \\
  b. & \text{ The truth value of the proposition } a \text{ goose is a bird } = 0.7 \\
  c. & \text{ The truth value of the proposition } an \text{ ostrich is a bird } = 0.5 \\
  d. & \text{ The truth value of the proposition } a \text{ butterfly is a bird } = 0.3 \\
  e. & \text{ The truth value of the proposition } a \text{ cow is a bird } = 0.1
\end{align*}

These values indicate the typicality degrees of the individuals or kinds denoted by the subjects in the predicate *bird*. More precisely, in such models, predicates are not associated with sets as denotations. Rather, for every predicate \(P\), a characteristic function, \(\text{deg}_m(., P)\), assigns to each entity \(d\) in \(D\), a value in the real interval \([0,1]\), its degree of membership in \(P\). Moreover, each predicate is associated with a prototype \(p\), i.e., the best member possible. Finally, a degree function \(\text{deg}_P\) (a distance metric) associates pairs of entities with values in the real interval \([0,1]\). If, for example, \(r\) is a robin, \(b\) a blue jay and \(o\) an ostrich, then \(\text{deg}_P(r, b) < \text{deg}_P(r, o)\) stands for the fact that \(r\) is less different, or more similar, to \(b\) than to \(o\).

The typicality of an entity \(d\) in \(P\) is represented as the difference, or distance, of \(d\) from the prototype \(p\) of \(P\): 
\[
\text{deg}_P(d, p) = \text{deg}_m(d, P) - \text{deg}_m(p, P)
\]

Typicality degrees are assumed to roughly correspond to objective probabilities of membership in the category. We can see that in the rules that predict the typicality degrees in complex predicates. There are three composition rules for \(\text{deg}_m\):

(77) 
\begin{align*}
  a. & \text{ The complement rule for } \neg: \quad \text{deg}_m(d, \neg P) = 1 - \text{deg}_m(d, P) \\
  b. & \text{ The minimal-degree rule for } \land: \quad \text{deg}_m(d, P \land Q) = \text{Min}(\text{deg}_m(d, P), \text{deg}_m(d, Q)) \\
  c. & \text{ The maximal-degree rule for } \lor: \quad \text{deg}_m(d, P \lor Q) = \text{Max}(\text{deg}_m(d, P), \text{deg}_m(d, Q))
\end{align*}

Consider, for instance, the complement rule for negated predicates in (78a). The degree of a goose in *not-a-bird* is assumed to be the complement of its degree in *bird* (e.g., 1 - 0.7). This rule is directly inspired by the idea that the probability that \(p\) is the complement of the probably that *not-p*. Similarly, the minimal-degree rule for conjunctions in (78b) states that an item’s degree in a modified noun like *brown apple* is the minimal degree among the constituents, *brown* and *apple*. This rule, and other versions of the rule for conjunctions and modified nouns in fuzzy models, are directly inspired by the fact that the (objective) probability that \(p \land q\) cannot exceed the probability that just \(p\), or just \(q\).
3.3.1.2 Problems with fuzzy models

Osherson and Smith (1981) show a variety of shortcomings of fuzzy models. Following them, Kamp and Partee (1995) argue at length against such models. The main problem is that they generate wrong predictions.

Consider, for example, the minimal-degree rule in (78b). This rule predicts that the typicality degree of, e.g., brown apples, cannot be bigger in brown apple than in apple. Hence, this rule fails to predict the empirically well-established conjunction effect, to which (Smith et al. 1988) or fallacy (Tversky et al. 1983) alluded in 2.2 – the finding that, according to speakers' intuitive judgments, both the typicality degree (Smith et al. 1988), and the likelihood of category membership (Tversky et al. 1983), of brown-apples, is bigger in brown apple than in apple.

The minimal-degree rule is most problematic when it comes to contradictory and tautological predicates. Intuitively, the degree of all entities in \( P \land \neg P \) and \( P \lor \neg P \) ought to be 0 and 1, respectively. But fuzzy models fail to predict this. For example, if a goose is a bird to degree 0.7, then according to the complement rule, a goose is not a bird to degree 0.3. Given this, the minimal degree rule predicts that a goose is a bird and not a bird to degree 0.3, rather than to degree 0.

Another problem has to do with the fact that the degree function in these models is total, while knowledge about typicality is often partial. For example, if one bird sings and the other flies, which one is more typical? We cannot tell out of context. This problem highlights the need for more context dependency in the representation of typicality. Kamp and Partee (1995) argue at length for the importance of this aspect (yet, we will see that their proposal is also insufficient in this respect).

I would like to point out a further problem with the complement rule, which usually goes unnoticed. It is indeed true that the typicality orderings of negated predicates are essentially the reverse of the orderings of the predicates that are being negated (see, for instance, the findings reported in Smith et al. 1988). However, exceptions to this rule are quite common (Giora, Balaban, Fine and Alkabets 2005). Why? Because negated predicates are often contextually restricted. For example, the set of non-birds is frequently assumed to only consist of animals. In such contexts, non-animals are intuitively assigned low typicality degrees both in the predicate bird and in the negated predicate non-bird (rather than a low degree in bird and a high degree in non-bird, as predicted by the complement rule). This judgment is not captured because the relevant contextual factors are not represented.

The features of the similarity function \( \text{deg}_p \) were criticized by Tversky (1977) and Tversky and Gati (1978) on empirical grounds. For example, these researchers show that human judgments of similarity are not symmetric. For instance, speakers often judge the sentences in (79) to be true. Less typical or salient category instances are judged to be more similar to the prototypical items than vice versa. Following this criticism, other models which use a geometric distance (or similarity) function were modified to correct these assumptions (Nosofsky 1991).

(78)  

a. Mexico is more similar to USA than the USA is similar to Mexico  
b. An ellipse is more similar to a circle than a circle is similar to an ellipse.

To conclude, we see that multiple truth values or probability degrees as the means of defining typicality degrees are problematic in many respects. An alternative to the Fuzzy models is the Supermodel Theory (Kamp and Partee 1995). This analysis uses the same types of mechanisms, namely – a membership degree function \( \text{deg}_m \), a prototype \( p \), and a typicality degree function \( \text{deg}_p \). However, it also differs in two crucial respects. First, it replaces fuzzy
logics with three valued logics. Second, the typicality degrees are not always coupled with the membership degrees. With these two differences, the analysis is claimed to be significantly improved. However, while indeed improved in some respects, we will see that this analysis is still highly problematic in other respects.

3.3.2 The supermodel theory: Kamp and Partee (1995)

A supermodel, $M_c$, is a simplified vagueness-model, in that it consists of but one partial context $c$ (the ground context) and a set of total contexts $T_c$, as discussed in 3.2.1, and as demonstrated in Figure 3 (repeated in Figure 6 below). The ground context is thought to represent the 'real' context (the world knowledge and the information that was accepted as true by the discourse participants in the context). The total contexts are thought to represent different standards for membership in the denotations of the vague predicates (Lewis 1979). For example, in some of them only very tall things are considered tall, in others more things are considered tall, etc.

We saw that a well established tradition views such vagueness models as useful for the representation of gradability (Lewis 1970; Kamp 1975; Fine 1975; McConnell Ginnet 1973; Klein 1980; Landman 1991, etc.) The gist of theories like Kamp (1975) and Fine (1975) is that a comparative statement like Dan is taller than Sam is considered true in $c$ (or, in fact, in $M_c$) iff Dan is tall relative to more standards (total contexts in $T_{c1}$), compared to Sam (80).

$$[[\text{Dan is taller than Sam}]]_{M_c} = 1 \text{ iff:}$$
$$\{t \in T_{c1} | [[\text{Sam}]]^t_1, \in [[\text{Tall}]]^t_1 \} \subset \{t \in T_{c1} | [[\text{Dan}]]^t_1, \in [[\text{tall}]]^t_1 \}$$

Figure 6: The context structure in a vagueness model with ground context $c_1$

Given the basic assumption in (80), in the supermodel theory, each predicate $P$ maps each entity $d$ to a membership degree, $\deg_m(d,P)$, representing the size, or measure, of the set of total contexts in which $d$ falls under the predicate. For example, the membership degree of an entity $d$ in chair, $\deg_m(d,\text{chair})$, reflects the size, or measure, of the set of total contexts in which $d$ is a chair, $m(\{t \in T_{c1} : d \in [[\text{chair}]]^t_1 \})$. If $d$ is a chair in all the total possibilities, its membership degree is 1; if $d$ is a chair in no total possibility, its degree is 0; if $d$ (say – a stool, or another borderline case) is a chair in a third of the total contexts, its degree is $1/3$, etc., as stated in (81) and demonstrated in Figure 7.

$$[[\text{Dan is taller than Sam}]]_{M_c} = 1 \text{ iff:}$$
$$\{t \in T_{c1} | [[\text{Sam}]]^t_1, \in [[\text{Tall}]]^t_1 \} \subset \{t \in T_{c1} | [[\text{Dan}]]^t_1, \in [[\text{tall}]]^t_1 \}$$

Figure 6: The context structure in a vagueness model with ground context $c_1$

a. $m$ is a measure function from sets of total models to real numbers between 0 and 1. That is, $m$ is a function which satisfies the following constraints (Kamp and Partee 1995, p. 153): $m(T) = 1$; $m(\{\}) = 0$ and:

$$\forall T_1, T_2 \subseteq T, \text{ s.t. } T_1 \subset T_2; m(T_2) = m(T_1) + m(T_2 - T_1), \text{ etc.}$$

b. Each predicate $P$ is associated with a function, $\deg_m$, such that for each entity $d$, the membership-degree of $d$ in $P$, $\deg_m(d,P)$, is given by the measure $m$ of the set of total models in which $d$ is $P$: $\deg_m(d,P) = m(\{t \in T : d \in [P]^t_1\})$
Moreover, in the supermodel theory, each predicate P maps each entity d to its *typicality degree* in P, deg_p(d,P). Typicality is taken to reflect similarity to the predicate’s prototype, which is defined to be the best possible P. Examples are given in Figure 8.

![Figure 7: A Supermodel](image)

**Figure 7: A Supermodel**

How is the typicality function, deg_p, determined? Kamp and Partee (1995) aimed at reconstructing it from the notion of membership degree, at least where typicality and membership seem to them to be related, namely in some vague nouns. The resulting taxonomy of predicates is given in (82), and is demonstrated in Table 3.

Figure 8: Examples of possible entity prototypes for *bird*, *male* and *nurse*.

<table>
<thead>
<tr>
<th>+/– Vague</th>
<th>+/– Prototype</th>
<th>+/- Typicality is coupled with membership, deg_m ≡ deg_p:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tall, wide, heavy, not red</td>
<td>Adolescent, tall tree</td>
<td>In vague predicates like <em>chair</em>, degree of membership is a matter of prototype resemblance (Kamp and Partee: 172). The same entity ordering is determined by the membership- and the typicality-function: deg_m ≡ deg_p. Thus, the prototype determines the denotation. Conversely, in non-vague nouns like <em>bird</em> (and in some vague concepts too), membership is viewed as independent of typicality: deg_m ≠ deg_p.</td>
</tr>
<tr>
<td>Even, odd, inanimate, non-bird</td>
<td>red, shy, chair</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Predicate types in Kamp and Partee’s (1995) analysis

Finally, Kamp and Partee’s (1995) membership degree function, deg_m(d,P), cannot represent the graded-membership effects in modified nouns. For example, intuitively, a brown
apple is regarded as more likely (as well as more typical) a brown apple than an apple. That is, for some entities d, the values of degm (and of degp) have to be bigger in brown apple than in apple. But brown apples are apples, so for any entity d, the set of total contexts in which d is a brown apple (\{t \in T_c; d \in [[brown apple]]^+\}) is always a subset of the set in which d is an apple (\{t \in T_c; d \in [[apple]]^+\}). So, degm(d,brown apple) is always smaller than degm(d,apple).

Consequently, Kamp and Partee associate modified nouns like brown apple with a modified membership-function, degm(d,brown/apple). The values of this function are basically given by the membership function of brown. The set of brown degrees which are assigned to the apples are stretched (by a linear transformation which is formally defined in (83)), so as to range from 0 to 1. The result stands for the degrees of apples in brown apple.

(82) \[ \text{Let } \delta_{\text{min}} \text{ and } \delta_{\text{max}} \text{ be the minimal and maximal brown degrees in } [[\text{apple}]]_c: \]
\[ \text{deg}_m(d,\text{brown/apple}) = (\text{deg}_m(d,\text{brown}) - \delta_{\text{min}}) / (\delta_{\text{max}} - \delta_{\text{min}}) \]

For example, a brown apple d may have degree 0.9 in brown; the minimal brown degree existing among the apples may be 0 (because some apples are not brown at all); the maximal brown degree existing among the apples may be 0.95 (assuming that no apple is maximally brown). Then, degm(d,brown/apple) = (0.9 – 0) / (0.95 – 0) = 0.974. The value 0.974 is indeed greater than d’s degree in brown, 0.9, and possibly also than d’s degree in apple, as desired. Principle (83) seems to produce correct membership degrees (and correct typicality degrees, in modified nouns for which membership and typicality are coupled).

There is no doubt that the supermodel theory is better suited to the representation of natural language than, for instance, fuzzy models. For example, intuitively, the degree of all entities in a contradictory predicate of the form P\(\land\)¬P and a tautological predicate of the form P\(\lor\)¬P ought to be 0 and 1, respectively. But fuzzy models fail to predict this. For example, if a certain stool is a chair to degree 0.7, then according to fuzzy models, it is not a chair to degree 0.3 (by the complement rule), and it is a chair and not a chair to degree 0.3 (by the minimal rule), rather than to degree 0 (Osherson and Smith 1981). Conversely, the supermodel theory derives the correct degrees. This intuitive trait is achieved by virtue of the fact that, in any total context, no entity falls under P\(\land\)¬P and all entities fall under P\(\lor\)¬P. Thus, even if, say, a certain stool is a chair to degree 0.7 and not a chair to degree 0.3 (due to being regarded as a chair in 70% of the total contexts, and not a chair in the rest), this stool is a chair and not a chair to degree 0, and a chair or not a chair to degree 1.

Despite this advantage, we will now see that the supermodel theory has serious problems.

3.3.3. Problems with the supermodel theory

3.3.3.1 Membership and typicality degrees of denotation members

Is the vagueness-based principle (80) (the basis for the definition of the membership degree) a good description of the connections between gradability and vagueness? In 3.2.1 it is shown that the most basic problem with this principle is that it can only apply to gap members, for otherwise it will deliver inadequate truth conditions. Why? The entities which are known to be tall in c are all tall in the very same set of standards (in all the total extensions of c). Thus, they are wrongly predicted to be all equally tall, tough intuitively, two entities can be tall without being equally tall.

The lack of an adequate account of the connections between vagueness and gradability is a general problem of semantic theories of gradability. Its manifestation in the supermodel theory is that the membership function, degm, assigns all the positive denotation members of vague predicates (say – all the known chairs) the maximal degree of membership, 1.
In vague nouns like *chair*, for which it is assumed that typicality and membership are coupled, this has the problematic consequence that also the typicality function, \( \deg_p \), assigns all the positive denotation members the maximal typicality degree, 1, though intuitively, some known chairs are not perfectly typical. Thus, we do not have an account of the typicality effect in these vague nouns, unless we postulate that their denotations are effectively empty in the ground context \( c \) (that they do not fully represent our knowledge about membership).

The situation is even more problematic in the so-called non-vague nouns. For example, the *bird* denotations are assumed to be completely specified in \( c \) (not to vary across different total contexts). This is the standard way by which semanticists represent the fact that predicates like *bird* are not – or are much less – vague than predicates like *chair* or *tall*. However, this is also the reason for which the membership function cannot indicate typicality in non-vague predicates. Given that they are known to be birds in every \( t \) in \( T_c \), the membership degree of atypical examples like ostriches and penguins in *bird* is 1. And for non-birds, be they bats or cows, since they are members in \([\text{[bird]}})\) \( c \) (they are known not to be birds in every \( t \) in \( T_c \)), their membership degree in *bird* is 0. Intermediate typicality degrees in non-vague nouns cannot be based on \( \deg_m \) \( \neq \deg_p \).

Nevertheless, first, the separation of \( \deg_m \) and \( \deg_p \) forces us into an inelegant theory, which stipulates as primitives two unconnected sets of values for \( \deg_m \) and \( \deg_p \). Second, it fails to account for the empirical findings that show that typicality and membership are tightly coupled even in predicates like *bird* (cf. 2.2). And third, no mechanism for calculating typicality degrees (other than the membership degree) is given. Thus, we do not have an account for the typicality effects in non-vague nouns, unless we give up the assumption that their interpretation is non-vague already in \( c \) (cf. (82a)), and postulate that their denotations are effectively empty in \( c \) (and that typicality and membership are coupled).

### 3.3.3.2 The sub-type and conjunction effects

Even if we assume that the denotations of all the predicates are empty in the ground context \( c_1 \), the degree functions would still fail to correctly predict the graded-membership (or typicality) judgments.

Consider, for example, modified nouns (Kamp and Partee’s 1995 account of the so-called conjunction fallacy). Brown apples, for instance, are allowed to have greater degrees in *brown apple* than in *brown* or in *apple*, as desired, but they are ordered only by how brown they are. This yields incorrect degrees. For example, intuitively, an apple of an unusual shape or size, which is therefore assigned, say, degree 0.2 in *apple*, even if maximally brown (of degree 1 in *brown*), is considered a bad example of a brown apple, and not the best example (a brown apple of degree 1), contrary to what principle (83) of the supermodel theory predicts in that case (as demonstrated in (84)). Thus, the degrees which are assigned by the modified membership-function are incorrect (see also Smith et al. 1988).

\[
(83) \quad \deg_m(d,\text{brown/apple}) = \frac{\deg_m(d,\text{brown}) - 0}{1 - 0} = 1 / 1 = 1
\]

---

8 Kamp and Partee's proposal belongs to a family of *dual theories*, namely theories that postulate separate mechanisms to determine typicality and membership (Putnam 1975; Osherson and Smith 1981; Landau 1982; Armstrong et al. 1984; Smith et al. 1984; Keil 1994; Laurence & Margolis 2003 present a theory which assumes even more than two mechanisms). These theories view the mechanism assumed by the prototype theory as an *identification procedure* - a tool that helps identifying denotation members quickly, but not always accurately. With this tool the variety of typicality effects are accounted for. Another mechanism, a core structure, is being used to accurately to determine membership in the denotation (Armstrong et al. 1984). Like the super-model theory, these theories are less elegant than non-dual ones, and at the same time they are less explanatory (they fail to derive the tight connections that exist between typicality and membership judgments).
Finally, for similar reasons, Kamp and Partee’s (1995) membership degree function, \( \text{deg}_m(d,P) \), cannot represent the graded-membership effects in basic lexical items. We saw that manifestations of the so-called conjunction fallacy occur in basic lexical items, too. We called these manifestations sub-type effects. For example, intuitively, an ostrich is regarded as more likely (as well as more typical) an ostrich than a bird. That is, for some entities \( d \), the values of \( \text{deg}_m \) (and of \( \text{deg}_p \)) have to be bigger in ostrich than in bird. But ostriches are (usually) known to be birds already in the ground context, so for any entity \( d \), the set of total contexts in which \( d \) is an ostrich \( (\{t \in T_c : d \in [[\text{ostrich}]]^+ \}) \) is always a subset of the set in which \( d \) is a bird \( (\{t \in T_c : d \in [[\text{bird}]]^+ \}) \). So, \( \text{deg}_m(d, \text{ostrich}) \) is always smaller than \( \text{deg}_m(d, \text{bird}) \). Nor can the modified membership function, which Kamp and Partee add to the model in order to capture the conjunction fallacy (principle (82), help us here. Why? Because the minimal and maximal ostrich degrees in \( [[\text{bird}]]^+_c \) are 0 and 1. We can find both complete ostriches (of membership degree 1) and complete non-ostriches (of membership degree 0) among the birds. Consequently, \( \text{deg}_m(d, \text{ostrich} / \text{bird}) \) is identical to \( \text{deg}_m(d, \text{ostrich}) \):

\[
\text{deg}_m(d, \text{ostrich} / \text{bird}) = (\text{deg}_m(d, \text{ostrich}) - 0) / (1 - 0) = \text{deg}_m(d, \text{ostrich})
\]

Thus, we have to keep \( \text{deg}_m \) and \( \text{deg}_p \) separated in such lexical nouns. It is the values of \( \text{deg}_p \) which represent the intermediate typicality degrees and the subtype effect / fallacy in bird. But then again, Kamp and Partee do not specify how exactly the values of \( \text{deg}_p \) are determined when \( \text{deg}_m \) and \( \text{deg}_p \) are dissociated. Hence, subtype effects (the typicality judgments) in lexical nouns are not accounted for.

### 3.3.3.3 Partial knowledge

Another classical problem concerns the representation of context dependency in typicality judgments. Typicality judgments are culture-, language- and context-dependent. First, Malt and Sloman (2003) found that typicality judgments of non-experienced second-language learners diverge substantially from native responses and even the most experienced learners retain some discrepancies from native speakers' patterns. Second, knowledge about typicality is often partial even in native speakers. Which bird is more typical – an ostrich or a penguin? Is flying more important for birds than singing or nesting? Normally yes, but in certain contexts, (say –when vocal communication in animals is discussed or when the domain is confined to northern water birds), this may change. The importance of a dimension may vary with context. Third, this context dependency affects processing. For example, in the context the bird walked across the barnyard, a chicken (not a robin) is regarded as a typical bird, and categorization time is faster for the contextually appropriate item chicken, not for the normally typical but contextually inappropriate item robin (Roth and Shoben 1983). Thus, the representation of typicality judgments needs to be context-dependent and possibly partial. Formally, the typicality function should assign its values in each total context separately, like the intensity. But the values of the membership function, \( \text{deg}_m \), in Kamp and Partee (1995) are necessarily given once per supermodel (because these values represent the proportion of total contexts in which entities are predicate members). Thus, it is not easy to see how these values can be made to vary through total contexts, so as to correctly represent typicality.

### 3.3.3.4 Prototypes

The notion of a prototype is problematic in several respects.
One well-known problem concerning this notion is that it is exceptionally unfruitful when it comes to compositionality, i.e., in predicting prototypes of complex concepts from the prototypes of their constituents (Kamp and Partee 1995; Hampton 1997). Consider negations: What would the prototype of non-bird be: a dog, a day, a number? Similarly for conjunctions: What would the male-nurse prototype be, given that a typical male-nurse may be both an atypical male and an atypical nurse (Kamp and Partee 1995; cf. Figure 9).

![Figure 9: No prototypes for non-bird and male nurse](image)

Another problem has to do with predicates which are lacking a prototype. For example, there is no maximum tallness. But with no prototypes, the intuition that there are typical (and atypical) tall players, tall teenagers, tall women, etc., is not accounted for. The status prototypical, so it seems, ought to be given to an entity only within a context (a valuation); there are no context-independent entity-prototypes.

### 3.3.3.5 Dimension sets

Kamp and Partee avoid the notion of dimension set, which they see as ill defined (following Osherson and Smith 1981). But in eliminating the dimensions from the analysis, the supermodel theory is forced to remain silent about the type of properties that speakers regard as typical of a given predicate.

We saw in 3.2 that in other semantic gradability theories, the notion of an ordering dimension is hardly well-defined (for further discussion see Moltmann 2006). The same can be said also about Lasersohn (1999)'s Halo model, representing pragmatic looseness. Pragmatic looseness is the possibility of interpreting any predicate P in context c scalarly, such that even if P does not strictly apply to a certain argument x in c, P(x) can be regarded true enough in c if it comes "close enough to the truth for all contextual practical purposes". For instance, we loosely can say about the townspeople that they are asleep in contexts whereby only atypical townspeople (for instance, the city-guards or residents with exceptional biological clocks) are awake. Lasersohn associates with a predicate denotation a contextual halo that contains sets that differ from the actual denotation only along "pragmatically ignorable features". These can be identified with the typicality dimensions. But Lasersohn does not say how the pragmatically ignorable features are identified, and exactly how they constrain the halo structure in each context.

A more adequate semantic account ought to explain the precise conditions under which a property counts as a typicality dimension (or an ordering dimension) in a context.

### 3.3.3.6 The linguistic contrasts

Finally, the supermodel theory assumes a complicated taxonomy of predicate types, with different mechanisms in their meaning, as demonstrated in Table 3. Yet, despite its complexity, this typology does not capture the distinction between semantically gradable and
non-gradable predicates (predicates that can and predicates that cannot occur in the comparative without modification, etc.) The obligatory modification by typical of (or of) cannot depend on the presence or absence of a prototype, because, on the one hand, certain +Prototype predicates cannot occur in the comparative without modification (e.g., chair and bird), while others can (e.g., red), and on the other hand, certain –Prototype predicates cannot do so (e.g., non-bird), while others can (e.g., tall). In addition, this typology does not give a good explanation for the distinction between predicates whose dimensions can be accessed by grammatical operations (as in healthy in every respect) and predicates whose dimensions cannot be accessed (like tall and bird).

3.3.4 Conclusions of part 3.3

As it stands, the supermodel theory fails to derive the range of intermediate typicality degrees in denotation members. Hence, it fails to predict typicality in non-vague predicates - the most prominent examples of the prototype theory. In addition, the theory fails to correctly represent the conjunction effects, the inherent context dependency of typicality judgments, and the gaps in these judgments. The status "typical to degree n" ought to be given to an entity only within a context (supervaluation), not once per supermodel. Finally, the analysis fails to explain the linguistic contrasts between gradable and non-gradable predicates, and it is forced to remain silent about the conditions under which a property counts as a dimension of a predicate. Thus, we gave compelling arguments, showing that the typicality effects are not accounted for by this theory.

Chapter 4 presents the basic account of the typicality effects in cognitive psychology. We will see that this account correctly predicts a large number of typicality effects. Yet, we will see that an important feature of Kamp and Partee's (1995) theory (the use of a three-valued model) will in fact prove useful in solving some problems with the psychological mechanisms for the representation of typicality.
VAGUENESS, GRADABILITY, AND TYPICALITY:
THE PSYCHOLOGICAL PERSPECTIVE

The most influential approach in the psychological analysis of concepts is called the prototype theory. The origins of the prototype theory go back to Wittgenstein (1968 [1953]) in "Philosophical investigations", and it has been developed and experimentally supported within cognitive psychology due to the extensive work of figures like Eleanor Rosch, Tversky and their associates. The most central alternative to the prototype theory is formed by the exemplar theory. By and large, other cognitive approaches can be seen as branches of these two theories.

Murphy's (2002) seminal "big book of concepts" demonstrates well the wide range and richness of the empirical findings, whose discovery was triggered by the cognitive approach. At the same time, this book emphasizes the diversity of theoretical models in the field. The cognitive mechanisms that contemporary theories in cognitive psychology take to be part of the mental representations of concepts include a set of dimensions, a similarity degree function, a partial entity set (denotation) and a categorization criterion. The controversy among different cognitive theories hinges upon the precise characterization and role of each of these mechanisms. For example, prototype theories associate each concept with one set of dimensions, while exemplar theories associate it with many sets. Dimension-sets are represented as lists (Hampton 1979-1997), vectors in conceptual spaces (Gardenfors 2004), theories (Murphy and Medin 1985), frames (Smith et al. 1988), networks (Murphy and Lassaline 1997), and so on and so forth. In addition, scholars still hardly concur about the precise definition of the similarity function (Murphy 2002; Ashby and Maddox 1993; Murphy 2002), and about the details of the categorization criterion (Ashby and Maddox 1993). Finally, facts concerning the notion 'dimension' are still somewhat mysterious: The ways dimensions are chosen and assigned attentional weights (Armstrong, Gleitman and Gleitman 1983; Murphy 2002), the way the dimensions themselves are acquired, the ways entities' degrees in the dimensions are determined, etc.

According to Murphy, more efforts need to be dedicated to the challenge of capturing generalizations within the data and theories. With the challenge in mind, the present chapter presents the main types of dimension-theories, drawing attention to some important open questions, and to some crucial problems that deserve more theoretical thinking. Chapter 5 presents a new model that attempts to give an answer to some of these open questions, and a consistent solution to some of these problems. For reasons of space and clarity, historical details (for instance, descriptions of classical implementations of the prototype theory such as Tversky's 1977 contrast model or Rosch and Mervis's 1975 dimension model), and many mathematical details (for instance, descriptions of mathematical dimensions of the similarity functions in different theories), were removed to the Appendix of chapter 4. The present chapter only represents inherent differences between salient contemporary theories.

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1 For a review of this work see Mervis and Rosch (1981). For a review of earlier studies which form the basis for this work see Lakoff (1987; chapter 2). Reviews of more recent developments and theoretical approaches are found in, for instance, Hampton (1997a) and Murphy's (2002) seminal "big book of concepts".

2 The origins of the exemplar theory are found in Medin and Schaffer's (1978) context model. Exemplar models include Hintzman's (1986) Minerva model; Nosofsky's (1988) generalized context model; Kruschke's (1992) ALCOVE model; Estes's (1994) array model, etc.
4.1 Basic notions

4.1.1 The Prototype Theory

4.1.1.1 Dimension sets, mean distance and similarity

In standard concept theories, concepts P (bird, robin, etc.) are associated with a set ('set'), F(P), of representative dimensions (typicality dimensions). For example, F(bird) consists of flying, singing, small size, etc. This set is often called the prototype or summary representation of the concept. Each dimension F in F(P) has an attention weight Weight(F,P). For example, Weight(flying,bird) tells us how important flying is in discriminating birds from non-birds. In addition, P has a selected value on F, deg(P,F). For example, deg(bird,size), represents the ideal size for birds.

Occurrences of individuals in time are also associated with dimension values, too. In entity occurrences, the values represent their actual value on the dimensions. For example, the size of Tweety at time point h (say, 20.09.80 4:00 PM), deg(<Tweety, h>,size), represents Tweety's size at 20.09.80, 4:00 PM. Atemporal representations of individuals also include dimension values (for instance, the value of Tweety on 'size', deg(Tweety,size)). These values represent either actual or ideal values (for instance, if I am characterizing Tweety as an adult bird, I may choose as a value her actual size as an adult. But for dimensions like calm I may choose a value that represents best adult occurrences of Tweety on time (in other words, Tweety may not be calm right now, and at the same time Tweety may be generally calm).

In the prototype theory, typicality in a concept is conceived of as similarity to its prototype. For example, the similarity of a robin to a bird is indicated by the extent to which it matches the prototypical values in the bird's dimensions.

Formally, the distance of an entity d from P along a dimension F, Dis(d,P,F), is the difference between d's and P's values on F (1). If d and P completely match on a dimension the distance is 0. Otherwise, the distance may in principle be infinite (but distance is usually modeled on a 0 to 1 scale). When the dimensions are treated as non-gradable (as mapping entities to either degree 1 or degree 0), distance in a dimension is, accordingly, either 0 or 1.

(1) The distance of d from P in a dimension F: Dis(d,P,F) = | deg(d,F) — deg(P,F) |

The mean distance of d from P (in the concept's dimension set) can be represented using the notion of a generalized weighted-mean, Dis(d,P,r), which is an abstraction of the different types of weighted means (Weidman 1993). The value of the exponent r determines the type of mean (2a). For example, for r = 1, we get the arithmetic mean: The sum of d's (weighted) degrees in every dimension (2b). The notion is defined only for cases where the weights are positive and sum up to 1.

(2) a. The generalized weighted-mean of the distance of d from P in F(P): 
\[ \text{Dis(d,P,r)} = \sqrt[r]{\sum_{F \in F(P)} (\text{Weight}(F,P) \times \text{Dis}(d,P,F))^r} \]
(where r≠0, the dimension weights are all positive, and they sum up to 1).

b. Arithmetic mean-distance (for F(P) = \{F_1, \ldots, F_n\}): 
\[ \text{Dis(d,P,1)} = (\text{Weight}(F_1,P) \times \text{Dis}(d,P,F_1)) + \ldots + (\text{Weight}(F_n,P) \times \text{Dis}(d,P,F_n)) \]
The similarity of \( d \) to \( P \), \( \text{Sim}(d,P,r) \), is assumed to be inversely related to \( d \)'s distance from \( P \). In contemporary prototype models (Hampton 1995; Smith and Minda 2002), similarity is assumed to be inversely related to distance by an exponential function:

\[
(3) \quad \text{The similarity of } d \text{ to } P: \quad \text{Sim}(d,P,r) = \frac{1}{e^{\text{Dis}(d,P,r)}}
\]

This exponential connection between distance and similarity is called "the universal law of psychological generalization", as it provides good match to similarity-based responses in both humans and animals (Shepard 1987). It has the advantage of a multiplicative function of mean similarity (\( \text{deg}(d,F_1)^{\text{Weight}(F_1,P)} \times \ldots \times \text{deg}(d,F_n)^{\text{Weight}(F_n,P)} \)); cf. Medin and Schaffer 1978; Nosofsky 1992). Multiplicative (but not additive\(^3\)) similarity functions capture the fact that the most radical decrease in similarity levels is between entities which perfectly match \( P \) in all the dimensions and entities with mismatch in one dimension. Even instances which match in all the dimensions except for a 0.5 score in one dimension, have low mean similarity, 0.5, because multiplication yields \( 0.5 \times 1 \times \ldots \times 1 = 0.5 \) (Murphy 2002). Two 0.5 scores yield mean similarity 0.25, etc.

The more typical instances of a concept are more similar to its prototype (their mean degree on the dimensions is higher). Thus, by identifying typicality with similarity to the prototype, the prototype approach derives the basic typicality effects, namely, the fact that speakers order entities by typicality (cf. 2.2.2), and the fact that the noun dimensions are ordering dimensions (cf. 2.2.3), which together help mapping entities to degrees.

4.1.1.2 Standard-based categorization

Categorization is assumed to be based on similarity to the prototype. Classically, prototype theories view categorization as based on a standard, formed by a certain similarity degree. An entity is classified as \( P \) iff its similarity degree reaches this standard. In other words, categorization is a process in which it is decided whether the mean distance of an entity from the concept's prototypical values on the dimensions is small enough (smaller than some standard mean distance \( n \)):

\[
(4) \quad \text{An arithmetic categorization criterion:} \quad [[P]] = \{ d \in D \mid (\Sigma_{F \in F(P)} (\text{Weight}(F,P) \times \text{Dis}(d,P,F))) \leq n \}
\]

Consider, for example, a concept \( P \) whose dimension set consists of two dimensions, \( F_1 \) and \( F_2 \). For each entity, its values on these dimensions can be represented by a unique point in a two dimensional space with axes \( F_1 \) and \( F_2 \). As demonstrated in Figure 10, the points solving the equation "(\( \Sigma_{F \in F(P)} \text{Weight}(F,P) \times \text{Dis}(d,P,F)) \leq n \)" (the categorization criterion in (4)) form a diamond shape in this space. The concept members are represented by the points in this diamond. The prototype is the middle point. In fact, it is often represented by some entity \( d_P \) whose values on the dimensions are identified with this point. Accordingly, in the discussion below, when more convenient (namely, in the discussion of exemplar theories), I use the notions which were defined for predicates (\( F(P) \), \( \text{Dis}(x,P,1) \), \( \text{Sim}(x,P,1) \), etc.) as notions that are defined for entities (the predicates' entity-prototypes), as in: \( F(d_P) \), \( \text{Dis}(x,d_P,1) \) and \( \text{Sim}(x,d_P,1) \), etc. Nothing else in definitions (1)-(4) needs to be modified in order to adapt for this change.

\(^3\) Classical prototype models (Rosch and Mervis 1975; Tversky 1977, etc.) assume that the inverse relation is linear (\( \text{Sim}(d,P,1) = 1 - \text{Dis}(d,P,1) \)), so similarity is predicted to be additive: \( \text{Sim}(d,P,1) = (\text{Weight}(F_1,P) \times \text{deg}(d,F_1)) + \ldots + (\text{Weight}(F_n,P) \times \text{deg}(d,F_n)) \) (for further discussion see the appendix).
Thus, according to the prototype theory, no particular dimension needs to be necessary for membership (to add categorization criteria). Rather, it is necessary for all the members that their mean on the dimensions will reach threshold (cf. 2.2.3). Hampton (1979) presented direct evidence for this view. He asked subjects to list category dimensions and then he asked other subjects to judge for each category member which dimensions it satisfies. The dimensions which all members satisfied were treated as necessary. Hampton found that these dimensions could not predict membership (they were not sufficient for membership: Some non-members satisfied all of them). However, the number of necessary and non-necessary dimensions which each item had, was sufficient to predict membership (the likelihood that people will treat the item as a category member). Thus, the true categorization rule is a requirement on the average in the typicality dimension set.

This standard-based categorization-principle accounts for many other typicality effects. First, it predicts the fact that likelihood of categorization is, by and large, monotonically related to similarity to the prototype (cf. 2.2.5). For instance, Hampton (1998) conducted a systematic examination of the relations between typicality judgments and judgments of membership likelihood based on data about 492 items in 18 categories (published by McCloskey and Glucksberg 1978). This data included the items' mean typicality ratings, the probability that they were categorized positively, and the degree of within-subject disagreement. Hampton found a very strong coupling between the mean typicality ratings of items and the probability that they were categorized positively. In addition, when deviations occurred, they were highly systematic.

Second, the existence of borderline cases (or a gap) in nouns is now predicted (cf. 2.2.1), because for some entities it may not be clear whether they reach the standard or not (if their degree is very close to the standard, or if the precise standard is unknown).

Third, by assigning important dimensions (like "horse genotype") a particularly high weight, this principle derives the intuition that they can almost count as necessary and sufficient for membership. They might be violated only if the values on the other dimensions may compensate (may be sufficient to reach threshold; Hampton 1979).

Fourth, the online processing effects are predicted, too (cf. 2.2.2). Within an experiment, the instances which are directly taught are categorized faster than new items. But already a week later, untrained typical instances are categorized earlier than less typical trained items (Posner and Keele 1970). This is predicted by the prototype theory, based on the fact that when the short term effects of recent exposure are weakened, the crucial factor which determines categorization rates remains to be that of similarity to the dimension set. Newly encountered typical instances are more similar to the dimension set than less typical old training items. Given that they satisfy many category dimensions, it is possible to determine that the typical instances reach the threshold for categorization relatively fast. Atypical instances violate many dimensions and, hence, more dimensions need to be considered before it becomes possible to determine that they reach threshold.

Finally, the set of known concept members, $[P]^+$, plays a crucial role when the concept standard is unknown. A criterion like (4) predicts that newly encountered entities, whose mean similarity is higher than that of already known members, can be automatically regarded as members. Thus, this theory allows for a finite memory representation for concepts (or predicate
intensions). It captures the fact that we can determine membership of infinitely many new instances under the concepts we are familiar with, on the basis of a finite set of known facts (dimensions and members).

4.1.1.3 Contrast-based-categorization

Furthermore, mostly exemplar models, but also Tversky (1977) and contemporary prototype models (Smith and Minda 2002), use another type of categorization criterion.

Concepts P are assumed to belong to a set of "contrast-concepts", \( K_P \). For example, the contrast set of \( \text{bird} \), \( K_{\text{bird}} \), may consist of the categories \( \text{mammals, reptiles, insects and birds} \), meaning that these categories are mutually exclusive and together they form a local domain, \( D_{\text{bird}} \).

(5) The contrast set:

Mutual exclusivity: \( \forall Q_1, Q_2 \in K_P, (\{[Q]\}_1 \cap \{[Q]\}_2) = \emptyset \)

Domain cover: \( D_{K_P} = \{ d \in \{[Q]\} \mid Q \in K_P \} \)

Items are assumed to be classified in the contrast concept they resemble most in \( K_P \) (Ashby and Maddox 1993). Formally, d's similarity to each contrast concept P is normalized. That is, d's similarity to P relative to \( K_P \), \( \text{Sim}(d,P,K_P) \), is the ratio between d's similarity to P and d's similarity to the contrast categories, as stated in (6a). When categorization is uncertain ('probabilistic'), it is assumed that the number of times an item d is classified as P is given by \( \text{Sim}(d,P,K_P) \) (this assumption is called "The Luce choice axiom"). When categorization is deterministic, d is assumed to be classified in the category it resembles most (7b) (Ashby and Maddox 1993).

(6)

a. Similarity to P relative to \( K_P \):
   \( \text{Sim}(d,P,r,K_P) = \frac{\text{Sim}(d,P)}{\sum_{Q \in K_P} \text{Sim}(d,Q,r)} \)

b. A deterministic categorization decision:
   \( \{P]\} = \{ d \in D_P \mid \forall Q \in K_P, \text{Sim}(d,P,r,K_P) > \text{Sim}(d,Q,r,K_P) \} \)

The principles in (7b) predict that, (though, this is not likely to occur often), membership likelihood may sometimes not be monotonically related to similarity. Consider for example, a set \( K \) of three contrast concepts P, Q and Z, and two items x and y whose values are as follows:

(7)

a. \( \text{Sim}(x,P) = 0.33 \) \( \text{Sim}(x,Q) = 0.33 \) \( \text{Sim}(x,Z) = 0.34 \)
b. \( \text{Sim}(y,P) = 0.42 \) \( \text{Sim}(y,Q) = 0.18 \) \( \text{Sim}(y,Z) = 0.40 \)

In this example, for each concept P and entity d, its normalized similarity \( \text{Sim}(d,P,r,K) \) equals its similarity \( \text{Sim}(d,P,r) \). We see that y is more similar to Z than x, but, crucially, y is much less similar to Q than x. Consequently, principle (7b) predicts that x would be categorized under Z (the category which x resembles most in K), whereas y would be categorized under P (the category which y resembles most in K).

The situation is a bit different with sets of two contrast categories. For example, given the similarity values in (9a-b), the normalized values are as in (9c-d):

(8)

a. \( \text{Sim}(x,P) = 0.49 \) \( \text{Sim}(x,Z) = 0.51 \)
b. \( \text{Sim}(y,P) = 2 \) \( \text{Sim}(y,Z) = 1 \)
c. \( \text{Sim}(x,P,K_P) = 0.49 \) \( \text{Sim}(x,Z) = 0.51 \)
d. \( \text{Sim}(y,P,K_P) = 0.666 \quad \text{Sim}(y,Z,K_P) = 0.333 \)

Before normalization, \( y \) is more similar to \( Z \) than \( x \), but principle (7b) predicts that \( x \) would be categorized under \( Z \) (the category \( x \) resembles most in \( K \)), whereas \( y \) would be categorized under \( P \) (the category \( y \) resembles most in \( K \)). However, in a binary contrast set \( K_P \), membership is coupled with normalized similarity\(^4\). Thus, newly encountered entities, which are more similar to \( P \) relative to \( K_P \) than directly known members (members of \([\{P\}]^3\)), can be automatically regarded as category members. The theory predicts our ability to categorize infinitely many new instances under the categories we are familiar with.

Recall that Hampton (1998) has presented robust evidence for standard-based categorization (tight coupling between the mean typicality ratings of items and the probability that they were categorized positively). However, systematic deviations occurred, too. One of the three main reasons for deviations was the existence of contrast concepts, as predicted by principle (7). For example, both the category kitchen utensil and the category furniture were part of the stimuli. This fact has reduced the likelihood of classification, but not the typicality of items like a refrigerator in the category furniture (see also 2.2.4). The stimuli in experiments supporting the exemplar theory normally consist of a set of invented-concepts that are mutually exclusive and that cover all the items presented in the experiment. Obviously, these circumstances encourage contrast-based categorization, but the significance of the effects of contrast-sets on categorization in natural concepts is yet to be discovered. In chapter 7, I show the relevance of these matters to semantics, by demonstrating that pairs of adjectives or of nouns that are part of a common contrast set are licensed in between predicate comparisons.

Finally, perhaps non-monotonic cases (where \( d_1 \) is more similar to \( Q \) than to \( P \), but is classified as \( P \)), may be represented by adding the negations of \( P \)'s contrast categories to \( P \)'s dimension set (at least in categorization judgment which are made in the presence of these contrast concepts). This will predict that being dissimilar to them will raise the similarity of entities to \( P \) and the likelihood of categorization under \( P \) in the context. If distance from a contrast category \( Q \) were to enter the calculation of similarity to \( P \), we would be able to say that \( d_1 \) is not more \( P \) than \( d_2 \), precisely because it is more \( Q \).

4.1.1.4 The main problem of prototype models: Linear separability

The prototype theory applies only to linearly separable categories. Category dimensions like shape and color are non-separable iff the similarity of entities to the concept, or the likelihood that they fall under it, depends on the combination of their values in these dimensions. For example, if being a red triangle or a green circle raises similarity to \( A \), but being a red circle or a green triangle reduces similarity to \( A \), the category A dimensions color and shape are dependent.

In geometry, when two sets of points in a two-dimensional graph can be completely separated by a single line, they are said to be linearly separable. In general, two groups of points are linearly separable in an \( n \)-dimensional space if they can be separated by an \((n − 1)\) dimensional hyper-plane ("linearly separable", Wikipedia, the free encyclopedia). For example, the sets of points A and B in Figure 11 represent 2 non-linearly separable classes (planes), because they cannot be separated by a single line (a single \((2 − 1)\) hyper-plane).

\(^4\) If \( x \)'s normalized degrees on \( P \) is higher than \( y \)'s, than \( x \)'s normalized degree on \( Z \) is smaller than \( y \)'s (this is the case because for any \( x \), \( S(x,Z,\{P,Z\}) = 1 − S(x,P,\{P,Z\}) \)). Thus, \( x \) cannot be categorized under \( Z \) and at the same time be less \( Z \) than \( y \) (relative to \( K_Z = \{P,Z\} \)).
Consequently, a category is not linearly separable if there is no way to assign weights to the dimensions such that classification will be based on average in the dimensions, (and hence on one line). For instance, in figure 11, it is not the case that the greater your degree in the vertical dimension is, the more likely you are to fall under category B. In particular, this is false for entities with high degrees in the horizontal dimension (entities which fall within the right side of Figure 11), and it is true for entities with low degrees in the horizontal (in the left side of Figure 11).

Next, we will discuss exemplar models. These models give an account that extends to non-linearly separable categories.

4.1.2 The Exemplar theory

4.1.2.1 Exemplar-based similarity

The most significant difference between exemplar and prototype models is that exemplar models assign a small (or no) role to the dimension set of the entire category (the summary representation). Exemplar models assign a large role to the separate dimension sets of each one of the entities or sub-categories of the category (for instance, to the dimension sets of robin, duck, chicken, etc., for the category bird), or to the separate dimension sets of each one of the temporal stages of the instances of the category (for instance, to the dimension set of each occurrence of bird). These items are called exemplars. An exemplar is represented as a set of separable dimensions and selected values in these dimensions, thus an exemplar has the structure of a linearly separable prototype-based category (this representation might change in time, because memory might degrade or strengthen). The exemplar approach argues that we categorize objects by comparing them to remembered exemplars. The more similar an object is to entities or sub-categories whose membership we have already encoded, the more likely it is to be a regarded as a member.

Let the set \([P]^+\) stand for the finite set of entities which are directly represented in memory as instances of P in a certain context. The typicality degree of x in a category P is given by the sum of its similarities to all the known exemplars y of P (Medin and Schaffer 1978; Nosofsky 1992). A given exemplar y may be weighed by its importance to P (Weight(y,P)):

\[
\text{(9) Similarity to a category P (Typicality):} \\
\text{Sim}(x,P,r) = \Sigma_{y \in [P]} (\text{Weight}(y,P) \times \text{Sim}(x,y,r))
\]

Since similarity to an exemplar is assumed to be multiplicative (cf. (3)), it is best to be very similar (near to 1) to but few items than somewhat similar (dissimilar in but few respects and
hence near to 0 similar) to many items. For example, one 1 score is equal to the sum of eight 0.125 scores. Thus, generally, if an item is highly similar to a known instance of P, it is highly similar to P. (However, if there are many highly similar exemplars, this raises the similarity score of newly encountered entities that are similar to them, and this raises their membership likelihood).

4.1.2.2 What do we achieve by extending the number of exemplar-representations?

First, the exemplar theory directly captures *exemplar effects*. For example, physicians who were taught about various kinds of diseases, made more categorization errors when the new items they were presented with were superficially similar to trained items in different categories (Brooks, Norman and Allen 1991). This effect was stronger when a set of objects with an irrelevant dimension were presented in a sequence, not separated by other objects. This shows that the presentation order may be encoded in memory. In addition, if an item is highly similar to a category member but similar no member of other categories it is likely to be positively categorized in the former category (Brook 1987). There are also related priming effects: When categorization is based on similarity to a specific object, exposure to this object facilitates categorization (Malt 1989).

Second, an extended set of exemplars helps to capture the *extent of variability allowed by a concept* (for example, if dolphins are classified as mammals, classification of whales is predicted to be easy because they are similar to existing members). Accordingly, the exemplar and prototype theories make different predictions with regard to objects which are not similar to the prototype but are highly similar to a category member.

Third, a separate representation for each category member helps to capture *correlations between dimensions*. For example, being white may raise one's similarity to pet-hares, but not to pet-birds. Similarly, usually, small birds, but not big ones, sing. Hence, it is predicted that the similarity of big entities which sing to the category bird would be considerably smaller than the similarity of small entities which sing or big entities which do not sing. Indeed, the main distinction between the prototype and the exemplar theory can be seen as stemming from the importance attributed to the dependency between dimensions. The exemplar theory treats each exemplar as a set of dimensions. This allows for an account to categories in which classification cannot be based on average in one summary dimension-set (non-linearly separable categories). For example, if being a red triangle or a green circle raises similarity to a category P, but being a red circle or a green triangle reduces similarity to P, P cannot be summarized by one prototype. A pure prototype approach amounts to saying that non-linearly separable categories cannot be learnt, which is clearly false. A weaker version of the prototype theory, according to which an exemplar strategy is employed only when the prototype strategy fails (Smith and Minda 1998; 2000), predicts faster learning of the linearly separable categories. But some experiments show that, contrary to this prediction, non-linearly separable categories may be learnt faster (Murphy 2002). This is predicted by the exemplar theory.

4.1.2.3 What do we lose by eliminating the summary representations?

Despite the advantage just discussed, we have reasons to be skeptic with regard to our capacity to learn and use non-linearly separable categories, compared to linearly separable ones. The invented concepts that typically form the stimuli in experiments of the exemplar theory are not similar to most natural concepts. They have very weak structures. For instance, these categories may contain two objects which are different in every respect (such circumstances are quite rare in real life when natural categories are learnt and used). Usually,
the entities in these categories are characterized by a very limited set of dimensions, the number of entities is small, and many of the dimensions function as noise – they are not diagnostic of any category. Clearly, the only way to classify new entities under a category which consists of entities with no common denominator (say, phonologists, cheese cakes and rational numbers), is by similarity to exemplars. But most natural categories do seem to have some common denominator. Indeed, in the experiments supporting the exemplar theory, there are significantly high rates of error (Murphy 2002: 104) and significantly high rates of subjects which fail to learn the categories even after a relatively large amount of training sessions (Murphy 2002: 88). For instance, when a non-linearly-separable category is not very small, many subjects fail to learn it even after extensive training (say – after seeing nearly 4000 exemplars over a full week of training; McKinley and Nosofsky 1995). But most of the natural concepts are not small. Thus, the exemplar models may be too powerful (Ashby and Maddox 2005).

In addition, contemporary exemplar theories lag behind prototype theories in certain matters, for which summary representations seem to be crucial (summary-representation effects). First, Hampton (1997) criticizes the exemplar theory for being unfruitful in predicting the typicality effects in complex predicates. For Hampton, this requires the construction of a composite dimension set (summary representation) for the complex concept. Second, Murphy (2002: 485-6) argues that the prototype theory fairs better in predicting knowledge effects. For example, when one learns that all birds have bird-genes one would presumably not update every single exemplar with this new fact. But if so, an exemplar model would fail to represent the knowledge that all birds have this property. In addition, certain inferences can be made about categories without familiarity with any of their exemplars (Murphy 2002: 268; 185). A pure exemplar approach amounts to saying that we only memorize entity stages and their dimensions (atemporal instances, sub-kinds and dimensions are linked to categories like bird only by inference, based on encoded facts about these entity-stages). This seems highly unlikely. Finally, the different types of exemplar models differ in their sets of predictions. Studies such as Palmeri (1999) show that no single model can capture the whole set of effects related to hierarchical classification (the fact that objects can be, for instance, an animal, a dog, and a bulldog).

4.1.2.4 Are exemplars-based and prototypes-based predictions inconsistent?

We have seen that prototype and exemplar models predict a large number of facts. But exemplar-based predictions and prototype-based predictions appear to be inconsistent.

When a category is associated with both a prototype and an exemplar set in one and the same context, decisions about membership of new instances need to conform both to the prototype-based categorization principle and to the exemplar-based categorization principle. That is, members need to be sufficiently similar to the prototype and to at least one exemplar, (more similar than non-members). But we have seen in 4.1.2.2 three cases in which these two requirements appeared to be inconsistent. Can we explain these cases in a coherent way? In the following, I argue that we do.

First, concerning the fact that entities are sometimes classified in a concept in virtue of an irrelevant dimension that they share with an existing member, Murphy (2002), argues that for certain category members an implicit automatic learning rule may be acquired based on some superficial dimension (say – their occurrence in a specific setting). Entities with this dimension may be immediately positively categorized based on this rule and not on an exemplar based similarity function.

Second, if in virtue of the fact that dolphins are classified as mammals, classification of whales becomes easy, this may show that we link mammal with an exemplar-set, and not a
prototype, because these objects are not similar to the (presumed) prototype, but are highly similar to a category member. Alternatively, two different interpretation-levels may be involved. Possibly, categories like mammal or bird have a summary representation at the level of sub-kinds (namely, sub-kinds of birds are more typical if they sing, fly, perch, etc.), but not at the entity-set level (that is, entities are classified as birds based on their similarity to a summary representation of at least one bird sub-kind). Consequently, whales may be atypical as mammalian sub-kinds (as predicted by the prototype theory), but relatively typical as mammalian entities (as predicted by the exemplar theory).

Third, usually, small birds, but not big ones, sing. But is there a correlation between these dimensions? First, one needs to establish that indeed big birds that sing are judged as less typical than big birds that do not sing. If this is indeed the case, and it is not simply due to unfamiliarity with the former bird-kinds, then it may show that there is no summary representation for birds (at the relevant level). But it may also show that the summary representation does not include singing and small size as separate dimensions, but rather, it includes more complex dimensions like "small and sings".

Finally, inconsistency may form evidence for multiple senses or interpretations. In chapter 5 and on, I propose a model that explicitly represents context. In different contexts, predicate interpretations (concepts) may be structured in differently ways, with or without a summary representation. In addition, I incorporate prototypes into the interpretation of predicates, and I draw a suggestion according to which, by explicitly representing the "sub-category" (or "sub-kind") relation, one can derive exemplar effects.

4.2 The objections to truth conditions

Characteristics of concepts which are denoted by complex predicates (see 2.2.7 and the discussion below) are taken by many psychologists to refute the idea that semantics may be based on truth conditions. These researchers identify truth conditional semantics with the classical view, which is taken to be incompatible with the phenomena, or hardly relevant for their account. Characteristic examples for this view are found in the following citations from Murphy (2002) and Lakoff (1987):

"...There have been a number of attempts to revive the classical view...This is a bit difficult to explain, because the view simply does not predict the main phenomena in the field...There is a beauty and simplicity in the classical view that succeeding theories do not have. It is consistent with the law of excluded middle and other traditional rules beloved of philosophers. To be able to identify concepts through definitions of sufficient and necessary properties is an elegant way of categorizing the world, and it avoids a lot of sloppiness that comes about through prototype concepts (like the intransitive category decisions, and unclear members). Unfortunately, the world is a sloppy place." (Murphy 2002: 39).

"...The [AI] models often have some assumptions that are psychologically implausible, for example, a reliance on predicate logic and classical rules in the knowledge component." (Murphy 2002: 196).

"The PC view of rigor leads to rigor mortis in the study of categorization. It leads to a view of the sort proposed by Osherson and Smith 1981 and Armstrong, Gleitman and Gleitman 1983, namely, that the classical view of categorization is correct and the enormous number of phenomena that do not accord with it are either due to an "identification" mechanism that has nothing to do with reason or are minor "recalcitrant" phenomena. As we go through this book, we will see that there seem to be more so-called recalcitrant phenomena than there are
phenomena that work by the classical view ...In concluding that categorization is not classical, the book implicitly suggests that the PC view of scientific rigor is itself not sufficiently valid." (Lakoff 1987: chapter 1).

If we are to hold that semantics theories are viable, we have to deal with the problematic findings. In addition, if we are to bridge the gap between psychology and semantics, we have to see whether psychological accounts for these findings are viable, and whether they are consistent or not with semantic theories (for instance, with semantic rules like the intersection rule). Representatives of semantic theories usually do not attempt to deal with these issues. Nevertheless, the following citation from Chierchia and McConnel Ginnet (2002) shows that the Prototype view is not taken as necessarily incompatible with the formal semantic theories.

"...For all we know, some version of this view may prove right, but we disagree sharply with Lakoff’s conclusion that truth conditional theories of semantics are thereby ruled out. As the rest of this section shows, it is quite possible to hold that prototypes and similarity relations of some kind organize conceptual categories associated with individual words and still maintain a truth conditional approach to semantics." (Chierchia and McConnel Ginnet 2002: 547).

In the following, I examine psychological accounts for facts concerning typicality and categorization in complex predicates, and their consequences.

4.2.1 The conjunction and sub-type effects

Recall that a brown apple may be regarded as more likely (and more typical of) a brown apple than an apple. This is an example of a conjunction fallacy.

How are conjunction fallacies predicted by the prototype theory? A brown apple is more similar to the prototype of brown apple than of apple because its mean on the dimension set of the former category (which includes the dimension brown) is higher than its mean on the dimension set of the latter category (which includes dimensions like red). Similarly, an ostrich is more similar to the prototype of ostrich than of bird, iff its mean is higher in the former category than in the latter one. In chapter 2, this example was classified under the name the subtype effect. Thus, these effects are captured.

They are also captured by the exemplar theory, because a brown apple is highly similar to most of the objects in [[brown apple]]^+, while being similar to relatively few unimportant objects in [[apple]]^+ (and the same is true of sub-type effects, too).

4.2.2 Failures of intersection inferences

The psychological theories reject the truth-conditional compositional theory, including the intersection rule (cf. 2.2.9). Thus, they explain in just the same way similar effects in categorization, like the finding that the denotation of modified nouns may be bigger than required by constituents’ intersection (‘overextended’). For example, according to these theories, blackboards may be classed as school furniture while not being classed as types of furniture. How? Blackboards may be sufficiently similar to the prototype (or to encoded exemplars) of school furniture (by virtue of their high value on the dimension "things in school"), while at the same time they may not be sufficiently similar to the prototype or to encoded exemplars of furniture, which do not have this dimension (Hampton 1988, 1997; Costello 2000).
This explanation extends also to modified nouns like *sports which are games*. According to these theories, then, the set \([\text{[sports which are games]}]\) may be a subset of neither the set \([\text{[sports]}]\) nor the set \([\text{[(things) which are games]}]\).

The theory predicts also the fact that the set \([\text{[sports which are games]}]\) may appear not identical to the set \([\text{[games which are sports]}]\). This is predicted based on the fact that often the prototype of a modified noun appears to be such that dimensions which are inherited from the modifier are assigned higher weights than dimensions which are inherited from the head noun (Hampton 1997). Thus, typicality and categorization in, for instance, *sports which are games*, and *games which are sports*, are based on different prototypes.

In sum, the psychological theories directly derive an account for facts that appear to form counter-examples to the truth conditional compositional theory (the intersection rule). In chapters 7 and 8 I show that these phenomena are in fact compatible with the intersection rule, and they are even expected, if certain pragmatic constraints are taken into account.

### 4.2.3 A composite-prototype representation

From the psychological perspective, semantic theories are identified with the classical view of concepts as sets of necessary and sufficient conditions for categorization. Perhaps because of that, they are (or the intersection rule is) identified with the wrong prediction that the dimension set of a modified noun is composed by applying the Boolean union operation on the sets of the parts \((F(P \land Q) = F(P) \cup F(Q))\). Abundant counter-examples to this union-rule were found. The dimension set is not completely determined by (union of) the dimension sets of the parts and certain effects on the weights of dimensions are hard to predict in a systematic way. These findings are taken by opponents of the formal theory to be counter-evidence to it.

Hampton (1987) analyzed the dimensions of modified nouns and their constituents (dimensions that subjects listed and ranked for relative importance). He asked subjects to list dimensions for modified nouns (for instance, *pets which are birds*, *pets* and *birds*). Then he asked other subjects to rate the relative importance of the dimensions in the modified nouns and in their constituents. A dimension which was judged to be *necessarily true of all the concepts' instances* was coded numerically by the number 4; a dimension which was judged to be *a very important part of the definition* was coded by 3; a dimension which was judged to be *a fairly important part of the definition* was coded by 2; typically true but not very defining was 1; *not usually true* was \(-1\) and *necessarily false of all possible instances* was \(-2\).

The mean rated importance for each dimension across the subject group was between \(-2\) and 4. Mean reliability in a split group analysis was 0.85 for the conjunctions and 0.89 for the conjuncts. This value suggests that, generally, subjects are in agreement about the concepts' dimensions. In 17 of the 24 dimension lists, mean rated importance also correlated significantly with production frequency.

The union rule could account for about 80% of the dimensions which were produced for the modified nouns. Dimensions rated as necessary for one constituent were also rated as necessary to the modified noun. For example, *pets* must *have an owner* and *fish* cannot be *warm and cuddly*, and hence, so are *pet-fish*. In fact, modified nouns had on average 2.4 more important-dimensions (dimensions with mean importance 1.5) than their constituents had. Yet, crucially, modified nouns had on average about 3 important dimensions less than the union rule would predict. Dimensions which weighed little in one constituent could eventually not be inherited to the modified noun, refuting the union rule.

Only a special method could predict the weight of each dimension in a given modified noun, \(\text{Weight}(F, P \land Q)\), from its weights in the constituents, \(\text{Weight}(F, P)\) and \(\text{Weight}(F, Q)\). On the one hand, in this method, the weight of each dimension in a modified noun is indicated as a rising monotonic function of its weights in the constituents. On the other hand, if one of
these weights is very low / very high, the weight in the modified noun is also very low or even nullified / very high (dimensions which are necessary for either constituent are necessary for the modified noun and dimensions which are impossible for either constituent are impossible for the modified noun). In other words, Hampton (1987) found that we can summarize the data by an equation of the form given in (10), where the weight of a given dimension \( F \) in a modified noun, \( \text{Weight}(F,P \land Q) \), is a function of \( F \)'s weights in the constituents, \( \text{Weight}(F,P) \) and \( \text{Weight}(F,Q) \). In other words, Hampton discovered that there is a function \( f \), such that for some constant weights \( W_P \) and \( W_Q \), (10) holds true for every dimension \( F \) in the union of dimension sets of \( P \) and \( Q \). For example, for \textit{pets which are birds}, \( W_P = .46 \) and \( W_Q = .61 \) and for \textit{machines which are vehicles} \( W_P \) did not enter significantly and \( W_Q = .865 \). At least half of the inheritance failures were due to low importance for the constituents.

(10) \[
\text{Weight}(F,P \land Q) = f(W_P \times \text{Weight}(F,P) , W_Q \times \text{Weight}(F,Q))
\]

Hampton does not give details about the function \( f \). He refers to it as an averaging function, though, clearly, it is not identical to any of the forms of the generalized mean function.\(^5\) At any rate, these findings are taken as counter-evidence to the semantic theory, and positive evidence for the creation of a composite prototype for modified nouns, based on a special (non-Boolean) criterion for selecting and weighing dimensions of the constituents (Hampton 1997).

Hampton's findings can be taken as establishing the fact that modified nouns are associated with composite prototype representations. But, obviously, the composition criterion (the function \( f \)) cannot derive the weights that are assigned to new (emergent) dimensions that do not characterize any of the constituents (cf. 2.2.7; like \textit{lives in cages} and \textit{can talk for pet birds}, like \textit{wooden for big-spoon versus metal for small spoon}, and like \textit{hard for boiled eggs versus soft for boiled potatoes}). In addition, the function \( f \) may give good match to the weights of inherited dimensions, but it is still not very explanatory. For instance, the assumption that the intersection rule is not valid, turns puzzling the fact that necessary dimensions of the constituents tend to remain necessary for the modified noun. Why should they remain necessary? In chapter 5 and on, I propose a theory that takes both the intersection rule and the prototype theory to be valid, and in chapter 8 I show that such a theory gives motivation to such facts and to both failures of inheritance from the constituents and emergence of dimensions. Thus, this theory explains why we have to resort to functions roughly like the one discovered by Hampton (1987) for the composition of composite prototypes. In that I show that a theory that incorporates the logical rules is more explanatory than a theory that does not.

### 4.2.4 Constituent-based predictions

Hampton (1987, 1997) analyzed the categorization judgments and typicality ratings of a list of entities in modified nouns of the form "Ps which are Qs" (like "pets which are birds") and in their constituents (e.g. 'pets' and 'birds'). The scale for category members ranged from 3 (very typical) to 1 (very atypical). The scale for non-members ranged from −1 (related) to −3 (unrelated). The value 0 indicated borderline cases. The typicality ratings for each item \( d \) were averaged across subjects. The following patterns emerged.

First, for any item \( d \), it was possible to predict \( d \)'s typicality rating in a modified noun, \( \text{deg}(d,P \land Q) \), from \( d \)'s ratings in the constituents, \( \text{deg}(d,P) \) and \( \text{deg}(d,Q) \), by an equation like

---

\(^5\) In principle, this function may assign a dimension \( F_1 \) a higher value than a dimension \( F_2 \) even if the sum of \( F_1 \)'s weights in the constituents is smaller than the sum of \( F_2 \)'s weights. For instance, this may happen if \( F_1 \) is necessary for one constituent or \( F_2 \) is impossible for one constituent.
(11a). $W_P$ and $W_Q$ represented the constituents' weights, and $W_{P\cap Q}$ represented the weight of the constituents' interaction (the weight of the product $\deg(d,P) \times \deg(d,Q)$). For example, the values for "pets which are birds" were: $W_{\text{pets}} = .30$, $W_{\text{birds}} = .78$, and $W_{\text{pets}\cap\text{birds}} = .10$.

Second, the typicality ratings in modified nouns with negated constituents, $\deg(d,P \land \neg Q)$, were predicted by adding a negative sign to the weight of the negated constituent ($-W_Q$). The interaction term was also negative, when significant (11b). For example, for "pets which are not birds" the weights were: $W_P = .32$, $W_{\neg Q} = -.75$, and $W_{P\cap\neg Q} = -.11$. Why? Because the better an item is as an example of Q, the worse it is as an example of not Q.

\[
\begin{align*}
(11) \quad a. \quad \deg(d,P \land Q) &= W_P \deg(d,P) + W_Q \deg(d,Q) + W_{P\cap Q} (\deg(d,P) \times \deg(d,Q)). \\
b. \quad \deg(d,P \land \neg Q) &= W_P \deg(d,P) - W_Q \deg(d,Q) - W_{P\cap\neg Q} (\deg(d,P) \times \deg(d,Q)). \\
c. \quad \deg(d,P \lor Q) &= W_P \deg(d,P) + W_Q \deg(d,Q) - W_{P\lor Q} (\deg(d,P) \times \deg(d,Q)).
\end{align*}
\]

Third, given the logical connections between disjunctions, conjunctions and negations ($P \lor Q = \neg (\neg P \land \neg Q)$), and the fact that negation affects the equation by changing the coefficient sign, it was predicted that the typicality ratings in disjunctions like "hobbies or games", $\deg(d,P \lor Q)$, would be given by adding a negative sign to the interaction term ($-W_{P\lor Q}$). Why? The value $\deg(d,P \lor Q)$ ought to be identical to $\deg(d,\neg (\neg P \land \neg Q))$, which, in turn, should be given by an equation in which a negative sign is added to the weight of each negated constituent (namely by the equation: $-(\neg W_P \deg(d,P) - W_Q \deg(d,Q) + W_{P\lor Q} (\deg(d,P) \times \deg(d,Q)))$. After the elimination of double negation signs, this equation reduces to the one in (11c), with the negative interaction-weight. And indeed, using (11c), Hampton (1988) could predict the typicality ratings in disjunctions from the ratings in the disjuncts. In this case, the logical rules have triggered the discovery of the disjunctive pattern.

Patterns similar to those summed by the equations in (11) emerged also when the probabilities of classification of items in modified nouns (the number of their positive ratings in all the subjects) were compared to probabilities of classification in their constituents.

However, the most crucial observation has been that the typicality ratings in modified-nouns are better fitted by composite-prototype representations (cf. 4.2.3), where the weight of each dimension is adjusted by a special function (compared to the more coarse constituent-based equations in (11)).

For example, negated constituents sometimes had a decreased weight. This seems to have happened because some dimensions were treated as characterizing both the predicate and its negation. For example, 'animate' often characterizes both birds and entities which are not birds, and 'birdhood' characterizes both robins and non-robins.

This finding is important. Researchers frequently assume that the composition of a typicality scale for negated predicates is based on an inverse rule. In every context c, the typicality ordering of $\neg P$ is the reverse of P's typicality ordering: $[\leq \neg P]_c = [\geq P]_c$ (cf. Zadeh 1965; Kamp 1975; Smith et al. 1988, etc.). In fact, Smith et al. (1988) found that the ordering of instances in non-red fruit is essentially the reverse of that in red fruit (p. 511). However, exceptions to this pattern have been observed, too, which show that indeed, the generalization in (12) is a more precise description of the facts:

\[
(12) \quad \text{For every predicate } P, \text{ the ordering of } P \text{ and } \neg P \text{ are similar along dimensions that contextually characterize both } P \text{ and } \neg P. \text{ They are reversed only along }
\]

\[\text{\textit{6} Costello (2000) and (2002) has predicted typicality in modified nouns by multiplying the degrees in the constituents (deg}(d,P \cap Q,K) = \deg(d,P,K) \times \deg(d,Q,K)), but this has required a certain manipulation of the value of the contrast set, K.} \]
dimensions of \( P \) that are not taken to characterize \( \neg P \) in the context

This phenomenon of contextually restricted negations was noticed back by Hegel (1892, in Horn 1989:64) for whom a pure negative statement like the rose is not red suggests that a different predicate from the same semantic class (e.g., that of colors), applies to the subject. So the rose is not red still implies that it is colored. A variety of effects which result from the existence of non-negated (retained) dimensions are described in Hampton (1997).

We see that in negated predicates, the typicality ratings are best fitted using a composite prototype. But how is the prototype of a negated predicate figured out? This last question has remained open. In chapter 8, we will propose a general strategy for the selection and weighing of dimensions in either simple or complex predicates. In the following, I discuss leading proposals about dimension selection in (the concepts denoted by) lexical predicates.

4.3 The representation of knowledge about prototypes

Obviously, the set of possible dimensions for each concept is infinite, whereas the sets which are encoded in memory are finite. Thus, theories which associate categories and individuals with dimensions seek to determine how the dimensions of basic concepts are selected, how selected values on these dimensions are chosen, and how these dimensions are assigned weights. This task was discovered to be highly difficult. We will discuss two leading criteria, the probabilistic criterion and the knowledge-based (or theory-based) criterion.

4.3.1 The probabilistic criterion for selecting and weighing dimensions

4.3.1.1 The probabilistic criterion

Classically, prototype theories assume that dimension sets emerge from observed co-occurrences of properties in the world (Tversky 1977; Rosch 1973; Rosch 1978; Rosch and Mervis 1975). According to this probabilistic (or empirical) view, the weight of a dimension is monotonically related to its frequency in the concept, and is inversely related to its frequency outside the concept.

This principle aims at predicting that, for example, most bird kinds are small and can fly. In addition, it aims at predicting that violating dimensions which are common in contrast categories (satisfying their negations) raises the typicality of entities. For example, if two items are equally good in the bird dimensions, the one which is less good in the dimensions of other animal types (such as mammals or reptiles) is regarded as more typical in bird (Rosch and Mervis 1975).

We can formalize the probabilistic view by stating that the weight of a dimension \( F \) in a category \( P \), \( \text{Weight}(F,P) \), should equal the ratio (the difference) between the number of \( F \) instances within the category, \(|\mathcal{P}\cap\mathcal{F}|/|\mathcal{P}|\), and the number of \( F \) instances outside the category, \(|\mathcal{F}\cap\neg\mathcal{P}|/|\neg\mathcal{P}|\). Following Costello (2000, 2002), I see this principle as reflecting the extent to which \( F \) is necessary (how many entities in \([P]\) are also in \([F]\)), and the extent to which \( F \) is sufficient (how many entities in \([\neg P]\) are also not in \([F]\)). For example, this principle predicts that the weight of the dimension divisible by 2 without a remainder for the predicate even will equal 1 (cf. (13a)) and the weight of this dimension for the predicate odd will equal –1 (cf. (13b)). But \( \text{Weight}(\text{flies}, \text{bird}) \) will be less than 1 and more than –1, because some birds do not fly and some non-birds do fly.

\[
(13)
\]

a. \( \text{Weight}(\text{divisible by 2 without a reminder, even}) = \)
\[
\begin{align*}
([\text{even}] & \cap [\text{divisible}]) / [\text{even}] = 1 \\
([\neg \text{even}] & \cap [\text{divisible}]) / [\neg \text{even}] = -1 
\end{align*}
\]

b. Weight(divisible by 2 without a reminder, odd) =
\[
\begin{align*}
([\text{odd}] & \cap [\text{divisible}]) / [\text{odd}] = 1 \\
([\neg \text{odd}] & \cap [\text{divisible}]) / [\neg \text{odd}] = -1 
\end{align*}
\]

However, Costello (2000, 2002) calculated dimension weights without taking into account entities which are neither P nor F. Rosch and Mervis (1975) calculated dimension weights without taking into account any entity outside \([P]\) \((W(F,P) = | [P] \cap [F] |)\). For practical purposes, this gave a good enough estimation of dimension importance (but the formulation we gave demonstrates well enough the kind of ways the probabilistic view is implemented in practice).

In addition, according to the probabilistic view, the selected value of each dimension stands for its average value in the category – the value which maximizes the similarity of the category members to each other, that is, the value which minimizes the distance between them in the dimensions. For instance, the prototypical size, \(\text{deg(bird,size)}\), stands for the average size of the category members (the average size among the birds). In practice, usually the degrees in dimensions like flying are reduced to degree 1 if most members fly and 0 if most members do not fly. At any rate, the selected values in the dimensions are thought of as standing for a central tendency within the category.

4.3.1.2 Evidence for the importance of statistical regularities

In support of the probabilistic principle, Hampton (1987) shows that dimensions denoting necessary conditions for membership in a category weigh more heavily than dimensions which are typical but not necessary of category members. However, not all the necessary dimensions are alike: too general dimensions, like animate for bird, are not useful in distinguishing one bird from another, and in practice they usually restrict the contrast set as well. That is, we normally use the term non birds to refer to animate beings, and indeed, subjects often omit this dimension or consider it non diagnostic.

The probabilistic view appears to play a crucial role in both comprehension and production of noun-noun compounds (Costello 2002a). For example, dimensions like has tusks can be taken to be highly diagnostic of elephants because people can base their categorization of animals as elephants based on this dimension (it is taken to be sufficient, or nearly sufficient, for elephant-hood). Participants preferred to interpret compounds such as an elephant pig by such diagnostic properties (pigs that have tusks, pigs that are big) than by other properties (pigs that are grey, pigs that are endangered species). In addition, participants more frequently proposed to name objects by the compound an elephant pig if they were described to them by diagnostic properties (like pigs that have tusks), than if they were described to them by non-diagnostic properties.

Tversky (1977) presented direct evidence for dimension diagnosticity and its context dependency. Subjects that were presented with the country list: Austria, Sweden, Poland and Hungary, judged Sweden to be most similar to Austria, and they grouped Austria with Sweden and Poland with Hungary. Subjects that were presented with the country list: Austria, Sweden, Norway and Hungary judged Sweden to be most similar to Norway, and they grouped Austria with Hungary and Sweden with Norway. The dimensions east / west Europe were more diagnostic of the grouping in the first list than in the second list, while the dimensions Scandinavian and non-Scandinavian were more diagnostic of the grouping in the second list. This shows how highly context dependent diagnosticity is. Adding or eliminating one object
to/from the domain may change the grouping of the domain into categories and the
diagnosticity of dimensions in the categories.

4.3.1.3 Problems with the probabilistic view

Problem 1: The probabilistic criterion ignores the gradedness of the dimensions

The probabilistic criterion presupposes that the category dimensions are not graded, or that
the entities' degrees in the dimensions do not affect the dimensions' weights. This seems
inadequate given that the weight of a dimension F in P should reflect the extent to which
entities' degrees in F affect their degrees in P. The weight should be determined by the extent
of overlap between (or the relative size of the intersection of) F's orderings, ≤F, and P's
ordering, ≤P. The larger the set of pairs in identical ordering relations in P and in F, and the
smaller the set of pairs in inverse ordering relations in P and in F, the more diagnostic F is of
the ordering of P (≤P and >P). For example, the ordering of bird is identical to the ordering of
small (which in the context of bird means a robin-sized-bird), with only few exceptions (cf.
Figure 12), so this dimension's weight is big. It plays a central role in ordering entities by
typicality in bird. (Category members may play a bigger role in determining the weights of
dimensions, than non members).

![Figure 12: High overlap between the typicality-ordering of bird and of small / flies.](Image)

Exceptions (non-overlaps) are marked in red circles.
(Birds in the same block are, roughly, equally typical).

Problem 2: The probabilistic criterion is too extensional

The probabilistic criterion seems to be too extensional. It weighs dimensions within each
context separately, while ignoring the possibility that certain dimensions might be tightly
connected to the meaning of a word or a concept, and hence be taken as characterizing the
concept even in contexts which violate the required statistical regularities. Indeed, despite of
the abundant supporting evidence for the probabilistic criterion, several findings form
counter-examples to it. They show that, as it stands, it does not represent correctly the
conditions under which a property counts as a dimension of a category. Let us shortly review
some of the problematic findings.

First, dimensions of old categories appear to be treated as atypical of new categories
regardless of frequency. This phenomenon is called the base rate neglect. When one category
is much larger than others, it is learned faster and is associated with distinct dimensions
(which are common within it and infrequent outside of it, as predicted by the probabilistic
model). However, when attention is shifted to the learning of a second (smaller) category,
person tend to associate it with dimensions which are not highly distinctive of the first
category, but are relatively common in the second, even if in practice the dimensions are more
common (more objects have it) in the first category than in the second. So a dimension is
treated as a better predictor of membership in the second category even when it is actually a
better predictor of membership in the first category (Kruschke 1996). This problem indicates
that dimensions may become identified with a category regardless of the constraint on their frequency in or outside the category.

Second, frequency cues (coverage) play a minor role in induction. Rips (1975) and Osherson et al. (1991) systematically studied the conditions under which people conclude that an item possesses a certain property (say – having sesmoid bones or an ulnar artery), given a premise about another item possessing that property. Given the probabilistic criterion, it was expected that the more diverse the premise categories are (e.g. if they contain robins and ostriches rather than just robins and sparrows) and the more premises you add (the more bird types possess the dimension by premise), the stronger the argument is. These two generalizations together were supported in western adults. They directly represent the effect of within category dimension frequency (the degree of coverage of the category by the property). However, Lopez et al. (1992) found that children in kindergarten do not rely on coverage. At 8 years old (second grade) they only use coverage in certain situations (in inferences about all animals but not about an individual animal). In addition, Lopez et al. (1997) illustrate that adults in different cultures do not use coverage. Thus, perhaps coverage-based induction is an acquired cultural practice, not a cognitive universal that everyone eventually develops (Murphy 2002: 252).

Third, the view that category sets are inferred from observed co-occurrences of dimensions in the world triggered an empirical examination of whether people are indeed sensitive to correlation between dimensions. Certain subsets of a set may contain dimensions which more highly correlate with each other. For example, large beaks correlate with fish eating. More generally, the dimensions characterizing each sub-kind (or exemplar) of each category highly correlate with each other. The question is whether people attend to these correlations. This is of particular importance also because while averaging on an item's degrees in the dimensions in the calculation of its typicality (its similarity to the category dimensions or instances), most prototype theories treat the dimensions as independent, while exemplar theories treat them as non independent. Contrary to the predictions of the probabilistic view, attention to dimension correlation tends usually to be rather poor. Experiments show that, normally, people who learn several categories simultaneously do not attend to dimension correlations (see for instance, Malt and Smith 1984), unless the task directly requires it (Chin Parker and Ross 2002). In addition, when people possess a priori knowledge about causal or logical connections between dimensions, they judge items which satisfy both dimensions as better than others, even if in effect, the dimensions do not co-occur often (Murphy and Wisniewsky 1989). Furthermore, prior knowledge about connections between dimensions and categories may be more important than observed statistical regularities. To take a simple example, most vehicles are less than ideal people-movers, so the prototype (the ideal vehicle) cannot be based on the averaged (or common) value in [[vehicle]] on each dimension. Finally, experiments show that knowledge may also raise expectations about frequencies (Barsalou 1985; Wisniewsky and Medin 1994; Murphy 2002).

In sum, the main problem with the probabilistic view is that, sometimes, dimensions are taken to be characterizing concepts even in contexts which violate the required statistical regularities. These findings form the basis for the knowledge theory (or 'theory' theory) that views concepts as constituents in a large knowledge structure (Murphy 2002: 487-9).

7 Other inductive effects which were expected given the probabilistic criterion were supported empirically, but these effects may have explanations which are not frequency based (cf. 2.2.63-2.2.6.4).
4.3.2 The 'knowledge' criterion for selecting and weighing dimensions

4.3.2.1 The 'knowledge' theory

The knowledge (or theory) theory (Murphy and Medin 1985; Barsalou 1985; Keil 1989; Wisniewsky and Medin 1994; Gopnik and Meltzoff 1997; Murphy 2002) is the view that our knowledge representations are organized around naive and possibly fallacious theories (folk psychology, biology, physics etc.), and that the properties of concepts are determined by the roles they are playing within these theories. It is assumed that children's representations of the world are also organized around theories and that developmental changes are revisions in these theories, which are precisely of the same type as scientific theory revisions (Gopnik and Meltzoff 1997). The theory view puts strong emphasis on abstract "hidden" dimensions, rather than on observable properties. For instance, for the purpose of reference determination, the entities falling under the concept bird are better characterized by some bird essence (properties assumed by some folk genetics theory) rather than by properties such as small, sing, have wings, flies, nests in trees etc. The latter properties may take part in the representation of the concept's structure, but not as a simple list. Rather, each property is linked to the others by causal or goal directed connections, depending on the theory (e.g. birds have wings to enable them to fly, they fly to escape predators etc.) Similarity still plays a crucial role in categorization in this approach (Hann and Chater 1997). However, similarity is taken to be influenced by our theories of the world, rather then by sets of unrelated dimensions, perceptual in essence.

For example, Barsalou (1985) argued that abstract dimensions which determine the category's goal or use (say – transportation for vehicles) are often hidden (not mentioned by subject), but they are most important in determining category structure. Indeed, Barsalou found that the items' degrees in the goal derived dimensions account for 0.45 of the correlation between the typicality and family resemblance ratings (the weighted mean in the dimensions) and that these dimensions affect the weighing of other dimensions. For example dimensions like tend to jog weighed (i.e. affected typicality) more heavily, in the category teachers of physical education than in the categories teachers of current events / computer languages, regardless of their equal co-occurrence frequencies. Barsalou (1985) observed that in complex ('ad-hoc') categories like things to take from home in case of a fire or food for diet there are no perceptual similarities at all between the members. Typicality in these categories is determined solely based on the goal derived dimensions.

Representatives of the knowledge theory distinguish these dimensions from the normal typicality dimensions, by viewing them as part of domain general knowledge structures. While the typicality dimensions which are induced empirically stand for some average value in the category, the knowledge-based dimensions stand for ideals. For example, the ideal vehicle is not the average value of all vehicles, because most vehicles are less than ideal people movers. If people relied on the vehicles they had seen, they would find only moderately effective people movers to be the most typical vehicles. Thus, typicality is not always based on statistical regularities (family resemblance scores). Some dimensions are not selected based on an empirical examination of their frequency of co-occurrence with category members, but based on prior knowledge (Wisniewsky and Medin 1994: 267; Murphy 2002).

Crucially, the knowledge called domain general can often be described as a set of highly important, or, practically, necessary dimensions. For example, Lin and Murphy (1997) taught subjects about an unfamiliar artifact in a foreign country named tuk. For some subjects the tucks were described as hunting tools, which are used by slipping the loop on the tuck's top over the animal's head. For others the same objects were described as fertilizing tools, which hold a fertilizing liquid in the tank at the middle of the tuck and use the loop at the top to hang
up in storage. The first group did not categorize items as tucks if the loop at the top was missing, while the second group did. Hence, the loop was treated as practically necessary for membership (as highly important typicality dimensions). The dimensions which describe the category's use or goal are treated that way, at least in tool categories (Murphy 2002).

Most importantly, the knowledge-based dimensions affect online categorization rates. Consequently, they affect the category structure (Wisniewsky and Medin 1994). If so, possibly what we call domain general knowledge is not different from what we call domain specific, in that both are represented as part of concepts' structure, or, alternatively, we have one large knowledge structure which the concepts are its constituents (Murphy 2002: 487-9). But how should a knowledge structure be represented? Let us review classical and contemporary representations of knowledge about prototypes.

4.3.2.2 The classical representation of knowledge about prototypes

Classical dimension theories are static. They presuppose that competent speakers possess complete knowledge about the natural concepts, and they view dimensions as innate or directly given concepts with no internal structure (say – sensory properties; Locke 1968 [1690]), that define all the other concepts. There are several problems with this classical view.

Problem 1: Dimensions and concepts are not treated alike

First, many dimensions (say, eats insects or has wings) are no more sensory (or otherwise simple) than the concepts of which they are dimensions (e.g., bird; Fodor et al. 1980). Second, many dimensions are not acquired earlier than the concepts of which they are dimensions. For instance, a concept like kill is acquired earlier than the concept cause, which is assumed to be one of its building blocks, given that we understand to kill as meaning to cause death. In general, the idea that sensory dimensions are more basic was empirically refuted within the study of perception and its development. Learning of the dimensions seems to occur in parallel with category learning, not as a (possibly innate) precondition (Wisniewski and Medin 1994: 265-267; Murphy 2002: 479).

The classical view of dimensions as inherently different from concepts (as building blocks for concepts) was objected also based on theoretical grounds, when Quine (1961 [1951]) has criticized the analytic versus synthetic distinction. Quine argued that we cannot distinguish between truths that are grounded in meanings independently of matters of fact, and truths that are grounded in fact. Quine argues that confirmation of a claim is holistic in that it always relies upon auxiliary hypotheses. For example, the truth of it does not rain depends on assumptions regarding the meaning of the negation, the law of excluded middle and so on and so forth. Given a counter-example, it is impossible to determine whether the claim or any of the auxiliary hypotheses ought to be abandoned. Any statement can be held true come what may, if we make drastic enough adjustments elsewhere in the system.

Accordingly, a holistic theory does not assume the existence of any a-priorily given sensory concepts, in terms of which other concepts are defined. The conceptual system is taken to be defined in a holistic way, like dictionaries are: Each word is defined by the other words. None has the status of a primitive. The dictionary's circularity is not problematic as long as concepts or expressions are also linked to external non-linguistic objects, namely, denotations. Accordingly, each word may function both as a category and as a dimension of other categories. A statement of the form F is a dimension of P expresses a connections between two concepts (or words) P and F. This connection constrains the possible ordering relations (similarity functions) and denotations of P and F.
Problem 2: There is no representation of context-dependency

Given that they regard speakers' knowledge as complete and static, the classical psychological theories cannot capture cases of instability (Laurence and Margolis 2003). One such case is the problem of ignorance, namely the fact that people are not always aware of some parts of the concepts' structure of others. Children and adults may learn the set and denotation gradually. Another case of instability is the problem of error, i.e. the fact that sometimes people make mistaken assumptions about concepts' structure. In addition, people can argue about a concept with other people who characterize the concept differently (a case of instability between people), and they may sometimes correct (or change) their assumptions (a case of instability within a person). For example, subjects sometimes characterize plums as sweet, sometimes as sour; sometimes they characterize tractors by two wheels, other times by four wheels etc. (Armstrong et al. 1983). Finally, the similarity degree functions in the classical psychological theories are usually defined to be total and context independent, though speakers' knowledge about typicality is not total. For example, is the property in the house characteristic of chairs or not? Well, in some contexts it is, in others (when restaurants, offices, clinics, etc. are discussed) it is not. So we cannot tell this fact, out of context. Sets of typicality dimensions are contextually given and may be partial, and so are the typicality functions (cf. Tversky's 1977 findings).

The static nature of the theories (the fact that there is no explicit representation of context) is manifested also in the following problems. It was observed that there are infinitely many potential dimensions (Murphy 2002: 456; 459). Crucially, dimensions can always be added to the similarity calculation. But then, if they are shared by the entity which is being rated, d, and the concept's prototype, d1, they raise the similarity of d to d1. Hence, in principle, similarity as defined by Tversky (1977), has no upper bound, even when the similarity of an entity or a concept to itself (of d1 to d1 or P to P) is considered. This is highly unintuitive (Hann and Chater 1997).

Conversely, given that the finite resources of human beings do not allow them to pay attention to infinite dimension sets simultaneously, some researchers limit the set of dimensions of each concept to a rigid finite set (such are, for instance, frame models, like the one in Smith et al. 1988, as described in the Appendix). The problem for these models is that one can always find a context, whereby a dimension which is not represented is relevant for the interpretation of the concept. It is easy to see this when one considers the frame-based compositionality theory (according to which modifiers modify the weight and selected-value of one of the dimensions in the noun prototype). If the set of dimensions of each noun is limited to a rigid finite set, the theory runs into troubles in representing modifications of the noun by adjectives encoding dimensions which are not assumed to be part of its prototype. For example, the attribute orientation is usually not assumed to be part of the prototype of fruit concepts, but it needs to be represented in order for combinations like upside-down fruit to be interpretable (Smith et al. 1988). Things only get worse when more abstract or complex concepts are considered such as creative or detailed, which can be manifested in many different ways (Wisniewski and Medin 1994: 268-269). (Similarly, the frame model as it is cannot represent gradable dimensions like size properly, because, in principle, there are infinitely many values in such dimensions.)

These problems can only be solved by an explicit representation of the context dependency and partiality of the information one possesses about concepts in each context. That is, a parameter which represents the contextual stage should be added to the representation, and the relation between different contextual stages (the structure of contexts) should be clarified. In chapter 5 and on, I propose a theory that incorporates dimension sets into such context structures.
The context dependency of the representation relates also to the averaging method that is used in calculating similarity. Different averaging strategies are used in different occasions (Smith and Minda 1998, 2000). Thus, the averaging method, like the dimension values and weights, should better be represented as fixed within context (or as semantic arguments of nouns which are not lexically expressed).

Another issue that is related to the lack of representation of context is the following. Frequently, psychological theories (for instance the frame theories described in the appendix) assume that a category like apples can be characterized by several non-overlapping (contradictory) dimensions (red, green, yellow etc.) simultaneously; albeit to different extents (e.g. being red may make you a better apple than being green). The theory does not specify the way each speaker actually derives these extents (the weights for the dimensions red, green, etc.) In practice, they are determined by the number of times each dimension is mentioned by a pool of subjects. Perhaps the number of times a dimension is mentioned in a pool of subjects corresponds to the number of times a single subject counts it as a dimension of the concept through contexts. In addition, perhaps this number is related to factors such as frequency of occurrence of the dimension in the denotation or the dimension's perceptibility (Smith at al. 1988). However, it is also reasonable to assume that subjects putting green in the dimension list of apple assume a different context than subjects putting red in that list. The latter, but not the former, may have assumed a context in which red apples are more typical than green ones. It is plausible to assume that within each particular context, at most one dimension (either red or green, or maybe red or green) are simultaneously taken to be part of the prototype of apple or of any apple exemplar. The representation of a set of inconsistent dimensions (a frame) may only be required in order to predict "context-independent" typicality judgments, namely, ratings of degrees which were produced by different subjects, or, equivalently, by one and the same subject in different contexts.

4.3.2.3 Contemporary representations of (the acquisition of) knowledge about prototypes

Contemporary models put forward a more dynamic view of the conceptual system, according to which people keep changing their assumptions about the structure of concepts in the exposure to new information. Dynamic models try to explain how prior knowledge affects the acquisition of new knowledge. Prior knowledge includes both knowledge about specific categories, and knowledge of general strategies that help to reduce the space of hypothesis during category learning.

For example, Heit (1997) predicted that judgments of the probability of a jogger to possess the dimension wears expensive running shoes are arrived at by summing up both the expected probability to possess this dimension as given by prior knowledge, and the observed probability in the new exposure to joggers.

In connectionist models (networks), prior knowledge equals to the strength of connections between input and output nodes, which is then adjusted according to new data (Lamberts and Shanks 1997: 23). Learning to make a categorization decision equals to assessing the levels to which output nodes would be activated, after a pattern of inputs is presented to the network. These models are dynamic in that they include an initial representation for a category, and an integration (or "anchor and adjust") process of revision of the initial representation according to new observations. For example, Kruschke (1992) modeled learning of dimension weights, using a connectionist network model. The network produced categorization decisions, and the weights that were initially assigned for dimensions were adjusted, given feedback about categorization errors. Ultimately, the dimension weights were chosen to be the values which produced the most precise categorization judgments.
However, Wisniewski and Medin (1994: 272-277) and Murphy (2002) argue that contemporary representations of knowledge and its gradual growth are still problematic. Let us see why.

4.3.3.4 Problems in contemporary representations of knowledge about prototypes

Problem 1: Dimensions and concepts are still not treated alike

In practice, current theories are still classical in the sense that they do not treat dimensions and concepts alike. First, they often model dimensions as binary, simple and perceptual, whereas natural concepts are claimed to be gradable, and in effect dimensions may be complex and abstract. Second, degrees in dimensions, unlike degrees in concepts, are taken to be free parameters which are not determined by the theory, and they are represented as completely given, while in effect they may be learnt together with the concepts’ degrees. Thus, dimensions still have the status of some kind of primitives. The conceptual system is not defined in a holistic way.

Many researchers infer (some of the) dimensions of a concept through a process called multidimensional scaling. This method and the standard critic of it can help clarifying the problem with the non-holistic nature of the representation. In this method, subjects rank the degree of similarity of each pair of category members (for example, robin and sparrow) and of each member and the concept itself (e.g. robin and bird). These degrees are then represented visually as distances between points in a two-dimensional or a higher space (Rips et al. 1973). For instance, robin and sparrow are usually ranked more similar than robin and duck, so the distance between the points representing the former pair ought to be smaller than the distance between the points representing the latter pair. The points representing the more typical sub-concepts are usually seted together and are closer to the point representing the concept. Crucially, the axes can then be taken to stand for meaningful typicality dimensions. For instance, it is possible to represent the similarity judgments in bird in a two dimensional space, whereby the horizontal axis represents size and the vertical one represents ferocity. The points representing typical birds are located within the zone of relatively small creatures, in the middle of the ferocity scale. In this method, the dimensions emerge from a set of similarity judgments which are empirically assessed. In fact, it is often criticized because the similarity judgments themselves are viewed as based on the set dimensions (subjects need to know the dimension set in order to generate similarity judgments; Hampton 1997a).

In my view, neither the dimension set nor the similarity judgments need to be more basic. Most likely, our (partial) knowledge about the dimension set of each concept and our (partial) knowledge about the similarity degrees of instances in the dimensions are simply mutually restricted, and the theory only needs to state how (for a proposal see chapter 5 and on).

In sum, psychological and semantic theories alike fail to represent partial information about gradability (about mappings of entities to degrees along ordering dimensions). More complete models need to be developed to describe this type of knowledge and its gradual growth. In chapter 5 and on, I propose such a model.

Problem 2: How do different sources of knowledge affect prototype formation?

Contemporary models do not clarify how different sources of information (statistical and knowledge-based) interact in the formation of dimension sets (Wisniewsky and Medin 1994: 269-272).

In addition, Heit (1997) argues that the current models do not address the processes by which a learner would determine which prior knowledge is relevant for the formation of a given
category (knowledge selection processes). Markman (1994) suggested that, while trying to discover the meaning of a word, children are biased to assume that the new word is more likely to refer to a whole object (say, a dog) rather than to its parts (a leg), to refer taxonomically (to a set of dogs) rather than to a scheme (a dog, an owner, a leash and so on) and to be mutual exclusive (i.e. to refer to new yet nameless objects). A bias of generalizing on the basis of shape, in contrast to texture or size, emerges through experience with shape-based categorization. Furthermore, Heit (1997) argues that the process by which prior knowledge is selected, which is relevant to the learning of a new category, may involve conceptual combination, if the new category is conceived of as a complex concept (for instance, when it is presented by a complex phrase). More complete models need to be developed to describe these processes.

In sum, a more complete dynamic model should clarify the ways prior knowledge affects the meaning of new and old categories, and the notion of domain general as opposed to domain specific knowledge (or inferred as opposed to directly given knowledge), about concepts' sets and denotations.

4.4 Apparent dissociations between judgments of typicality and membership likelihood

Some experimental results show that speakers' subjective probability judgments may be dissociated from their typicality judgments. For example, Teigen and Keren (2003) show that surprise judgments are sometimes linked to typicality judgments but not to judgments of probability or expectedness. This shows that typicality cannot be equated with subjective probability. Can the view that categorization is based on typicality be maintained? If, empirically, we find dissociation, that's a problem for that view, and may count in favor of theories that predict dissociation. Let us examine further a number of apparent counter-examples to the view that categorization is based on typicality. I would like to argue that they are only apparent, caused by easily identifiable and plausible factors. And, hence, this view can in fact be maintained.

4.4.1 Shifts of weights

Consider examples (14a-c).

(14) a. axat ha-tofaot ha-yoter-s’xixot ve-ha-lo-tipusiot hikocer nes’ima bemaa’mac lelo muaka baxaze
   'One of the more frequent and non-typical phenomena related to Angina Factoris; GS], is shortness of breath in effort without discomfort in the chest'.

b. Dan is more likely a Kibbutz member than Sam, though Sam is more typical of a Kibbutz member.

c. Some non-biological fathers are more typical fathers than some biological fathers.

They seem to show that, while using the operator typical, shifts in dimension weights typically occur, reducing the weight of dimensions that are normally regarded very important (effectively necessary for membership) in the category (Hampton 1998). For example, one may believe that Dan is more likely a Kibbutz member than Sam (because Sam is violating some properties which are effectively necessary for Kibbutz members), but nevertheless, that it is Sam who is more typical of a Kibbutz member. Indeed, in Hampton's (1998) systematic
examination of the relations between typicality and membership likelihood (based on the data published by McCloskey and Glucksberg 1978), many deviations were shown to occur due to shift of weights towards non-definitional perceptual criteria in typicality judgments, compared to membership judgments. This shift has caused an increase in the typicality of non-members.

Shifts of weights are in effect changes in the context of interpretation. However, shifts of weights between typicality and categorization tasks may originate from semantic differences between the ordering relation which is being used in the typicality versus likelihood judgments, rather than from mere context shifts. In fact, the psychological theories often ignore the fact that there are differences between different comparative relations like more P, more of a P, more typical of a P, more similar to P, more likely P, etc. If subjects implicitly use the modifier typical of when they make typicality judgments, then (apparent) dissociations between typicality and membership are more than expected (for proposals regarding the meaning of "typical of" see chapter 7-8). The ordering relation of typical of P and of P may both be monotonically related to membership, but each one is related to membership in a different set (namely, the set of instances which are typical of P and the set of instances which are P, respectively). Naturally, some things (for instance, biological fathers) may fall in the denotation of father, but not in the denotation of typical of a father. Thus, monotonicity requires that these things would not be 'less fathers' than non father (say – non-biologically fathers), but it allows that they will be less typical fathers than atypical fathers (such as the non-biologically fathers). Most likely, an implicit use of typical is triggered in comparative tasks, given the ungrammaticality of nouns in comparative structures. This explains the findings which are classified by Hampton under shifts of weights.

4.4.2 Comparisons by relative position

Rips (1989) argued for a dissociation between typicality and membership in cases where an object d, halfway between a quarter coin and a pizza, was categorized as a pizza, but was rated as more typical in (or similar to) a quarter. In such examples, speakers compare the status of entities in two different concepts.

We may assume after Hampton, that, again, a context shift occurred in the typicality ordering judgment. For example, the weight of the definitional criteria for quarters (a certain diameter size), which played a crucial role in the categorization task, decreased in the ordering task.

However, Rip's data do not contradict the assumption that typicality is associated with membership, even in the lack of context shifts. Perhaps, entities which are mapped to low degrees in the interval [0,1] are still part of the denotation of pizza, and so d is a pizza but not a typical pizza (there are many other types of pizza which are more typical than d). At the same time, perhaps only entities which are mapped to high degrees in the interval [0,1] are part of the denotation of quarter. Thus, d may not be an element of this denotation, despite of being rather typical of a quarter. Crucially, one and the same object can have an excellent position on the scale of a category to which it does not belong, if many other non-members exist, which are even worse than it in the category dimensions. Similarly, one and the same object can have a bad position in a category to which it belongs, if many members are even better than it in the dimensions. Thus, we do not expect a stimulus which is categorized as a pizza and not as a quarter to be rated as more typical in (or similar to) a pizza than to a quarter!

Interestingly, such dissociations completely disappear when a richer set of membership and ordering criteria is made salient (which may turn d less typical of a quarter or more typical of a pizza), and when subjects are not asked to explain their decisions by a conscious reflection (Smith and Sloman 1994).
4.4.3 Typicality effects in definitional concepts

In Armstrong, Gleitman and Gleitman (1983), typicality ratings were assessed also for categories which the authors take to be well defined (such as even number and geometrical figures, but also female). The results show that the typicality effects "crop up" even in well-defined concepts, where membership, the authors argue, cannot be graded. These results were replicated by Larochelle, Richard and Soulieres (2000), for a much longer list of (relatively) well-defined categories and a larger number of subjects. Armstrong et al. (1983) also demonstrated that online categorization time (verification time in sentences such as n is an odd number) was shorter in typical compared to atypical items. A differential categorization time effect was not dependent on a lack of a definition. Rather, it cut across the ill-defined versus well-defined distinction. Finally, graded typicality ratings were given even by subjects who said no when asked "does it make sense to rate items for degree of membership in the category?", namely, subjects who admitted that membership in the category was not graded. Armstrong, Gleitman and Gleitman (1983) concluded from these results that the typicality effects say nothing about category structure.

This conclusion is problematic given that, more often than not, typicality is coupled with categorization judgments (see 2.2.2 and Murphy 2002). Thus, more likely, the typicality effects in well-defined categories only show that typicality judgments are not inconsistent with definitions (cf. the discussion of Kamp and Partee's (1995) model in 3.2). Two items can be regarded as category members, and at the same time, have different degrees in the category. Moreover, the lack or existence of a definition cannot explain the variability in the percents of subjects admitting that membership is not graded. These percents ranged from 24 to 33, 43, 71, 86 and 100 in different categories (for data-tables see the Appendix). Most likely, in certain categories, it was easier for the subjects to see how membership could be graded (or what the ordering criteria were), while in others this was a harder task, regardless of the binary distinction ill- / well-defined. In addition, the mean typicality ratings (the tendency to judge examples as atypical) were slightly decreased when a category was said to be non-graded. A conscious reflection on the gradedness of membership affected the typicality ratings too. Thus, there is an association, not dissociation, between membership and typicality. This association also explains the fact that the mean typicality ratings of exemplars was significantly lower in the well-defined categories than in the ill-defined ones, in all the experiments (meaning that, on average, instances of well-defined categories are considered better examples of their categories compared to instances of ill-defined categories). Hence, these effects do say something about category structure. The so-called ill-defined categories have more typicality dimensions and less definitional dimensions than the well-defined ones. Thus, more of their members are considered worse examples.

In fact, the third type of deviations from the general pattern of coupling between typicality and membership likelihood reported by Hampton (1998) is due to unfamiliarity, namely, lack of knowledge about the dimensions (or dimension values) of members reduces their typicality. Indeed, Larochelle, Richard and Soulieres (2000) found that the familiarity gradience accounts for the typicality effects, (which for them are only apparent typicality effects) in definitional concepts. But an adequate account has to state precisely what familiarity is, and how it might affect judgments of goodness of example (typicality). In chapter 8, I propose a possible answer to these questions. Note that definitional concepts are characterized by the lack of a rich set of typicality dimensions. In 8.3 I propose that the coupling between familiarity and typicality in the lack of knowledge about the category dimensions is highly expected, and I explain why.
4.5 Linguistic distinctions between predicate types

The main problem with incorporating the psychological analysis of nouns into the linguistic theory is that important distinctions between predicate types might become blurred.

The first distinction is between nouns and gradable adjectives. The psychological theories fail to provide an explanation for the linguistic distinctions between adjectives and nouns, which were described in the first set of facts in chapter 2 (2.1). Namely, if nouns are indeed gradable predicates, it is not clear why they cannot felicitously occur in within-predicate comparisons (as in *Tweety is more (a) bird than Tan), superlative structures (*birdest; *most a bird), and other degree structures.

The second distinction is between nouns and multi-dimensional adjectives. The standard cognitive theory analyzes nouns as multi-dimensional. This creates a new problem. The linguistic differences between nouns and multi-dimensional adjectives are not explained. For example, we need to explain the fact that it possible to quantify over the dimensions of multi-dimensional adjectives (as in generally healthy/healthy in some/every respect, etc.), but it is usually impossible to do so in bare nouns (*generally a bird/*bird in some/every respect). We need to explain the fact that exception phrases are compatible with multi-dimensional adjectives (either in their positive form or when negated), but they are not compatible with nominal concepts (whether negated or not).

We see that in order to allow for psychological adequacy and in order to account for the fact that nouns do occur in certain degree structures (for instance, between predicate comparisons, cf. 2.2), nouns should be linked with a set of dimensions and degree functions that map entities into their mean on these dimensions. Yet, this also means that the infelicity of nouns in most of the degree structure should be predicted by some more subtle features of the nominal gradable structure. In chapter 7, I propose that, indeed, the linguistic contrasts between nouns and adjectives can be explained, based on the different degree functions they are associated with.

In a nutshell, I propose that a comparative morpheme like more (in a within-predicate comparison) is an operation on a single dimension. It is infelicitous if the dimension set of its predicative argument is not a singleton (namely, in nouns). The infelicity of nouns in comparative structures is due to the multidimensionality of nouns, together with the lack of a "with respect to" (wrt) argument role (this lack is demonstrated by the contrast between *is a bird wrt flying and is healthy wrt blood pressure.) A wrt-argument reduces the dimension set of multidimensional adjectives (like healthy) into a singleton. The lack of a wrt-argument role in nouns seems to be explained by the fact that the nominal dimensions (e.g. flying and singing for bird), normally do not function as categorization criteria. Conversely, the dimensions of multi-dimensional adjectives (e.g. normal pulse level and normal blood pressure level for healthy), normally do add categorization criteria. You cannot be considered healthy without falling within the normality range in all the contextually relevant respects of healthy. In addition, if before an operation, the doctor asks you whether you are healthy, this is understood as healthy in every respect, not in some respects. This account captures also other linguistic dimensions, such as the infelicity of quantification over the dimensions in nouns (e.g. *bird in some/every respect).

4.6 Conclusions of part 4

The dimension theories should be preferred to Kamp and Partee (1995), because they propose a mechanism that determines similarity to a prototype (nouns are linked with degree functions that map entities to their mean value on a set of ordering dimensions), and this mechanism derives correct typicality - and categorization - judgments in many cases with simple and
complex predicates. However, these theories fail to adequately represent the effects of background knowledge (context) on typicality judgments, and they do not treat dimensions and concepts alike. Finally, they do not provide explanation for the first set of facts (the linguistic contrasts between nouns and adjectives that are described in 2.1).

In the following, I present a theory that combines semantic and psychological mechanisms. I use a knowledge structure (context structure), that is standardly used in semantics, and I assume that in each context in this structure nouns are linked with dimension sets and mean functions. I show that this helps solving problems in both semantics and psychology.
Part III
A new Semantic Analysis
5 MOTIVATIONS FOR MY PROPOSAL

5.1 Typicality: representing gradability in nominal concepts

We have seen in 2.2 that nouns possess a gradable structure. First, robust empirical facts show that membership judgments in nominal concepts are gradable. Second, nominal concepts do combine with certain degree-morphemes (as in *Tan is more a cat than a bird; this is pretty much a chair*, etc.) As discussed in 4.1-4.2 and 3.2, respectively, the psychological theories, which treat nouns as gradable (as mapping entities to their mean on a set of dimensions), make correct predictions about a large number of typicality effects, and they should be preferred to Kamp and Partee's (1995) account of nominal gradability (as dependent on a membership function and an entity prototype, but not dimensions). Thus, nouns should be linked with a set of ordering dimensions and degree functions that map entities to their mean on these dimensions. However, as we have seen in 4.4, psychological theories fail to adequately represent partial knowledge about dimensions and dimension-sets. I propose that the psychological mechanisms (dimension sets, including weights and selected values for the dimensions, and mean functions) should be embedded within a formal semantic model (a context structure similar, but not identical to, the one Kamp and Partee 1995 use, as explained below). In the following chapters I formulate such a model. It allows a representation of effects of context and general knowledge on typicality judgments (the fact that we often cannot tell which one of two entities is a better example of a given nominal category; the fact that we sometimes cannot tell whether a dimension is typical of a category or not; the fact that often the answers for such questions vary between different contexts; the fact that our knowledge about a given dimension may be partial just as much as our knowledge about the predicate it is a dimension of is, etc.)

5.2 Stage 1: Representing information growth (vagueness removal)

We have seen in 3.2.1 that, intuitively, there are connections between gradability and vagueness (partial information about the denotation). Following van Fraassen (1969); Kamp (1975); Fine (1975); and many others, vagueness-based gradability theories usually model partial information (vagueness) by assuming that semantic interpretation is relative to information states (contexts). In partial contexts (as opposed to worlds), the interpretation of linguistic expressions is only partially specified (cf. 3.1 and 3.2.1). Each partial context c is accompanied with a set of total contexts, the set of all the possibilities of consistently completing the information in c. In total contexts the denotations of all the predicates are completely specified (there are no gaps). Vagueness-based gradability theories use simplified vagueness models with but one partial context c (cf. Figure 13). Each total context is thought of as representing a different cutoff point between the positive and the negative denotations of a given predicate (a different standard for membership). In fact, vagueness-based gradability theories assume that entities are classified under a given predicate in a given total context t iff they reach the standard for membership in the predicate.

(1) \( \forall P \in \text{CONCEPT}, \forall c \in C, \text{Categorization is standard-based:} \)
\( \forall t \in T_c: \quad [[P]]_t = \{ d_1 \in D \mid \text{deg}(d,P,M_c) \geq \text{Standard}(P,t) \} \)
Finally, these theories assume that, for instance, *Dan is taller than Sam* in a context c (or in fact in the model $M_c$), roughly, iff Dan is *tall* in more of the total extensions of c, compared to Sam (Kamp 1975). Similarly, degrees in predicates like *tall* are assumed to reflect the proportion of total contexts in which entities are *tall* (Kamp and Partee 1995):

\[
(2) \text{ Entities’ degrees in } P \text{ represent the proportion of total contexts in which they are } P: \\
\begin{align*}
\text{a. } \forall d \in D: & \quad \text{deg}(d,P,M_c) = m(\{t \in T_c: \ d \in [P]_t\}) \\
& \quad (\text{where } m \text{ is a measure function from subsets of } T_c \text{ to numbers between 0 and 1}) \\
\text{b. } \forall d_1,d_2 \in D: & \quad <d_1,d_2> \in [\text{is more } P]_{M_c} \quad \text{iff} \\
& \quad \{t \in T_c: \ d_2 \in [P]_t\} \subset \{t \in T_c: \ d_1 \in [P]_t\}
\end{align*}
\]

However, we have seen that vagueness-based gradability theories such as Kamp (1975) and Kamp and Partee (1995) fail to adequately represent the connections between gradability and vagueness (cf. 3.2.1).

First, these theories fail to represent partial knowledge about degrees and ordering relations. They wrongly predict that the truth values of comparative statements do not vary between different total contexts, whereas, as a matter of fact, the relations that hold between two entities may be unknown, especially in comparatives such as *more normal, more interesting, or more typical*, whose interpretation is highly context dependent.

Second, these theories fail to represent the connection between partial knowledge about the denotation and knowledge about degrees or ordering relations. They wrongly predict that all the denotation members of a given predicate have the same degree (say, they are *equally tall*), because they are all denotation members (*tall*) in all the total extensions of t. But, intuitively, two entities can be, for instance, *tall*, without being *equally tall*. I attempt to solve these two basic problems.

These problems result from the use of simplified vagueness models, like the one in figure 13, with just one partial context (the ground context c). As argued in Landman (1991) and Sassoon (2002), the problems are solved by adding back the larger set of partial contexts which is part of full vagueness models, like the one in figure 14.

Figure 13: The context structure in a simplified vagueness model $M_c$

Figure 14: The structure of contexts in a vagueness model $M_{c_C}$, with a set of partial contexts $C$
Such models represent not only the partiality of information, but also the gradual process (stages) of its growth up to completeness. They do that by including in the context structure a context of zero information, many intermediate contexts of partial information, a monotonic relation of information extension between contexts, and a set of contexts of total information. Thus, one important respect in which the new model that I introduce differs from the standard model for the analysis of gradability is that it is a full vagueness model (Veltman 1984; Landman 1991; Sassoon 2002).

In order to solve the first problem, we need to assign comparative predicates different ordering relations in different total contexts (so that the truth of comparative statements will vary through total contexts and will be unknown in some partial contexts). In order to do that we have to associate gradable predicates with different degree functions in different total contexts. In standard vagueness-based models this is not the case, only the predicates’ standards vary across total contexts, not the degree function itself. I propose to incorporate degree functions into the knowledge-structure, by treating them on a par with denotations (Sassoon 2002). On my proposal, each predicate $P$, in each total context $t$, is associated (relative to an assignment $g$) with a degree function, $\text{deg}^+(P,t,g)$, such that for each partial context $c$, and each entity $d$, $d$’s positive degree in $P$ in $c$ is either unknown (if for at least two total extensions $t_1$ and $t_2$, $\text{deg}^+(P,t_1,g)(d) \neq \text{deg}^+(P,t_2,g)(d)$), or a given real number (the number that forms $d$’s degree on $P$ in every total extension $t$ of $c$). Furthermore, in each total context $t$, $\text{deg}^-(P,t,g)$ assigns any $d$ the set of real numbers that do not form $d$’s degree in $P$ in $t$ ($R - \text{deg}^+(P,t,g)(d)$). In each partial context $c$, the set of numbers that are already known not to form $d$’s degree in $P$ in $c$ ($d$’s negative-set of degrees in $P$) is represented as the intersection of the number-sets assigned to $d$ by $\text{deg}^-(P,t,g)$ in each extension total $t$ of $c$. More generally, I propose that not only the standard of membership, but also any notion that is involved in representing gradable structures of predicates (dimensions-sets, dimension-weights, domains, or any other mechanism that affects or is affected by the degree function or the standard of membership), should be represented as a positive and a negative notion within a given partial context, on a par with the representation of denotations. Thus, my analysis can represent the fact that our knowledge about entities' degrees is often partial. In chapter 7, I examine closely the partial knowledge that we do have about the graded structure of different types of predicate. I state the things that are context dependent and the things that are not. In chapter 9, I show that these observations (and their representation) are helpful for accounting for the polarity effects (they have to do with the fact that our knowledge about degrees of negative predicates is inherently partial).

In addition, we can no longer say that, e.g., *Dan is taller than Sam* is true in a context $c$ iff Dan is *tall* in more of the total extensions in $T$ (in $M_C$), because this will not give us different truth values in different total contexts. Nor can we say that Dan is *tall* in more of the total extensions in $T_c$ (the total contexts above $c$). Imagine that Dan is tall in all the total contexts in $T_c$, and Sam is tall in but one total context, $t_1$, above $c$. It will wrongly follow that in $t_1$ Dan and Sam are equally tall. Thus, we need to abandon principles (2a-b). By abandoning these principles, the second problem is solved, too. If entities can be, e.g. *taller*, without being *tall* in more total contexts, then any two denotation members can be mapped to different degrees (so as to stand in the relation *taller*).

But intuitively, there are connections between partial knowledge about the denotation and knowledge about degrees or ordering relations, and we need to represent them. We have presented in 2.2 and 4.1 robust psychological evidence showing that denotation membership of

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1 In the following sections I refer to the value $\text{deg}^+(P,t,g)(d)$ with the notation $\text{deg}^+(d,P,t,g)$. 

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entities with higher degrees in concepts is learnt earlier (we have called these effects the learning-order effects). Only the use of a full vagueness model allows representing learning. Using a full vagueness model, Sassoon (2002) argues that the gradable structure of predicates like tall and bird reflects the order in which entities are learnt to be denotation members (e.g., tall or not-tall), directly or by inference (by epistemic necessity), through contexts and their extensions. Sassoon (2002) has represented gradable structures with ordering relations. On the current proposal, I represent gradable structures with degree functions (see section 5.4 below), but I still argue that degrees represent the order in which instances are learnt to be denotation members (or non-members), either directly or by inference, through contexts and their extensions. In chapter 8, I show that this learning principle accounts for the connections between typicality and learning effects. I also show that the learning principle helps speakers in acquiring the degree functions of multi-dimensional predicates, and it may help explaining certain familiarity effects. In addition, we can derive from the learning principle intuitive predictions concerning degrees in complex predicates (for instance, negated and modified nouns). Finally, the learning principle does not suffer from the problems of earlier gradability theories. First, two denotation members may not be equally good relative to a given predicate if one is added to its denotation in an earlier stage. So on this account gradability can characterize predicates even if their denotations are completely known. This paves the way for assigning a graded structure to nominal concepts and absolute predicates. Second, the order in which entities are added to the denotations of a given predicate in the partial contexts under a total context \( t_1 \) may be different from that order in the contexts under another total context \( t_2 \). Thus, it is predicted that the ordering between entities may be context dependent and sometimes unknown (for further discussion see chapter 8).

In sum, the use of a full vagueness model allows us to describe in a precise way the connections between partial knowledge about the denotation and knowledge about degrees or ordering relations, and to account for a variety of facts.

5.3 Stage 2: Representing partial information about proper names

Previous vagueness based models, like standard semantic models, assume that proper names are rigid designators. In all worlds (or total contexts) \( t \), a proper name like Dan is linked to one and the same individual, say – \( d_1 \). David Lewis (1968) argues that that is not appropriate. Lewis argues for a theory where a proper name like Dan denotes different individuals in different worlds, with these individuals being bound to each other by the counter-part relation. The intuition is that if in one world the proper name Dan denotes an apple and in another world it denotes a chair (or in one world it denotes an object that is 1.87 meters tall, and in another it denotes an object that is 1.86 meters tall), it does not make sense to say that this is the same individual. I agree with this intuition; I really do not see how we could consider them "the same". As a consequence, I have to reject the assumption of rigid designation.

The total contexts in an information structure represent all the ways our partial knowledge about the 'real' world (presuming that it exists) can be completed. In other words, when our information is partial, we do not yet know which one of the possible worlds is the actual one. Each total context represents a possible world. If in a given partial context \( c \) we do not know whether a proper name like Dan denotes an apple or a chair, to me that means that we do not yet know which individual in the world that name refers to. Which I would like to represent by saying that the positive denotation of Dan is empty in \( c \); in two total extensions of \( c \), the positive denotation of Dan consists of two different individuals, even if the two individuals are bound by
a counter-part relation. I will assume then that proper names are represented by positive and negative denotations. Their positive denotations are either empty or singleton sets, which may consist of different entities in different total contexts. In chapter 6, I explain again the reasons for and consequences of this move.

I said that if the name Dan denotes in one world an object that’s 1.87 meters tall and in another an object that’s 1.86 meters tall, it doesn’t make sense, intuitively, to say that this is the same individual. I would now like to note that there is a sharp contrast between that and the following case. Suppose that in two worlds the proper name Dan denotes an object that is 1.87 meters tall, but in one world this object is considered tall and in the other it is considered not tall (and the references of Dan in the two worlds are identical in all other respects). The intuition now is that we definitely can say that this is the same individual. I would like to represent this intuition, and account for the contrast between the two cases.

The crucial difference between the two cases seems to lie in whether it is ‘real’ properties that are being compared across worlds, or only the way we humans categorize the object’s properties. I therefore assume the following. Two individuals in two total contexts are the same iff they have the same length, width, color, intelligence, and so on and so forth. Furthermore, the two individuals are the same in these two contexts regardless of whether or not their length, width, color, intelligence and so on are sufficient for them to count as long, wide, red, intelligent, etc. in these contexts. In short, an object should be identified with its ‘real’ properties (so to speak), but not with its ‘linguistic’ or ‘conceptual’ properties (so to speak).

Here is how I propose to do that. First, I will adopt an assumption which is common in gradability theories (cf. Kennedy 1999), namely that the ontology consists of a set of entities D and a set of functions that map entities from D to real numbers (a comparative ordering of a predicate is partial when you do not know with which degree function the predicate is linked). Then, I will assume that individuals are distinguished by their property values (the values that the degree functions assign to them). For instance, if the referent of Dan in a context t₁ is 1.87 meters tall, and the referent of Dan in context t₂ is 1.86 meters tall, I will say that the name Dan refers to two different individuals in these two contexts. Suppose, in contrast, that in two contexts t₁ and t₂ the referent of Dan is 1.87 meters tall, and identical in all the other property values. Even if 1.87 counts as ‘tall’ in t₁ but not in t₂, I will still say that the name Dan denotes the same individual in these two contexts (it is only our interpretation of the word tall that has changed).

This proposal captures the fact that even for predicates like tall, whose ordering relation depends on a conventional well-known measuring system, it is not the case that we know for any two referents of proper names whether they stand in the relation taller or not. I do take individuals to be real entities, identified with their ‘real’ properties. So it is invariably and unambiguously determined for each two individuals in D what their heights are, and hence also how their heights compare. However, when we use two proper names, we do not know exactly which individuals in D they refer to, and in particular, we may not know what the heights are of these individuals – so we may easily not know how their heights compare. (We say Dan, but we do not know exactly which possible individual it is we are referring to, since we do not know all of its property values. It may be any one of several possible individuals that agree on many property values, but not on their height.)

Thus, I follow Lewis (1986) in taking different worlds to be distinguished by the reference of proper names, but, unlike Lewis (1986), I take the domain of (possible) individuals to be common to all possible worlds. In formal semantics, we usually take an individual to have a property iff it is in the denotation of that property. I say, in contrast, that an individual’s ‘real’ properties are
represented by property measures. And then I can take different worlds to (also) be distinguished by the cutoff points for the predicate denotations in them. This enables me to say that in different worlds, the same individual, with the same property values, may still have different ‘linguistic’ or ‘conceptual’ properties, represented as belonging to the denotation of some property in one world but not in the other.

5.4 Numerical degree functions

We have seen in 3.2.2 and 4.1 that analyses of, e.g., the semantics of statements with numerical degree phrases (as in *two meters tall*), comparative relations (like *more than two meters shorter*), and nouns (predicates whose graded structure depends on the weighted mean of entities on several different dimensions), have all pointed at one direction: grammar links predicates with (at least) interval scales, i.e., scales where the underlying primitives are numerical degrees, and the notion of distance between any two degrees is well defined (the mathematical operations – and + and relation ≥ are defined for these degrees). This assumption accounts for phenomena like the ones mentioned above. Yet we have also seen that it leaves many questions open.

We have seen in 3.2-3.5 that for each gradable adjective $P$, existing semantic theories define its scale to be a set of degrees which are ordered along some ordering relation (cf. Rullmann 1995; Landman 2005). These theories assume that this ordering relation is not identical for all the predicates $P$, and so it has to be stipulated as part of $P$'s interpretation (let us call it $\geq_P$). So a pair of entities $<d_1, d_2>$ is assumed to fall under a derived comparative like *more $P$* iff, roughly, $d_1$'s degree is higher relative to $\geq_P$ than $d_2$'s degree:

$$\forall P \in \text{CONCEPT}, \forall c \in C: [[\text{more or equally } P]]_c = \{ <d_1, d_2> \in D^2 | \text{deg}(d_1, P, c) \geq_P \text{deg}(d_2, P, c) \}$$

For positive predicates like *tall*, these theories often assume that the scale consists of degrees of height, and $\geq_{\text{tall}}$ is $\geq$ (the 'bigger or equal' relation for the real numbers). For negative predicates like *short*, these theories assume that they share the scale with their positive antonyms, but $\geq_{\text{short}}$ is assumed to be $\leq$ (the 'smaller or equal' relation for the real numbers). For yet other predicates, like *healthy wrt blood pressure*, the degrees are blood-pressure degrees, but the theories do not explicite how the ordering of the degrees is fixed. We have seen that in such predicates, the connections between the degrees and the ordering is not trivial (entities with higher blood pressure degrees are neither necessarily healthier nor necessarily less healthy than entities with lower blood pressure degrees).

In chapter 7, I propose a theory that does not associate the degrees in predicates with predicate specific ordering relations, $\geq_P$, at all. I propose that there are relatively few types of simple operations, which apply to degrees of given degree functions and produce degrees for new functions. For example, the degrees of negative predicates are produced by reversed functions. A reversed function takes a degree 'n' of a positive predicate, and returns a degree of the form '−n', or 'Tran − n' (where Tran is a number, a transformation value). The bigger $n$ is, the smaller '−n' is. Thus, a reversed function produces reversed orderings, in virtue of its use of the difference operation '−'. A function of a predicate of the form 'P wrt Q' (*healthy wrt blood pressure*) takes a Q degree 'n', and returns a P degree of the form '−|Value − n|', where Value is the ideal blood pressure value. So the smaller the distance between $n$ and this value, $|Value − n|$, the bigger one's
degree of health \( -|\text{Value} - n| \). I propose that since this function uses the difference operation twice, healthy is grasped as positive (in sick wrt blood pressure, the number of reversals is odd). Finally, nominal functions take a set of degrees \( n_1 \ldots n_m \) and values \( \text{Value}_1 \ldots \text{Value}_m \), and they return the (reversed) weighted mean of the distances \( |\text{Value}_1 - n_1| \ldots |\text{Value}_m - n_m| \) (so the smaller the distances of, e.g., a bird from the ideal values for birds on the bird dimensions, the bigger its degree in bird).

In sum, previous theories describe graded structure focusing on constraints on their scales. I shift the focus to the characterization of the degree functions themselves. I use operations to produce degree functions for all the predicates. So on my proposal the degrees of all the predicates are naturally ordered by the standard "bigger or equal than" relation for the real numbers. The interpretation of a predicate becomes more economic – we do not need to stipulate a degree-relation \( \geq P \) for each and every predicate. So the current proposal is more economic. At the same time more facts are explained (such as the connections between \( P \) and \( Q \) in predicates of the form \( P \text{ wrt } Q \), and the fact that typicality degrees correspond to means on a set of dimensions). As a consequence, the semantics of the comparative morphemes is simplified too (it does not use the predicate specific relation \( \geq P \)).

\begin{equation}
\forall P \in \text{CONCEPT}, \forall c \in C:

[[\text{more or equally } P]]_c = \{<d_1, d_2> \in D^2 | \text{deg}(d_1, P, c) \geq \text{deg}(d_2, P, c)\}
\end{equation}

Furthermore, we saw in 2.1 and 3.2.5 that clausal comparatives, but not phrasal comparatives, license negative polarity items (as in today was hotter than it ever was in this area, versus *today was hotter than any day). In order to account for this, Landman (2005) has enriched the interpretation of predicates with supremum and difference operations. In chapter 9, I show that given my analysis of negative predicates as denoting reversed functions, Landman's (2005) supremum theory can be incorporated without adding these operations to the interpretation of predicates.

Finally, on my proposal we do not need to stipulate any arbitrary constraint on the scale of a predicate. The constraints are derived from the nature of the degree function. For example, full is linked with a function of the form \( -|\text{Value} - n| \)’, where 'Value' represents the volume of a container (the referent of the argument of full, for instance, the glass in The glass is full) and 'n' the volume of the contained substance. As no entity exists whose volume is smaller than the volume of its contained substance (there are no such possible entities), the scale of full is bound (when n = Value, \( \text{deg}^+_{\text{full}} \) gives 0, and when n < Value, it gives smaller numbers). Given this view, I propose, in chapter 7, that standards of absolute predicates like full and open (i.e. predicates whose standards are thought to be the maximum or minimum points on a given bound scale; cf. 3.2.1.2, problem 5) are not determined directly by a given bound scales (as, for instance, in Winter and Rotstein 2005). They are determined based on domains, just as standards of relative predicates are. I show that my proposal is helpful in accounting for cases in which for-phrases modify absolute predicates (as in empty for a Hollywood film theatre; cf. Kennedy 2001). I define the notion of a domain and explicate its roles in nominal concepts.

5.5 Negative predicates: The quantity metaphor and transformation values

The distinction between negative and positive predicates is puzzling (cf. 3.3-3.4). First, why are some predicates regarded as positive (tall) while others as negative (short)? Second, why can
the positive form of positive, but not negative, predicates, combine with numerical degree phrases? (as in \textit{two meters short} versus \textit{two meters tall}). What is more, numerical degree phrases are always fine in the comparative (as in \textit{two meters shorter}), and they also occur in argument position (as in \textit{Dan is shorter than two meters}). These facts still await an explanation. Third, often the positive form of negative predicates does not combine with ratio phrases (cf. \textit{? twice as short} versus \textit{twice as tall}). Fourth, negative and positive antonyms are not good in between-predicate comparisons (\textit{*Dan is taller than Sam is short}), unless the negative predicate comes first, and the positive predicate is uttered with no emphatic stress (\textit{The ladder is shorter than the house is high}), etc. Using the reversed function proposal and the transformation value, I propose an explanation for these facts.

Let me explain why we need the transformation value.

In standard gradability theories, the ontology consists of a domain \(D\) and also a set \(D_f\) of mappings of (tuples of) entities to degrees (functions from \(D^n\), for any natural number \(n\), to the set of real numbers \(R\): \(D_f \subseteq \{f: D^n \rightarrow R, \text{ for some } n \in \mathbb{N}\}\)).\(^2\) These functions are often referred to as the ordering dimensions of gradable predicates. Each function is thought of as designating something (a quality that human beings can detect). For example, a function can map entities to their heights, to their weights, and in principle, it can even map entities to \(1\) iff they have ID number 095689897 and to \(0\) otherwise. The domain of individuals, \(D\), consists of individuals that may form the reference of proper names like \textit{Dan} and \textit{Sam}. But parts of these individuals are also individuals in their own right (Sam's leg, Dan's happiness, Dan's height, etc.) They, too, are part of \(D\), and the mapping functions associate them, too, with values (Dan's happiness may be deep to different extents, surprising to different extents, related or not to his diet, etc.; Moltmann 2006).

How can we tell that a certain function \(f\) represents a certain quality, say happiness or height? Comparative morphemes have quantity readings (as in \textit{more boys than girls came to the party}). Semanticists often assume that their extent reading (as in \textit{Dan is happier than Sam}) can be described in terms of a quantity metaphor, too. For example, Moltmann (2006) argues that the extent to which entities satisfy an adjective like \textit{happy} reflects the quantity that they possess of the thing denoted by the adjective's nominalization, \textit{happiness} (recall that the happiness in an entity is an element of \(D\), just like the leg of an entity, any other of its parts, or the entity itself).\(^3\) In accordance with this view, when semanticists discuss gradable predicates, they often treat their nominalizations as ordering-dimensions ('height' for \textit{tall}, 'temperature' for \textit{hot}, 'happiness' for \textit{happy}, etc.) Furthermore, quantity functions are \textit{additive}. For example, the number of apples in two baskets together equals the sum of numbers of the apples in each basket separately. In

\(^2\) An example of a predicate that is not one-place, but it is associated with a mapping function is \textit{Love}: For example, we can say that Dan loves to eat a croissant in the morning more than Sam loves to eat bagels in the afternoon (the degree to which the tuple \(<[[Sam]],[[eating a croissant in the morning]]>\) exemplifies the property \textit{love} is greater than the degree to which the tuple \(<[[Dan]],[[eating bagels in the afternoon]]>\) exemplifies this property).

\(^3\) Perhaps the quantity metaphor can be supported by the fact that statements like (b) are felt to be stronger than statements like (a):

\begin{enumerate}
  \item The conference was successful
  \item The conference was a success
\end{enumerate}

Intuitively, the adjectival statement in (a) can be said to be true iff the quantity of success in the conference was sufficient for it to count as successful, but the nominal statement in (b) can be said to be true iff the conference was itself a quantity of success (it \textit{is} all success). If such an overstatement is grasped as appropriate, this tells us that the conference was very successful.
accordance with the quantity metaphor, semantic theories postulate that predicates’ degree functions are additive (wrt their nominalization). A good example for this is Klein (1991). Consider, for instance, the predicate long. It can be seen as measuring quantities of length. Klein (1991) symbolizes the concatenation (placing end to end) of two rods $d_1$ and $d_2$ as $d_1 \oplus_{\text{length}} d_2$. In Klein (1991), the degree function of long, $\text{deg}_{\text{long}}$, is additive in the sense that it adequately represents the fact that the length of two rods when placed end to end equals the sum of lengths of the two separate rods. The function $\text{deg}_{\text{long}}$ in Klein (1991) represents this fact by mapping these entities to values such that:

$$\text{deg}(d_1 \oplus_{\text{length}} d_2, \text{long}) = \text{deg}(d_1, \text{long}) + \text{deg}(d_2, \text{long}).$$

In particular, $\text{deg}_{\text{long}}$, adequately represents the fact that the length of a concatenation of two entities with equal (quantities of) length equals twice the length of each entity. The function $\text{deg}_{\text{long}}$ represents this fact by mapping these entities to values such that:

$$\text{deg}(d_1, \text{long}) = \text{deg}(d_2, \text{long}) \text{ iff } \text{deg}(d_1 \oplus_{\text{length}} d_2, \text{long}) = 2 \times \text{deg}(d_1, \text{long}).$$

In sum, the values of functions which are additive wrt a quality $Q$ represent the ratios between the quantities of $Q$ in different entities:

(5) **Additivity (Klein 1991):**

A function $f \in D_f$ is additive wrt a quality $Q$ iff $f$ conforms to these two principles:

a. for any two entities $d_1$ and $d_2$, $f$ maps their concatenation wrt $Q$, $d_1 \oplus_{Q} d_2$, to the sum of the values to which it maps $d_1$ and $d_2$:

$$\forall d_1, d_2 \in D: \ f(d_1 \oplus_{Q} d_2) = f(d_1) + f(d_2)$$

b. $d_1$ and $d_2$ possess equal quantities of $Q$ iff $f$ maps $d_1 \oplus_{Q} d_2$ to twice the value of $d_1$:

$$\forall d_1, d_2 \in D: \ f(d_1) = f(d_2) \text{ iff } f(d_1 \oplus_{Q} d_2) = 2 \times f(d_1)$$

Some functions from $D^n$ to $R$ (for any number $n$) are not additive. Consider, for example, the function $f_{-1}$ that maps each $d$ in $D$ to the value $(f_{\text{length}}(d) - 1)$ and the function $f_{1-f}$ that maps each $d$ in $D$ to the value $(1 - f_{\text{length}}(d))$. They are not representing adequately quantities of length (that thing quantities of which $f_{\text{length}}$ represents) – they are not additive wrt length. For example, if for two entities $d_1$ and $d_2$, $f_{\text{length}}(d_1) = f_{\text{length}}(d_2) = 5$, then by additivity $f_{\text{length}}(d_1 \oplus d_2) = 10$. But by the definition of $f_{-1}$:

$$f_{-1}(d_1) = f_{-1}(d_2) = 4 \quad \text{and} \quad f_{-1}(d_1 \oplus d_2) = f_{\text{length}}(d_1 \oplus d_2) - 1 = 9 \neq 2 \times 4$$

Similarly, by the definition of $f_{1-f}$:

$$f_{1-f}(d_1) = f_{1-f}(d_2) = -4 \quad \text{and} \quad f_{1-f}(d_1 \oplus d_2) = 1 - f_{\text{length}}(d_1 \oplus d_2) = -9 \neq 2 \times (-4).$$

We see that functions that map entities to their length quantity transformed by a constant are not additive wrt length. The ratios between the degrees that these functions assign to entities (like $d_1$ and $d_1 \oplus d_2$) do not adequately represent the ratios between the quantities of length in them (e.g.
the ratio between the degrees of $d_1 \odot d_2$ and $d_1$ is $9/4$ and the ratio between their quantities of length is $8/4$).

Because of the quantity metaphor, semantic theories postulate additivity. But there are two respects in which the quantity metaphor is sometimes misleading. I discuss one respect now, and the other in section 5.7.3.

Let us call a context $c$ in a context structure $M_C$ actual iff it represents the linguistic and world knowledge of a given competent speaker (or community of competent speakers) out of the blue (in the lack of a specific context). The notion of actual contexts corresponds to Kamp’s (1975) notion of a ground-context (cf. 3.1-3.2), except that in most theories (Kamp (1975); Kamp and Partee (1995), etc.), the ground context functions as the minimal context in the context structure of a model, while in the current proposal, the minimal context in the structure contains less information than actual contexts contain (the structure represents information growth). Let us call total contexts actual (wrt $c$) if they are extensions of a given actual context $c$ (otherwise let us call them counterfactual contexts (wrt $c$), given that they contain some piece of knowledge that is already known to be false in $c$).

I agree with Klein that in every total extension of an actual context $c$ predicates like long are linked with additive functions. However, one respect in which the quantity metaphor is misleading is that many natural language predicates are not additive wrt their (presumed) dimension (or nominalization). In particular, when you look at negative predicates like short, their values are often taken to depend on quantities of height (cf. Landman 2005), or on quantities of height that entities do not possess (cf. von Stechow 1984; Kennedy 1999). But, I submit that the mapping of entities to degrees in short is not additive wrt these quantities in every $t$ and $g$.

How do I know that? Let us see what we know about the degree function of short. We know (or we have a very strong intuition) about the entity ordering of short that it is reversed compared to that of tall (Dan is taller than Sam iff Sam is shorter than Dan). Thus, the degrees are reversed (if Dan is mapped to a higher degree in tall Sam is mapped to a higher degree in short). But, crucially, that is about all that we know about these degrees. In other words, we know that they are produced by a reversed function, but we do not know which reversed function. There are many candidates. For any $g$ and $t$, let $f^+(d,\text{tall},t,g) \in D^f$ be the function that is linked with tall in $t$ and $g$. For any constant $\text{Tran} \in \mathbb{R}$, a function $f_{\text{Tran}}$ that assigns any $d$ the degree $(\text{Tran} – f^+(d,\text{tall},t,g))$ can do the job of reversing the degrees. But when the constant Tran is not zero, these functions are not additive wrt height, as we have just demonstrated with $f_1 – f^+$. Do we have intuitions that tell us that $\text{Tran}_{\text{short}}$ is zero (in any actual context $c$)? Well, I do not think so. This can be tested by checking our intuitions concerning the value of individuals with zero height (abstract entities; surfaces; points). If for any $g$, in an actual context $c$ we know (in any total extension $t$ of $c$ it holds true) that $\text{Tran}^+(\text{short},t,g) = 0$, and $f^+(\text{tall},t,g)$ is additive (it maps entities with no height to 0), then the degree of entities with no height in short should be known to be zero (in any such $t$ and $g$ it should be zero). But is it? I do not know. Nor do my very intellectual neighbors know. Maybe they are mapped to a number that approximates infinity? This view is endorsed by some well-known semantic theories (cf. von Stechow 1984; Kennedy 1999, 3.2.4). But if so, then the mapping function of short is not additive, it is a function that transforms height quantities by a non-zero constant, $\text{Tran}_{\text{short}}$. To be honest, we do not know anything about the constant; it may be any real number from zero to infinity. It should be represented as a value that varies between total contexts, $\text{Tran}^+(\text{short},t,g)$, and that may be unknown in a partial context.

To summarize, I propose that the interpretation of a predicate $P$ in each context $t$ and assignment $g$, includes in addition to the positive denotation, a positive degree function $f^+(P,t,g)$
(this function corresponds with the notion ‘dimension’ as it is used in semantics, but not as it is used in the current proposal, as I explain in 5.6 below), and a positive transformation value \( \text{Tran}^+(P,t,g) \), such that for any \( d, t \) and \( g \), for any positive predicate \( P \), \( \text{deg}^+(d,P,t,g) = f^+(d,P,t,g) - \text{Tran}^+(P,t,g) \), and for any negative predicate \( P \), \( \text{deg}^+(d,P,t,g) = \text{Tran}^+(P,t,g) - f^+(d,P,t,g) \). But in most positive predicates, in actual contexts \( c \), the transformation value is known to be zero (in any total extension \( t \) of \( c \), \( \text{deg}^+(d,P,t,g) = f^+(d,P,t,g) \)), while in most negative predicates it is unknown in actual contexts (it is a different number in different total extensions of these contexts). We see that there is a sharp semantic distinction between positive and negative predicates. Negative predicates denote reversed degree functions, where the precise reversed function is not known, because the transformation value is unknown. In chapter 9, I show that by representing this distinction, we can explain the polarity effects (the differences between negative and positive predicates that are mentioned in the beginning of this section).

In a nutshell, roughly, first, if, for instance, \( \text{tall} \) maps an entity \( d \) to 2 meters, \( \text{short} \) maps \( d \) to \( \text{Tran}^\text{short} - 2 \), where the value \( \text{Tran}^\text{short} \) is unknown. This produces indeterminacy concerning the number set of \( \text{short} \), which is felt in the fact that numerical-degree modifiers (such as two meters) cannot be used with its positive form. In the lack of knowledge about \( \text{Tran}^\text{short} \), we cannot say which entities are mapped to two meters \( \text{short} \). Second, if \( d_2 \) has a double length compared to \( d_1 \), \( \text{tall} \) maps \( d_1 \) to \( n \) (say, 2 meters), and \( d_2 \) to \( 2n \) (say, 4 meters). Given that \( \text{short} \) reverses the degrees, in the given context \( \text{short} \) maps \( d_1 \) to \( n' = \text{Tran}^\text{short} - n \) (e.g., \( \text{Tran}^\text{short} - 2 \) meters), and \( d_2 \) to \( m' = \text{Tran}^\text{short} - 2n \) (e.g., \( \text{Tran}^\text{short} - 4 \) meters). But \( m' \) is not two times \( n' \) (unless \( \text{Tran}^\text{short} \) is set to zero). Thus, \( d_2 \) has a double length compared to \( d_1 \) iff \( d_2 \) is twice as tall, but not iff \( d_2 \) is twice as short. As a consequence, twice as \( \text{short} \) is less acceptable than twice as \( \text{tall} \). Third, when degree-differences are computed (as in \( \text{Dan is } n \text{ meters taller / shorter than } \text{Sam} \)) the transformation values of the two degrees cancel one another. For instance, \( d_2 \) has \( n \) meters more length compared to \( d_1 \) iff \( d_2 \) is \( n \) meters taller (iff \( \text{tall} \) maps \( d_2 \) to \( m \) and \( d_1 \) to \( m - n \)) and iff \( d_2 \) is \( n \) meters shorter. I.e. \( \text{short} \) maps \( d_2 \) to \( \text{Tran}^\text{short} - m \) and \( d_1 \) to \( \text{Tran}^\text{short} - (m - n) \), and the difference between these two degrees is still \( n \). (The difference is negative, \( (\text{Tran}^\text{short} - m) - (\text{Tran}^\text{short} - (m - n)) = -n \), because \( d_1 \) has a higher degree in \( \text{short} \) – it is shorter).

In sum, we need a context structure where different contexts differ not only in predicates’ standards for membership, but also in the value with respect to which predicates' degrees are transformed, \( \text{Tran}^P \). This explains the linguistic contrasts between positive and negative adjectives.

5.6 Dimensions and dimension sets

Speakers associate nouns like \( \text{bird} \) with multiple dimensions (\( \text{small, flying, singing, eating} \) \( \text{insects} \), etc.) But we have seen in chapter 4 that contemporary psychological representations of knowledge about dimension sets and their gradual growth, and about the dimensions themselves, are still problematic. Dimensions are often modeled as binary, simple or perceptual, and completely given, whereas dimensions are often gradable, complex, abstract, and are learnt together with the categories (cf. 4.3).
My model differs from standard dimension theories in psychology in the treatment of dimensions. Crucially, the dimensions are taken to be normal concept-names (predicates). Thus, categories and dimensions are treated completely alike. Dimensions are associated with degree functions \( f^+ \) just like predicates, and their interpretation is context dependent just like the interpretation of predicates. I incorporate dimension sets into the knowledge-structure, by treating them on a par with denotations (Sassoon 2002). Each predicate \( P \), in each context \( t \), is associated (relative to an assignment \( g \)) with a dimension set, \( F^+(P,t,g) \) (I have chosen the letter \( F \) because it is the initial of feature). In each partial context \( c \), a predicate like wooden is a dimension of a predicate like chair iff wooden is in \( F^+(chair,t,g) \) in any total context \( t \) of \( c \), and wooden is already known not to be a dimension of chair in \( c \) iff in no total extension \( t \) of \( c \) wooden is in \( F^+(chair,t,g) \). Otherwise, we do not yet know in \( c \) whether being wooden is characteristic of chairs or not.

Degree functions of dimensions (the values that they assign to entities) constrain the degree functions of the predicates they are dimensions of. The type of constraint depends on the type of predicate. Note first that in one-dimensional adjectives \( P \) (like tall, short and healthy wrt blood pressure) the notion of a dimension is rather redundant (for any \( t \) and \( g \), for any dimension \( Q \) of \( P \), and for any \( d \), \( f^+(d,P,t,g) = f^+(d,Q,t,g) \)).

Now, for nominal concepts (bare nouns and noun phrases, like bird, nocturnal bird, northern grey robin, etc.), I incorporate the prototype theory (the assumption that they map entities to their weighted mean on a set of dimensions). Put simply, for any nominal concept \( P \) (including complex nominal concepts), for any \( t \) and \( g \), \( f^+(P,t,g) \) takes the entities' degrees in the dimensions' \( f^+ \) functions and returns the mean degree. I also propose a general strategy for the acquisition of dimension sets in simple and complex nominal concepts based on the learning principle (cf. 5.2). The resulting theory improves upon contemporary psychological theories, and it is also far superior to contemporary linguistic theories in its psychological adequacy.(Regarding complex nominals, recall that people link gradable structures with very complex predicates (with relative clauses, disjunctions, negations, etc. as in pets which are not birds; cf. 4.3). I show that, contrary to the intuitions of cognitive psychologists, judgments of typicality and categorization in complex predicates are compatible with, and sometimes motivated by, semantic rules like the intersection rule, and by pragmatic rules. Concerning exemplars (briefly, dimension sets of sub-categories), I assume that they need not be represented as part of the interpretation of a predicate (they are simply the dimension-sets of its sub-categories). The exemplar effects (briefly, the fact that sometimes categorization is based on mean on the dimensions of a sub category, rather than the category itself) can be derived, if "sub-kind" relations between concepts are represented.

As for multi-dimensional adjectives, these require a different approach; I will propose a new analysis for them, as discussed below.

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5 As there is a one-one connection between predicates and their nominalizations, it makes no difference whether one represents a dimension using a nominalization or a predicate. Furthermore, there is a one-one connection between predicates and their nominalizations, but some predicates are multi-dimensional. Thus, the dimensions of predicates cannot be identified with its unique nominalization. Finally, degrees in happy can be said to reflect quantities of happiness (Dan is happier iff Dan has more happiness), but not degrees to which things exemplify the predicate happiness. To avoid confusion as to whether we relate to quantities or degrees of happiness, I take the dimensions of predicates like tall to be predicates (say, has height, or tall), not nominalizations (e.g. height).
5.7 Nouns, adjectives, and 'more'

One difference between nominal and adjectival concepts is that the dimensions of adjectival but not nominal concepts are accessible to grammatical operations like with respect to phrases (as in typical wrt flying / size versus # bird wrt flying / size), or quantifiers (as in typical in every / some respects versus # bird in every / some respects). Another difference between nominal and adjectival concepts is that adjectival, but not nominal concepts, are licensed in within-predicate comparisons (as in Tweety is more typical than Tan versus #Tweety is more a bird than Tan), while nominal, and not adjectival concepts, are freely licensed in between-predicate comparisons (as in Tweety is more a bird than (it is) a cat and This is more a table than a wall, versus # Sam is more tired than Dan is clever or # This book is heavier than it is old). Finally, some adjectives have unit names and they sometimes map entities to known numbers (as in Dan is two meters tall), but nouns never do. I propose an account of these facts.

5.7.1 The psychological reality of the one-dimensional versus multi-dimensional distinction

In contemporary theories in cognitive psychology, one-dimensional concepts (usually invented ones, but also real concepts like tall) are often called rule-based, because categorization under them does not involve averaging over dimensions. Categorization in multi-dimensional nouns like bird or house is called similarity-based, as it is based on a more complex categorization rule which involves averaging across the dimensions. This distinction is important because there is evidence that rule versus similarity based categorization tasks recruit different brain systems (Ashby and Maddox 2005) and their acquisition course seem to be different (perhaps due to late maturation of the rule-based brain system; Keil 1979; Zelazo et al 1996; Thomason 1994). This neuropsychological model describes rule-based tasks as requiring more working memory and recruiting mostly verbal or declarative systems (the prefrontal cortex), and similarity judgments as recruiting implicit or procedural learning systems (the inferotemporal cortex). The basal ganglia decides which strategy is most effective in a given situation. The difference in brain systems can be said to reflect the number of dimensions involved in categorization (Photos 2005). Evidence for the dual neuropsychological model is formed by brain deficits which selectively affect either rule or similarity based tasks (for discussion see Ashby and Maddox 2005). Further evidence is formed by the late development of the prefrontal cortex which explains why young children often base categorization on similarity (Keil 1979) and have difficulty in consistently using rules (Zelazo et al 1996; Thomason 1994).

In sum, the distinction between rule versus similarity based categorization (or put differently, between one-dimensional and multi-dimensional concept interpretations) seems to be a cognitively real distinction. I propose that it is this distinction that has been grammaticized into the two categories – (basic) nouns and adjectives, as explained below.

5.7.2 Multi-dimensional adjectives as conjunctions / disjunctions of one-dimensional concepts

I propose an analysis of multi-dimensional adjectives (like healthy, sick, normal, typical of a bird, similar to a bird, good etc.), which employs the hypothesis that they too are rule based. The relevant categorization rule for adjectives does not involve averaging on their dimensions, but

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6 Some speakers are willing to accept sentences like Tweety is more tall than (it is) heavy, while others completely reject them (for further discussion of the data see Kennedy 1999: 19-24).
rather checking the degree of the conjunction or disjunction of their dimensions. In other words, I argue that categorization under multi-dimensional adjectives depends on either universal quantification over dimensions (the entities are required to reach standard in all the dimensions, or in the dimension conjunction) or existential quantification over dimensions (the entities are required to reach standard in one of the dimensions, or in the dimension disjunction), not averaging (the mean degree of the entities in the dimensions need not reach standard).

Consider for example, the multi-dimensional adjective healthy. Intuitively the dimensions of healthy do add categorization criteria. For example, an entity which has cancer or has a heart attack is not healthy, even if its degree in these predicates is very low. Even less important dimensions like has the flu add a categorization criterion. Consider, for instance Dan. When Dan is healthy in any other respect except that he has the flu, we cannot strictly speaking say that he is healthy. Saying that he is healthy will be close enough to the truth in most of the contexts for most of the purposes (he might be able to go to work or to meet friends), but if we are to be completely precise, (say, if Dan needs to undergo an operation which is dangerous for sick patients, then we have to tell the doctor that), Dan is not healthy. Thus, categorization under multi-dimensional adjectives like healthy depend on whether the entity falls in the intersection of the dimensions' positive denotations or not, rather than to depend on whether the entity's average degree in the dimensions reaches threshold.

In addition, the denotation of some adjectives (usually the negative antonyms of positive multidimensional ones) is fixed by dimension disjunction, not conjunction. For example, intuitively, it is sufficient to violate but one property which functions as a healthy dimension in a context c in order to be considered sick in c. But in this case too, when sick (or healthy) is conceived of as associated with multiple dimensions (respects), entities seem to be added to the positive or negative denotation based on whether they reach the standards of the dimensions, and not based on a direct measurement of their degrees in sick (or healthy) and an independently known standard for sick and healthy (namely, not based on their mean in the dimensions).

I call adjectives whose interpretation involve universal quantification conjunctive and adjectives whose interpretation involve existential quantification disjunctive.

To take another example, clearly, the sentence Dan is good as a teacher does not entail that Dan is good, nor does Dan is good as a teacher and as a father entail that Dan is good, etc. When the respect in which Dan is supposed to be good is not specified, we cannot be sure that Dan is good is true, unless we know that Dan is good in any possible respect. This is of course quite implausible given the variety of respects which may be linked with the predicate good. Context-dependent information is needed in order to restrict the set of dimensions to those which are relevant, in each context. The meaning of multidimensional adjectives can be considerably weakened within context either by restricting the dimension-set using a with-respect-to (wrt) modification (as in healthy wrt the flu) or by using quantification over the dimensions as in generally healthy and healthy in most respects. It may also be implicitly conveyed that healthy is to be understood to mean generally healthy, healthy in most respects, or even healthy in some respect. Such implicit quantification on the dimensions turns entities that do not satisfy all the dimensions healthy, but only in some respects!

I propose, that, in particular, typical of P is a (conjunctive) multi-dimensional adjective. This proposal predicts the intuition that it's meaning is stronger than that of the noun P (it has more categorization criteria), although it is hard to put a finger on the exact dimensions which add criteria, since the possibility of implicit wrt-modification (like in typical wrt flying) or quantification (like in typical in some respects) is always available.
Since the meaning of multi-dimensional adjectives is often subject to context dependent restrictions, it is hard to empirically support or refute the hypothesis that the (contextually relevant) dimensions in each context \( t \) are all treated as necessary conditions which are jointly sufficient for membership in \( t \). Yet, the *except* test can form a natural empirical test for the hypothesis that the interpretation of the *conjunctive* adjectives involves universal quantification over dimensions (their instances have to reach threshold in *all* the dimensions), while the interpretation of the *disjunctive* adjectives does not. Exception phrases are only compatible with universal operators (as in *everybody / nobody except for Dan is happy*, versus *somebody except for Dan is happy*). The main evidence for my proposal is formed by the compatibility of multi-dimensional adjectives with exception phrases, either in their positive forms (in conjunctive adjectives), as in "*Dan is healthy except for his blood pressure*" or in their negative forms (in disjunctive adjectives), as in "*Dan is not sick except for his blood pressure*" (this is expected because a negated existential statement is a universal statement). Nouns do not combine with exception phrases, as they do not have quantifiers as part of their interpretation (as in *somebody except for Dan is happy*, versus *somebody except for Dan is happy*).

We see that adjectival predicates sharply differ from nominal ones – when a predicate is classified as adjectival, its dimensions are combined using quantifiers (or Boolean operations), not averaging operations. This proposal explains also the other semantic contrasts between them.

First, this proposal explains the fact that dimensions of adjectives but not nouns are accessible to grammatical operations (as in *typical wrt flying and typical in some respect*, versus *bird wrt flying / ? bird in some respect*). Intuitively, modifying a predicate \( P \) with a *with respect to* (wrt) phrase makes sense only when several dimensions are treated as necessary conditions for membership in either its positive or its negative denotation, and as a consequence, entities may indeed be regarded as \( P \) in one respect, and as "not \( P \)" in another respect. Thus, a predicate \( P \) can be modified by a wrt phrase iff either \( P \) is interpreted conjunctively (as in *healthy*) or \( P \)'s negation is interpreted conjunctively (as in *sick*). Conversely, in nominal concepts like *bird* or *not a bird*, the dimensions are normally not necessary for membership. At best, some of them are very important typicality dimensions. Thus, nouns normally do not license 'wrt' phrases. In fact, when the nominal dimensions do add categorization criteria, a wrt phrase *is* licensed. For instance, if an expert characterizes birds by the possession of, say, 100 separate genes, which all and only the known birds possess, she might indeed describe new species that possess only 50% of these genes, as "birds with respect to gene A, but not with respect to gene B". Finally, in the dimension set of one-dimensional adjectives like *tall*, we cannot find two different dimensions that form necessary conditions for *tall*, so the requirement for the licensing of a wrt phrase is not met. Thus, a wrt phrase can only be licensed in multi-dimensional adjectives.

Second, my proposal that the dimensions of adjectival and nominal concepts have different roles (in virtue of their being dimensions of *nominal* versus *adjectival* predicates) elucidates the reason for which we give them different names. While the adjectival dimensions can be called *respects*, the nominal dimensions can be referred to as *typical of the category* (as in *flying is typical of birds*; Adjectives do not combine with *typical* in this dimension-reading as in: *blood pressure is typical of healthy*).

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7 In the dimension-reading *typical* denotes the dimension set of its nominal argument (as in *flying is typical of a bird*), while in its example-reading it denotes typical instances or ordering of entities by typicality (as in *Tan is more typical of a bird than Tweety*).
Third, I propose that my analysis explains the fact that adjectival but not nominal concepts are licensed in within-predicate comparisons (as in *Tweety is more typical than Tan versus *Tweety is more a bird than Tan), but nominal, and not adjectival concepts, are freely licensed in between-predicate comparisons (as in *Tweety is more a bird than (it is) a cat versus *Tweety is more tall than (it is) old).

When but one predicate is explicitly mentioned in a comparison statement (as in "x is more P than y is"), the statement must be interpreted as a within-predicate comparison (the compared degrees ought to be assigned to x and y by one and the same degree function, as proposed by Kennedy 1999). Which degree function? Adjectives are either one-dimensional in the first place, or can easily become one-dimensional in virtue of an accommodation of a wrt-phrase. I show that when they are used in within-predicate comparisons, adjectives are always interpreted as one dimensional (Dan is healthier is interpreted as healthier in every respect or generally healthier). But nouns do not license a wrt argument. Thus, they are inherently multi-dimensional. Consequently, the (within-predicate) comparative operation is undefined with nouns. In chapter 9, I further explicate an inherent difference between one-dimensional and multi-dimensional concepts, due to which more in within-predicate comparisons cannot combine with the latter (see also 5.7.3).

Obviously, between-predicate comparisons (like "x is more P than y is Q") are not restricted to a one-dimensional comparisons (contrary to Kennedy's claims, they allow for comparisons between values of different degree functions as in more a chair than a wall, more a bird than a horse, etc.) But such a comparison makes sense only provided that the ranges of the two functions can be normalized so as to consist of the same bound interval. For example, in school, many different talents of students are normally measured on a shared scale. We can meaningfully say that, for instance, Dan is better in mathematics than in literature, if Dan's marks in these two fields are, say, 5 and 4, respectively, on a shared six-point scale. Crucially, the number-set of nominal degree functions is readily normalized for the purpose of averaging. Thus, they are optimal candidates for occurring in between-predicate comparisons. This is not the default case for adjectives, which in fact occur less freely in such comparisons. Adjectival scales are usually not bound on both sides. I discuss cases in which adjectives do occur in this structure and explain them in 7.4.

In conclusion, the new perspective seems to be promising and fruitful as a basis for explaining the semantic contrasts between nouns and adjectives. It improves upon contemporary psychological and linguistic theories in its linguistic adequacy.

5.7.3 One-dimensional versus multi-dimensional predicates and the quantity metaphor

We have seen that multi-dimensional adjectives like healthy have separate one-dimensional interpretations (like healthy wrt blood pressure, healthy wrt pulse, etc.), but nouns do not have such interpretations. This distinction seems to be important as it explains the conditions for the licensing of predicates in comparison statements. But the intuitive distinction between one-dimensional and multi-dimensional predicates is puzzling. Why can't we represent bird as a one-dimensional predicate, whose unique dimension is bird (or birdhood)? My theory aims at explicating the inherent distinction between one-dimensional and multi-dimensional predicates.

The quantity metaphor is important again, i.e. the view that the extent to which entities satisfy an adjective (e.g. happy) reflects quantity of something in them - the thing denoted by the adjective's nominalization (e.g., happiness). Recall that this view is misleading in that quantity
functions are additive (their values adequately represent the ratios between different quantities) while adjectival functions are not always additive. The functions of predicates like tall are additive wrt 'height' (they adequately represent the ratios between the quantities of height in different entities), but functions of predicates like short are not.

The second respect in which the quantity metaphor is misleading is that some predicates may well denote additive degree functions in any total context, but the thing wrt which their function is additive (quantities of which it represents) may vary between different total extensions of a given actual context c. This is typically the case in multi-dimensional predicates. Consider, for example, the predicate bird. The set of bird dimensions and their weights is partial. They vary considerably between contexts. The set of things quantities of which the function \( f(bird,t,g) \) measures and their relative importance varies between different total contexts t in \( T_c \) for any actual context c and assignments g. Hence, there is no one thing quantities of which the predicate bird can be said to represent in any actual c. Our knowledge about this predicate is like a recipe for a cake that does not specify quantities of ingredients. In one total context, the cake may include two cups of sugar, in another three cups, and in yet another no sugar at all. In one total context it may include chocolate, in another cheese. In a partial context below all these total contexts, this recipe does not adequately represent any cake. In chapter 9, I propose a theory that represents the distinction between predicates for which there is something they measure in any total extension t (of a given actual context c) and g (like tall and short), and predicates for which there is no such thing (like bird and healthy wrt blood pressure and pulse and ...). I define formal properties of degree functions that distinguish between these two types of predicates (one-dimensional and multi-dimensional predicates), and I show that multi-dimensional predicates (predicates such that the quality their degree function designates is unknown) violate the licensing conditions of comparative morphemes. Put simply, I argue that one can assert that \( x \) is more P than y iff one knows quantities of what P's degree function represents. Multi-dimensional adjectives occur in (within-predicate) comparison statements iff they are (implicitly or explicitly) modified by a with respect to phrase that reduces their dimension set to a unique one-dimensional dimension that is a predicate such that the quality its degree function designates is known. Nominal concepts cannot combine with wrt phrases so they remain inherently multi-dimensional, and so they cannot occur in (within-predicate) comparisons.

Finally, why do nouns not have units? Let us call a function a 'Q function', \( f_Q \), iff it is additive wrt Q (for example for any extension t of an actual context c and any g, \( f_Q(long,t,g) \) is \( f_{length} \); in the contexts that represent the knowledge of competent speakers long never designates happiness or health; it always represents length\(^8\)). Note that many functions in \( D_f \) (the functions' domain) may be additive wrt the same quality Q. If \( f_1 \) is additive wrt to Q, then any \( f_2 \) that maps entities to two times the values that \( f_1 \) maps them to is additive wrt Q, too (its values too adequately represent the ratios between the quantities of Q in entities). Thus, a whole set of functions in \( D_f \) are length functions. We can tell that a predicate like long is linked with a length function, but we cannot tell which function. So even our information wrt deg\(_{long} \) is partial. This is a correct result, given that length is a mass noun – it denotes stuff, so to speak (we cannot tell for a given quantity of length the number that represents this quantity). We can only tell facts about ratios (we can tell that one quantity of length is two times another, etc.) However, in actual contexts, unit names like meter denote an agreed upon set of objects that share the amount of length (there is no actual total

\(^8\) I regard long in the sense of temporal duration, as a separate lexical entry. I do not think that we do not know whether long measures length or duration. I think the predicate long has two senses and we have partial knowledge about each one of them separately.
context where \textit{meter} denotes centimeters or objects with equal quantities of happiness). Given that we know that \textit{long} measures (quantities of the quality) length in any actual \textit{c}, and we know the objects that are part of the denotation of \textit{meter} in \textit{c} (for any person or object that we see we can take a ruler and tell whether it is one meter or not), we can map any entity to a known number, the number that represents the ratio between this entity's length and the meter-units' length. This number is the same for all the length functions in all the total contexts above \textit{c}.

In sum, we can map any entity to a known number in a predicate \textit{P} in an actual context \textit{c} iff \textit{we know the quality \textit{P} represents and we have an established unit for \textit{P}}. In predicates like \textit{happy} we do not have established units. In predicates like \textit{bird} we do not know the quality they designate (the set of \textit{bird} dimensions and their weights vary between contexts). So we cannot map entities to known numbers.\footnote{In predicates like \textit{short}, we know the quality and the unit, but the unknown transformation value interferes – the degree function does not represent correctly the ratios of height quantities in entities (cf. chapter 9). Yet, in the comparative, where differences between degrees are concerned (for instance, \textit{in Dan is two meters shorter than Sam}, the difference between Dan's and Sam's height are concerned), the transformation values in the degree of Dan and of Sam cancel one another, so the result does represent adequately a quantity of height, and we can say that this quantity is exactly the quantity of height in two meter units (for detailed explanations concerning the use of units with negative predicates or other predicates whose functions are transformed by a constant see chapter 9).}

5.8 The degree function of multi-dimensional adjectives

Recall that I use the notion 'typicality' to refer to gradability in \textit{nominal concepts} (to speakers' numerical ratings of degrees to which entities exemplify nominal concepts). I use the term 'typicality' to refer to means on dimension sets (given that speakers' degree ratings significantly correlate with such means). Quite unrelated to the above technical use of the term \textit{typicality}, I argue that natural language expressions of the form \textit{typical of \textit{P}} are adjectival, and as such, they do not denote mean functions, but other degree functions. Which?

The ordering in multi-dimensional adjectives is not (directly) given by averaging on the dimensions. For example, consider again the adjective \textit{healthy}, in a context in which health is measured by the dimensions \textit{blood pressure, pulse} and \textit{fever}. Imagine that Dan has the maximal degree in two of these dimensions but he is not within the norm in the third. Conversely, imagine that in all these dimensions, Sam's levels are within the normative range, but they are the lowest possible, so Dan's mean degree in the dimensions is higher than Sam's. Intuitively, in this scenario, \textit{Sam is healthy}, but \textit{Dan is not}, because Sam, but not Dan, reaches the norm in all the contextually relevant respects. Because of that, intuitively, \textit{Sam is healthier than Dan}. But this shows that it is not the case that we directly compare Sam's mean degree in the \textit{healthy} dimensions to Dan's mean degree in the \textit{healthy} dimensions. Had we done that, we would have judged \textit{Dan} to be \textit{healthier than Sam}.

How do we compare Sam's health with Dan's, then? I propose the following. First, we fix negative and positive denotations for \textit{healthy}, based on dimension intersection (for the positive denotation we select entities that reach the standard in all the dimensions). Second, we fix the ordering relation to be such that positive denotation members would always be healthier than negative denotation members. Only then (if at all) do we allow comparisons based on mean degrees, such that among the positive denotation members, those which average better in the dimensions are regarded as healthier, and among the negative denotation members, those which average better are regarded as healthier.
To take another example, imagine that Dan and Sam are school students. Dan is an excellent student (his mark is 100) in 2 subjects and he is a failure (his mark is 50) in all the rest, while Sam is a good, but not a brilliant, student in all the subjects (all her marks are 75). We can say that as a student, Sam is good (but not more than that), but Dan is (very) good wrt two fields but is not good wrt the other fields. Thus, we can say that Dan as a student is good on average, or good in some respects, but we cannot say just that he is good (without restricting ourselves, be it explicitly or implicitly, to a certain subset of the fields of study). We can say that in some respects he is better than Sam, and maybe even that he is on average better (if, say, there are only three fields of study), but it is not clear whether we can simply consider him a better student.

Similarly, if someone is talented in math but not generally talented then we cannot say that he is just talented, or that he is just more talented than someone which is less talented in math but more generally talented. If someone is normal wrt language development but not wrt emotional development, then we cannot just say that she is normal (without implicitly restricting normal to language development) or that she is more normal than someone less normal in language but more normal emotionally (without implicitly restricting normal to language development or hedging it with an implicit on average modifier).

These judgments arise due to the fact that the ordering of entities in conjunctive and disjunctive concepts does not correspond to the order given by averaging on the dimensions. In adjectives P like healthy, which (I argue) are interpreted conjunctively (as healthy wrt cancer and healthy wrt blood pressure and...), unlike nouns, only the values assigned by the degree function of the dimension conjunction deg_{\wedge F(healthy)} (e.g., by deg(blood pressure and pulse and...) indicate the entities’ degrees. But what is this degree function? The denotation of a conjunction is built compositionally from the denotation of the conjuncts, but what about the degree function? There are no compositional rules for degree functions. I propose that when adjectives are interpreted conjunctively or disjunctively, their degree function is based on the order in which their denotation is learnt (the learning principle, cf. 5.2).
In the following, I present a full vagueness model with degrees. In the discussion of vagueness theories in 3.1, I have not explained how the interpretation of complex predicates (predicates that may have as constituents variables, proper names, connectives, etc.) is derived compositionally from the interpretation of their parts, and I have ignored the role of variable assignments (I have directly discussed constraints on the interpretation of a predicate P in a context c, \([P]_c\)). In 6.1, I give the syntax of the language. In 6.2, I give a significantly more detailed representation of the recursive semantic interpretation of linguistic expressions in this language in partial contexts. Most of these rules are standard, and where they are not (in the interpretation of proper names) I discuss the reasons that forced me to make a different choice. I also give a systematic way of incorporating degree functions, dimension-sets (including weights and selected values), transformation values, domains, and membership standards into the context structure. This chapter is mostly technical, but to the best of my knowledge, the technique for representing partial knowledge has never been employed for representing partial knowledge about gradability notions (with the unique exception of standards for membership). Thus, in this respect, this chapter presents novel aspects of the formal model.

In chapters 7, 8 and 9, I present the core of my theory and its consequences. In chapter 7, I present the different types of degree functions that are associated with different types of predicates. I show that my proposals predict the basic semantic distinctions between different predicate types. In chapter 8, I give the learning principle, which elucidates the connections between gradability and vagueness regarding the denotation. Finally, in chapter 9, I present a detailed account of the polarity effects and of the licensing conditions of within-predicate comparison statements (including compositional derivations for statements with numerical-degree predicates as in 2 meters shorter and shorter than 2 meters, difference modifiers like more and less, and ratio modifiers as in twice as tall as and four times heavier).

6.1 The syntax of the language

The vocabulary of the language consists of the lexical items given in (1), classified by their syntactic categories:

(1) **Vocabulary:**

a. Let the set IND-VARIABLE consist of pronouns and indexical expressions like he, she, this, they, now, I, and variables \(x_1, x_2, \text{etc.}\) (we will be using the standard shorthand \(x, y, z, \text{etc.}\)).

b. \(\forall n \in \mathbb{N},\) let the set \(P^n\)-VARIABLE consist of \(n\) place anaphoric/indexical expressions (for example, so, as in so is Bill, is in \(P^1\)-CONCEPT) and \(n\)-place predicate variables \(F_1, F_2, \text{etc.}\).

c. Let the set PROPER-NAME consist of proper names (individual concepts like Sam and Dan).

d. VARIABLE = IND-VARIABLE \(\cup\) \(P^n\)-VARIABLE
e. Let the set CONCEPT-NAME consist of lexical predicates, including n-place predicates such as the following:
   * tall, healthy, bird, walk $\in$ CONCEPT-NAME\(^1\);
   * love, sister of $\in$ CONCEPT-NAME\(^2\)
   etc.

f. Let the set of logical constants consist of
   a set of CONNECTIVES (including $\neg$, $\land$, $\lor$, and $\rightarrow$);
   a set of QUANTIFIERS (including $\exists$ and $\forall$);
   the operator $\lambda$;
   and the definite article the.

Let SENTENCE be the set of well formed sentences in the language. The sentences are constructed by the following syntactic rules.

(2) Sentences:
   a. $\text{PROPER-NAME} \cup \text{IND-VARIABLE} \subseteq \text{TERM}$
   b. $\forall n \in \mathbb{N}, \text{CONCEPT-NAME}^n \cup \text{P}^n\cdot\text{VARIABLE} \subseteq \text{CONCEPT}^n$
   c. $\forall n \in \mathbb{N}, \forall \alpha_1 \ldots \alpha_n \in \text{TERM}, \forall P \in \text{CONCEPT}^n$:
      $P(<\alpha_1\ldots\alpha_n>) \in \text{SENTENCE}$.
   d. $\forall \varphi, \psi \in \text{SENTENCE}, \forall \alpha \in \text{VARIABLE}$:
      $\neg\varphi$, $\varphi \land \psi$, $\varphi \lor \psi$, $\varphi \rightarrow \psi$, $\exists \alpha(\varphi, \psi)$, $\forall \alpha(\varphi, \psi)$, $\in \text{SENTENCE}$
   e. $\forall \alpha_1, \alpha_2 \in \text{TERM}, (\alpha_1 = \alpha_2) \in \text{SENTENCE}$

(3) Complex concepts (complex predicates):
   a. $\forall n \in \mathbb{N}, \forall \alpha_1 \ldots \alpha_n \in \text{IND-VARIABLE}, \forall \varphi \in \text{SENTENCE}$:
      $\lambda \alpha_n \ldots \lambda \alpha_1 . \varphi \in \text{CONCEPT}^n$
   b. $\forall n \in \mathbb{N}, \forall \alpha \in \text{TERM}, \forall P \in \text{CONCEPT}^n$:
      $P(\alpha) \in \text{CONCEPT}^{n-1}$

(4) Definite terms:
   a. $\forall P \in \text{CONCEPT}^1$, the(P) $\in \text{TERM}$

(5) Degree functions and degree terms:
   $\forall n \in \mathbb{N}, \forall P \in \text{CONCEPT}^n, \forall \alpha_1, \ldots, \alpha_n \in \text{TERM}$:
   a. $\text{deg}(P) \in \text{FUNCTION}$
   b. $\text{deg}(<\alpha_1, \ldots, \alpha_n>, P) \in \text{TERM}$

The expression $\text{deg}(P)$ is an object-language name of the (final) degree-function associated with the predicate P, as will become clear in chapter 7. The expression $\text{deg}(\alpha, P)$ (for a $P \in \text{CONCEPT}^1$) may be read as ‘the degree of $\alpha$ in $P$’. 
6.2 Vagueness models with degree functions

A model for this language is a vagueness model with degree functions:

\[(6) \text{A vagueness model with degree functions is a tuple:} \quad M_C = \langle D, D_f, C, \leq, c_0, T, \text{Extension}, G, I \rangle \]

In the following sub-sections I define the elements in $M_C$ (the ontology, $D$ and $D_f$; the context structure, $C, \leq, c_0$ and $T$; the representation of facts about the world, Extension; the representation of discourse facts, $G$, and the representation of additional parts of predicates' interpretation, $I$).

6.2.1 A degree-ontology: $D$ and $D_f$

The ontology (presupposed by this language) includes a set $D$ (of all the possible entities). In addition, for any natural number $n$, let $D^n$ be the set of $n$-tuples of elements of $D$. As in standard gradability theories (cf. 3.2), I assume that the ontology includes, also, for any natural number $n$, a set $(D^n)_f$ of mappings of ($n$ tuples of) entities to degrees (functions from $D^n$ to the set of real numbers $\mathbb{R}$, $(D^n)_f \subseteq \{f: D^n \rightarrow \mathbb{R} \}$). Let us call the union of all the function-sets $D_f (D_f = \cup \{(D^n)_f | n \in \mathbb{N} \})$. Each function in $D_f$ designates something (a quality that human beings can detect). For example, a function can map entities to their heights, to their weights, it can map entities to 1 iff they have ID number 095689897 and to 0 otherwise (there is only one actual individual, but many more possible individuals, with this ID number – some are 1.87 meters tall, some are 1.86 meters tall, etc.).

$D$ contains individuals that may form the reference of proper names like Dan and Sam. Parts of these individuals are also individuals in their own right (Sam's leg, Dan's happiness, Dan's height, etc.) They, too, are members of $D$, and they, too, are associated with a rich set of values (Dan's happiness may be deep, surprising, related to his diet, etc.; Moltmann 2006). Collections (sums) of individuals are also individuals in their own right, so they too are elements of $D$. That is, $D$ is closed under sum. For any subset $A$ of $D$, the sum of $A$, $\text{sum}(A)$, is an element of $D$ (sums are plural individuals, which are defined roughly as follows: $\forall d \in D: d \in A$ iff $d$ is 'part of' $\text{sum}(A)$; For complete definitions and discussion see Landman 2004: 1-18).\(^1\) Finally, concatenations of individuals are in $D$ as well (cf. Klein 1991). For example, the concatenation $d_1 \oplus \text{length}d_2$ (the result of placing end to end two rods $d_1$ and $d_2$) is an object in $D$ (and for any $n$, concatenations of $n$-place tuples of individuals are themselves $n$-place tuples, elements of $D^n$).

How can we tell that a certain function $f$ represents a certain quality, say height? Following Klein (1991), we can state that a degree function $f$ designates a quality $Q$ iff it is additive wrt $Q$:\(^1\)

---

\(^{1}\) Plural individuals form the denotations of kind-referring expressions (Chierchia 1998; Krifka 2004; Dayal 2004), like the nominal subjects in *the grey robin is extinct* and *tomatoes contain vitamin A*. I adopt this analysis in order to represent the fact that some typicality judgments are related to kinds like *robin, ostrich* etc., not to simple entities, as discussed in chapter 8.

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Additivity:
∀n ∈ N, a function f ∈ (D^n)_f is additive wrt a quality Q iff f conforms to these two constraints:
  a. f maps the concatenation wrt Q of any two n-place entity-tuples a_1 and a_2, a_1 ⊙_Q a_2, to the sum of their values:
     ∀a_1, a_2 ∈ D^n: f(a_1 ⊙_Q a_2) = f(a_1) + f(a_2);
  b. a_1 and a_2 possess the same quantity of Q iff f maps a_1 ⊙_Q a_2 to twice a_1's value:
     ∀a_1, a_2 ∈ D^n: f(a_1) = f(a_2) iff f(a_1 ⊙_Q a_2) = 2 × f(a_1)

We will say that a function f represents quantities of a quality Q iff f is additive wrt Q. For example, let deg^+(long,t,g) stand for the degree function of long in t and g. Let deg^+(d,long,t,g) stand for the value this function assigns to d (deg^+(d,long,t,g) = deg^+(long,t,g)(d)). For any t and g, deg^+(long,t,g) (the degree function of long in t and g) is additive wrt length. It adequately represents the fact that the length of two rods when placed end to end equals the sum of lengths of the two separate rods. For any t and g, deg^+(long,t,g) represents this fact by mapping these entities to values such that: deg^+(d_1 ⊙_length d_2,long,t,g) = deg^+(d_1,long,t,g) + deg^+(d_2,long,t,g). In particular, in any t and g, deg^+(long,t,g) adequately represents the fact that the length of a concatenation of two entities with equal (quantities of) length equals twice the length of each entity. It represents this fact by mapping these entities to values such that: deg^+(d_1,long,t,g) = deg^+(d_2,long,t,g) iff deg^+(d_1 ⊙_length d_2,long,t,g) = 2 × deg^+(d_1,long,t,g). In sum, the values of additive functions represent the ratios between the quantities of Q in different entities.

Let us constrain D_f by stating that all the functions in D_f adequately represent (are additive wrt) some quality.

Additivity is based on the notion of a concatenation, which is hard to define (what is the concatenation of two entities wrt happiness? How do we concatenate parts like happiness instantiations?). I will simply take it as an axiom that the functions in D_f designate some quality that we can detect. That is the thing that they do by definition.

Recall (from chapter 4) that given a degree ontology, any (possible) individual (or tuple of individuals in D^n, for any n), can be identified with a point on a multi-dimensional space whose axes are the ranges of the degree functions in (D^n)_f. Each (possible) individual (or tuple of individuals) can be described as a unique maximal (consistent) assignment of degrees, the degrees that all the possible degree functions assign it (Leibnitz's law of identity of indiscernible, cf. Loemker 1969: 308).

For technical reasons, I assume that D does not include the undefined symbol ⊥ (this assumption is not necessary but it simplifies the interpretation rules).
6.2.2 The context structure: \( C, \leq, c_0, \) and \( T \)

\( M_C \) is a full vagueness model. I.e. it contains a context structure, which is defined as follows.

\[
\text{(10) The context structure} \\
\begin{align}
\text{a. } & C \text{ is a set of (possibly partial) contexts } C \\
\text{b. } & \leq \text{ is a partial order on } C \text{ (a meet semi-lattice)} \\
\text{c. } & T \text{ is the set of maximal contexts under } \leq \text{ (} \forall t \in T: \forall c \in C, \neg(t < c) \text{).} \\
\text{d. Totality: } & \text{ Every context is extended by a non-empty set of total contexts} \\
& \quad \quad \quad \quad \quad \quad (\forall c \in C: T_c = \{ t \in T: c \leq t \} \neq \emptyset) \\
\text{e. Monotonicity: } & \text{ A context } c_1 \text{ is monotonically extended by a context } c_2 \text{ }(c_1 \leq c_2) \text{ iff} \\
& \quad \quad \quad \quad \quad \quad \text{ The set of total extensions of } c_2 \text{ is a subset of the set of total} \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{ extensions of } c_1 \quad (\forall c_1, c_2 \in C: c_1 \leq c_2 \text{ iff } T_{c_1} \supseteq T_{c_2}) \\
\text{f. Zero information: } & c_0 \text{ is the minimal context under } \leq: T_{c_0} = T \text{ (i.e. } \forall c \in C: c_0 \leq c) 
\end{align}
\]

What is the intuitive idea underlying the proposed knowledge structure? First, any intermediate context \( c_1 \) in \( M_C \) represents a possible 'real' context (say, the knowledge shared by the participants in a certain discourse). Second, the contexts extended by (leading to) \( c_1 \) represent the way in which this knowledge has been gradually accumulated, and the contexts extending \( c_1 \) represent the possible ways in which this knowledge may be gradually completed. The rest of the structure includes counterfactual contexts (sets of facts about alternative realities).\(^2\) For example, in a partial context \( c \), the positive denotation of tall, \( [[\text{tall}]]_c \) (see definitions below), may consist of very tall items, and the negative denotation, \( [[\neg\text{tall}]]_c \), may consist of very short items, but \( c \) is extended by a set of total extensions, \( T_c = \{ t \in T: c \leq t \} \), that represent all the possibilities of specifying the complete sets of tall and non-tall things which are consistent with the information in \( c \). In each \( t \) in \( T_c \), each item is either in the positive or the negative denotation of tall, bald, clever, love, Sam and all the other proper-names and concept-names. As a result, every statement is either true or false (the recursive semantic rules in total contexts \( t \) in \( T \) relative to an assignment \( g \) are classic). Let us see how this works step by step.

6.2.3 The extension-assigning function, \( \text{Extension} \)

The \( \text{Extension-Assigning Function} \), is a total function from pairs of an expression in CONCEPT-NAME \( \cup \) PROPER-NAME and a total context \( t \) in \( T \) to an extension in the appropriate semantic domain, as follows:

\(^2\) The primary purpose of contexts is to represent knowledge states about \textit{the actual world} (assuming that there is one). However, each total context \( t \) in \( T \) can also represents a total set of facts about some other world (or about a set of worlds that are indistinguishable in terms of the interpretation of the linguistic expressions in them). Such structures were proved useful in the analysis of topics such as epistemic modality, conditionals, domain restriction, genericity, etc. (see, for instance, Veltman 1984, Landman 1991, and Sassoon 2002).
The extension-assigning function \( \forall t \in T: \)

a. \( \forall k \in \text{PROPER-NAME}, \ \text{Extension}(k,t) \in D \)
b. \( \forall n \in \text{N}, \ \forall P \in \text{CONCEPT-NAME}^n, \ \text{Extension}(P,t) \subseteq D^n. \)

6.2.4 The set of assignment functions, \( G \)

The extensions represent (partial sets of) facts about the world, but there are also discourse-facts, which consist of the interpretation that is given in each discourse stage for the set of linguistic variables. For an elaborate dynamic discourse model see Groenendijk, Stokhof and Veltman (1996). I use a simplified representation of discourse.

Let \( G \) be a set of total functions \( g \) from pairs of an expression in \( \text{VARIABLE} \) and a total context \( t \) in \( T \) to an extension in the appropriate semantic domain, as follows:

(11) Variable-assignments:

An assignment function \( g \in G \) is a function from variables to extensions, as follows:

a. \( \forall \alpha \in \text{IND-VARIABLE}: g(\alpha) \in D \)
b. \( \forall n \in \text{N}, \ \forall F \in \text{P}^n-\text{VARIABLE}: g(F) \subseteq D^n \)
c. \( \forall \alpha, \beta \in \text{IND-VARIABLE}, \ \forall \alpha, \beta \in \text{P}^n-\text{VARIABLE}, \ \forall \alpha = \beta \in D^n: g(\alpha/\beta)(\alpha) = g(\beta)(\beta) \)
d. \( \forall n \in \text{N}, \ \forall F_1, F_2 \in \text{P}^n-\text{VARIABLE}: \forall A \subseteq D^n: g(F_1/A)(F_1) = A \) and \( g(F_1/A)(F_2) = g(F_2) \)

In the following, we first define truth for total contexts, and then, based on this, we define truth ("super-truth") in partial contexts using the super-valuation method (van Fraassen 1969). The notion of super truth in \( c \) is based on truth in every total extension.

6.2.5 Semantic values relative to a context \( t \) in \( T \) and an assignment \( g \) in \( G \)

Let \( [[\ ]]^+ \) be a function from triples of an expression \( \alpha \) in the language, a total context \( t \) in \( T \) and an assignment \( g \) in \( G \) into the positive semantic value of \( \alpha \) in \( t \) and \( g \). I also define \( [[\ ]]^− \) to be a function from triples of an expression (a term or concept name) \( \alpha \), a total context \( t \) in \( T \) and an assignment \( g \) in \( G \) into the negative semantic value of \( \alpha \) in \( t \) and \( g \).

First, the interpretation of variables relative to a context \( t \) and assignment \( g \) is given by \( g \), while the interpretation of proper names and lexical concept names is given by the 'extension' function.

(13) The semantic values of terms in \( t \) and \( g \): \( \forall g \in G, \ \forall t \in T: \)

a. \( \forall \alpha \in \text{IND-VARIABLE}: \ [[\alpha]]^+_{tg} = g(\alpha) \)
   \( [[\alpha]]^-_{tg} = D - \{g(\alpha)\} \)
b. \( \forall n \in \text{N}, \ \forall F \in \text{P}^n-\text{VARIABLE}: \ [[F]]^+_{tg} = g(F) \)
   \( [[F]]^-_{tg} = D - g(F) \)
c. \( \forall k \in \text{PROPER-NAME}: \ [[k]]^+_{tg} = \text{Extension}(x,t) \)
   \( [[k]]^-_{tg} = D - \{\text{Extension}(k,t)\} \)
d. \( \forall P \in \text{CONCEPT-NAME}: \ [[P]]^+_{tg} = \text{Extension}(P,t) \)
   \( [[P]]^-_{tg} = D - \text{Extension}(P,t) \)
Second, we can now formulate recursive interpretation rules for complex expressions in the language relative to a context \( t \) in \( T \) and an assignment \( g \) in \( G \). The semantics in total contexts is classic (every sentence is either true or false. Sentences are interpreted by the predication rule.

(14) Sentences \( \forall g \in G, \forall t \in T, \forall \alpha_1 \ldots \alpha \in \text{TERM}, \forall P \in \text{CONCEPT}^n:\)

a. \([P(<\alpha_1\ldots\alpha_n>)]_{t,g}^+ = 1 \text{ (true in } t,g)\) \( \iff \) \([\alpha_1]_{t,g}^+ \ldots [\alpha_n]_{t,g}^+ \in [P]_{t,g}^+\)
\([P(<\alpha_1\ldots\alpha_n>)]_{t,g}^- = 0 \text{ (false in } t,g)\) otherwise, i.e.
\( \iff \) \([\alpha_1]_{t,g}^+ \ldots [\alpha_n]_{t,g}^+ \not\in [P]_{t,g}^-\)
or for some \( j, 1 \leq j \leq n \) \([\alpha_j]_{t,g}^+ = \bot\)

b. \([\forall \alpha (\varphi,h)]_{t,g}^+ = 1 \text{ iff} \) For all \( d \in D \) s.t. \([\varphi]_{t,g,d}^+ = 1, [\psi]_{t,g,d}^+ = 1,\)
0 otherwise

c. \([\exists \alpha (\varphi,h)]_{t,g}^+ = 1 \text{ iff} \) For some \( d \in D \) s.t. \([\varphi]_{t,g,d}^+ = 1, [\psi]_{t,g,d}^+ = 1,\)
0 otherwise

The language contains definite terms whose semantic value may be undefined. If the interpretation of a term \( \alpha \) is undefined, then \([\alpha]_{t,g}^- = \bot\), as stated and explained below. Since \( \bot \) is not in \( D \), the truth value of any sentence of the form \( P(\alpha) \) in \( t \) and \( g \) is false. Thus, no predicate can truly apply to a definite term (or a degree term with a definite term) that is already known to be a term that fails to denote. The object \( \bot \) is neither in the positive nor negative denotation of predicates so sentences like \text{the king of France is cold} and \text{the king of France is warm} are both false (and in accordance, sentences like \text{the king of France is not cold} and \text{the king of France is not warm} are both true), even if \text{warm} and \text{cold} are interpreted as complementary (as covering the whole domain \( D \)). There is no contradiction because we use an object \( \bot \) which is not part of \( D \), such that any statement of the form \( P(\alpha) \) where some argument denotes this object is always false. This technique leaves total contexts classical models where every statement is either true or false. Strawson’s intuition that these sentences lack a truth value is indirectly represented by the fact that, really and truly, the value \text{false} represents two very different cases. One in which the arguments’ referents are part of \( D \) but not of \([P]_{t,g}^+\) (they are part of \([P]_{t,g}^-\)), and one in which at least one argument is undefined, i.e. not part of \( D \) (so the arguments' referents are neither in \([P]_{t,g}^+\) nor in \([P]_{t,g}^-\)).

In particular, identity statements are also part of \( \text{SENTENCE} \), and they are true iff the referents of the two terms are not \( \bot \), and are identical. They are false iff either the referents of the two terms are not \( \bot \) and are not identical, or either term fails to denote (its referent is \( \bot \)).
Identity sentences:

a. \( \forall \alpha_1, \alpha_2 \in \text{TERM}, (\alpha_1 = \alpha_2) \in \text{SENTENCE} \)

b. \( \forall g \in G, \forall t \in T: \)

\[
\begin{align*}
[[\alpha_1 = \alpha_n]]_{t,g}^+ = 1 & \quad \text{(true in } t, g) \quad \text{iff} \quad [[\alpha_1]]_{t,g}^+ = [[\alpha_n]]_{t,g}^+ \neq \bot \\
[[\alpha_1 = \alpha]]_{t,g}^- = 0 & \quad \text{(false in } t, g) \quad \text{otherwise, i.e.} \\
\end{align*}
\]

If \( [[\alpha_1]]_{t,g}^+ \) or \( [[\alpha_n]]_{t,g}^+ = \bot \) or both, or \( [[\alpha_1]]_{t,g}^- \) or \( [[\alpha]]_{t,g}^- \) or both.

The set CONCEPT is closed under the linguistic operations. For example, we can compose expressions like brave cats that eat this stuff. In our (half formal) language this predicate translates into \( \lambda x. \text{cat}(x) \land \text{brave}(x) \land \text{stuff}(y) \land \text{eat}(x, y) \). Complex concepts are interpreted by the following rules:

Complex concepts:

a. \( \forall \alpha_1, \ldots, \alpha_n \in \text{IND-VARIABLE}, \forall \varphi \in \text{SENTENCE}: \)

\[
\begin{align*}
[[\lambda \alpha_1 \ldots \lambda \alpha_n \cdot \varphi]]_{t,g}^+ = \{<d_1, \ldots, d_n> \in D^n: [[\varphi]]_{t,g(\alpha_1/d_1) \ldots (\alpha_n/d_n)} = 1\} \\
[[\lambda \alpha_1 \ldots \lambda \alpha_n \cdot \varphi]]_{t,g}^- = \{<d_1, \ldots, d_n> \in D^n: [[\varphi]]_{t,g(\alpha_1/d_1) \ldots (\alpha_n/d_n)} = 0\}
\end{align*}
\]

b. \( \forall \alpha \in \text{TERM}, \forall P \in \text{CONCEPT}: \)

\[ [[P(\alpha)]]_{t,g}^+ = \{<d_1, \ldots, d_{n-1}>, [[\alpha]]_{t,g}^+ \in [[P]]_{t,g}^+\} \]

\[ [[P(\alpha)]]_{t,g}^- = \{<d_1, \ldots, d_{n-1}>, [[\alpha]]_{t,g}^- \in [[P]]_{t,g}^-\} \]

Since \( [[P]]_{t,g}^- \subseteq D^n \), if \( \alpha \) fails to denote (denotes \( \bot \)) everything in \( D^{n-1} \) is in \( P(\alpha) \)'s negative denotation.

For any one place concept \( P \), the\((P) \) is a term such that the\((P) \) denotes an element \( d \in D \) iff \( d \) is known to be the unique element in \( P \)'s positive denotation, and the\((P) \) denotes \( \bot \) otherwise (iff \( P \)'s positive denotation has more than one element or it is empty). Let \( \sigma \) be an operator that maps each singleton set to its unique element and is otherwise undefined:

Definite terms:

(1) \( \forall A \subseteq D: \quad \sigma(A) = a \quad \text{iff} \quad A = \{a\}, \quad \sigma(A) = \bot \quad \text{otherwise} \).

(2) \( \forall g \in G, \forall t \in T: \quad [[\text{the}(P)]]_{t,g}^+ = \sigma([[P]]_{t,g}^+) \)

\[ [[\text{the}(P)]]_{t,g}^- = D - \{\sigma([[P]]_{t,g}^-)\} \]

For any \( n \) place concept \( P \), the expression \( \text{deg}(P) \) in FUNCTION denotes a degree function from \( D^n \) to \( R \). \( [[\text{deg}(P)]]_{t,g}^+ \in R^P \), is "the final degree function of \( P \) in \( t \) and \( g \)."

In accordance, for any term \( \alpha \) and one-place concept \( P \), \( \text{deg}(\alpha, P) \) ("the final degree of \( \alpha \) in \( P \) relative to \( t \) and \( g \)") is a term such that in any context \( t \) and assignment \( g \) \( [[\text{deg}(\alpha, P)]]_{t,g}^+ \) is \( \bot \) if \( \alpha \) is a definite term that fails to denote, and otherwise, \( [[\text{deg}(\alpha, P)]]_{t,g}^+ \) is a unique real number (\( \alpha \)'s
The expression ‘deg’ is not assigned its own semantic value, but really what it is, is a function that takes an n place predicate and returns a member of $\mathbb{R}^n$.

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For example, different assignments may assign the occurrence of *this* in "brave cats that eat this stuff" (or y in "\(\lambda x. \text{cat}(x) \land \text{brave}(x) \land \text{stuff}(y) \land \text{eat}(x,y)\)"") different values (\(g_1\) may assign y an entity which in t is a piece of cat food, \(g_2\) may assign y an entity which in t is a piece of dog-food, etc.) In such a case, the denotation of "\(\lambda x. \text{cat}(x) \land \text{brave}(x) \land \text{stuff}(y) \land \text{eat}(x,y)\)" in t may be undefined, as the sets of cats that like cat food may be rather different from the set of cats that like dog-food. Similarly, even tough t is a context of total information, the statement *Dan is a brave cat that eats this stuff* may have different truth values relative to \(g_1\) and \(g_2\) in t. If, for instance, the unique element in Extension\(^*\)(Dan,t) is a brave cat that eats dog food, not cat food in t, the statement may not be defined in t independent of an assignment function.

### 6.2.6 Super semantic values relative to a context \(c\) in \(C\) and an assignment \(g\) in \(G\)

We have not directly assigned interpretation to expressions in partial contexts \(c\) in \(C\). The interpretation of expressions relative to a partial context \(c\) and an assignment \(g\), \([\![\ ]\!]_{c,g}\), is given by the super-valuation technique (van Fraassen 1969; Veltman 1984). The notation for super-truth, \([\![\ ]\!]_{c,g}\), is identical to the notation of truth, \([\![\ ]\!]^+_{c,g}\), except that it lacks the superscript \(^*\). The semantics in partial contexts is three valued, because the truth value of statements in partial contexts may be unknown – neither (super-) true nor (super-) false. A statement \(\varphi\) is *super-true* (it is known to be true either directly or by inference) in a partial context \(c\) in \(C\) relative to an assignment \(g\) (\([\![\varphi]\!]_{c,g} = 1\)) iff \(\varphi\) is true in all the total extensions \(t\) of \(c\) relative to \(g\). A statement \(\varphi\) is *super-false* (known to be false directly or by inference) in \(c\) and \(g\) iff \(\varphi\) is false in all the total extensions \(t\) of \(c\) relative to \(g\). Its truth value is unknown in \(c\) otherwise.

(20) **Supertruth:**

\[
\forall g \in G \quad \forall c \in C, \quad \forall \varphi \in \text{SENTENCE}: \\
\begin{align*}
\text{a. } & \quad \lfloor \varphi \rfloor_{c,g} = 1 \quad (\text{supertrue in } c, g) \quad \text{iff} \quad \forall t \in T, \ t \geq c: \lfloor \varphi \rfloor_{t,g}^+ = 1 \\
\text{b. } & \quad \lfloor \varphi \rfloor_{c,g} = 0 \quad (\text{superfalse in } c, g) \quad \text{iff} \quad \forall t \in T, \ t \geq c: \lfloor \varphi \rfloor_{t,g}^+ = 0 \\
\text{c. } & \quad \lfloor \varphi \rfloor_{c,g} = \text{unknown in } c, g \quad \text{otherwise}
\end{align*}
\]

A statement is *super-true* in a partial context \(c\) in \(C\) (\([\![\varphi]\!]_{c} = 1\)) iff it is true in any total extension \(t\) of \(c\) relative to any \(g\). A statement is *super-false* in \(c\) iff it is false in any total extension \(t\) of \(c\) relative to any \(g\). The truth value is unknown in \(c\) otherwise.

\[
\begin{align*}
\text{a. } & \quad \lfloor \varphi \rfloor_{c} = 1 \quad (\text{supertrue in } c) \quad \text{iff} \quad \forall t \in T_c, \ \forall g \in G: \lfloor \varphi \rfloor_{t,g} = 1 \\
\text{b. } & \quad \lfloor \varphi \rfloor_{c} = 0 \quad (\text{superfalse in } c) \quad \text{iff} \quad \forall t \in T_c, \ \forall g \in G: \lfloor \varphi \rfloor_{t,g} = 0 \\
\text{c. } & \quad \lfloor \varphi \rfloor_{c} = \text{unknown in } c \quad \text{otherwise}
\end{align*}
\]

For example, the fact that Dan is 1.90 meters tall in a given partial context \(c\) may be directly given to us, if a person (an object in D) stands in front of us, tells us that his name is *Dan*, and we measure it with a ruler showing that the person's height is 1.9 meters. This fact is formally represented in our model as follows. Relative to any assignment \(g\), in any total context \(t\) above \(c\),
[[Dan is 1.9 meters tall]]\textsuperscript{t\_lg} is true. I.e. in any total extension t of c, whatever the referent (extension) of Dan in t is, it is in the denotation of tall in t.

We can now use a classic definition of entailment. A statement $\phi$ entails a statement $\psi$ iff any total context that verifies $\phi$ verifies $\psi$. It follows that $\phi$ entails $\psi$ iff any partial context c that verifies $\phi$ verifies $\psi$ (because all its total extensions verify $\phi$ and hence by the definition of entailment they also verify $\psi$, and so, $\psi$ is (super) true in c.)

\begin{align*}
(21) \quad \phi \implies \psi \quad \text{iff} \quad \forall M_c, \forall t \in T_{M_c}: \text{if:} \quad &[[\phi]]^t_{c,g} = 1, \text{then:} \quad [[\psi]]^t_{c,g} = 1. 
\end{align*}

While Veltman (1984) and van Fraassen (1969) represent epistemic necessity at the level of truth conditions of statements, I represent epistemic necessity also at the level of constituents of statements. For example, for any predicate P, the set of Ps (entities whose membership in the positive denotation of P is positively known) is represented by the notation $[[P]]_{c,g}$, the positive super denotation of P in c and g. The sets of non-Ps (entities whose membership in the negative denotation of P is positively known) is represented by the notation $[[-P]]_{c,g}$, the negative super denotations of P in c and g.

\begin{align*}
(22) \quad \text{Super-denotations relative to a partial context } & c \in C \text{ and an assignment } g \in G: \\
& \text{a. } \forall c \in C, \forall g \in G, \forall k \in \text{TERM}: \quad [[k]]_{c,g} = d \text{ iff } \forall t \geq c, [[k]]^t_{c,g} = d \\
& \quad [ [k] ]_{c,g} = \text{unknown otherwise} \\
& \quad [[-k]]_{c,g} = \cap \{ [[k]]^t_{c,g} \mid t \geq c \} \\
& \text{b. } \forall c \in C, \forall g \in G, \forall P \in \text{CONCEPT}: \quad [[P]]_{c,g} = \cap \{ [[P]]^t_{c,g} \mid t \geq c \} \\
& \quad [[-P]]_{c,g} = \cap \{ [[P]]^t_{c,g} \mid t \geq c \} \\
\end{align*}

\begin{align*}
(23) \quad \text{Super-denotations relative to a partial context } & c \in C: \\
& \text{a. } \forall c \in C, \forall k \in \text{TERM}: \quad [[k]]_c = d \text{ iff } \forall g \in G, \forall t \geq c, [[k]]^t_{c,g} = d \\
& \quad [ [k] ]_c = \text{unknown otherwise} \\
& \quad [[-k]]_c = \cap \{ [[k]]^t_\neg_{c,g} \mid t \geq c, g \in G \} \\
& \text{b. } \forall c \in C, \forall P \in \text{CONCEPT}: \quad [[P]]_c = \cap \{ [[P]]^t_\neg_{c,g} \mid t \geq c, g \in G \} \\
& \quad [[-P]]_c = \cap \{ [[P]]^t_\neg_{c,g} \mid t \geq c, g \in G \} \\
\end{align*}

Semantic theories often assume that the interpretation of proper names is rigid (does not vary across different alternatives of reality). I represent partial information about proper names, so I do not assign them rigid interpretations. For example, if in c we do not yet know the value that some function in D assigns to the unique individual in D to whom Sam refers, (for instance, we do not know Sam's height), this means that we simply do not yet know which unique individual in D Sam refers to in c. In that situation, several individuals in D that differ in height may still be the unique element to whom Sam refers – the unique element of the positive extension of Sam in the ‘real’ total context t, Extension\textsuperscript{t}(Sam,t). So in c itself, the positive denotation of Sam, $[[\text{Sam}]]_c$, is still empty. (In fact, not being omniscience, we never actually know the referent of a proper name!) If we know that Sam's height is 1.86, but we do not know the extent to which Sam loves Dan, or what her I.D. number is, the positive denotation is empty, but the negative denotation, $[[-\text{Sam}]]_c$,}
consists of all those entities whose height is not 1.86. Thus, the negative denotation encodes information about individual concepts.

There is no vagueness concerning the values of individuals in D. Vagueness pertains only to word meaning. Sure we may be aware of the existence of an individual, while some of its values may not be accessible to us (if, say, it is covered by a blanket). This means that we know that the statement An individual exists under this blanket is true in c, but we do not know which one of the possible individuals it is (we do not know which one of the possible individuals the discourse referent this individual denotes). We can refer to this individual by saying this or the individual under the blanket but the interpretation of these expressions will still be empty.4

Consider, for example, tall. We know how entities should be mapped to degrees in tall (by their height; you take a ruler and…) For any two possible individuals we can say which one is taller. Their values on any degree function f that adequately represents height stand in the bigger relation. The denotation of taller than is therefore completely known. But this knowledge is trivial, it is not empirical (it is knowledge about imaginable entities). Concerning actual things that we see or speak about (discourse entities, referents of terms, e.g. Dan and Sam), we can only say that they stand in the relation taller than (that the statement Dan is taller than Sam is true) iff we have knowledge about their heights (if, say, we see them and measure them with a ruler). And even then, we cannot add them to the denotation of taller than, because we do not know which individuals in D they are. We can only tell that in any t above c, all those pairs of entities d1 and d2 which are not part of \([taller than]^c\), cannot form the referents \([Sam]^c\) and \([Dan]^c\), respectively. Had we represented proper names as having a completely known interpretation, it would have wrongly followed that we also know the truth value of any statement of the form "Sam is taller than Dan". But we do not. For example, do you know who is taller, Adar or his father? No. How come? Because you do not know their heights. You do not know which possible individuals (degree-sets) they are supposed to refer to – which possible individuals they would be mapped onto, were we to reach complete knowledge about Adar and his father).

On this account identity statements (like Dan is Mr. Cohen) are highly informative (if \([Mr. Cohen]^c\) = \([Dan]^c\), we can add all the entities that are known not to be Dan to \([−Mr. Cohen]^c\), and all the entities that are known not to be Mr. Cohen to \([−Dan]^c\), which means that we may gain a lot of information about both).

Concerning predicates like tall, if in a context c we do not yet know that tall is related to the dimension has height (we do not yet know the degree function of tall), then we cannot classify individuals in D as tall or not. If we know this fact but we do not yet know anything about the standard of membership in tall (for a formal definition of this notion see next section), we still cannot classify individuals in D as tall or not. If we know that the height of entities who are 1.70

4 When we say ‘I’ (or this, pointing to something), we never know what actual d in D is the value of this expression – different assignment functions give it a different value. We only have information that constrains the assignment functions that might still be actual – assignments such that it is still possible that they describe correctly the objects we are pointing at (my model does not represent this kind of discourse knowledge; in this note I only intend to clarify the intuitive idea as to how things should work if such a representation is incorporated). That means that if we point at a ball and say this (ball) is red, that does not allow the hearer to put any particular d in D into the denotation of red. After all, we do not know which d the ball we were shown is. We only know that if something has the color of this d (no matter which d it actually is) that thing is red, and whatever the value of this ball is, it is an object which is red. In other words, everything that is not red is added to the negative denotation \([−[this ball]]^c\).
meters tall is below the standard, and the height of entities who are 2 meter tall is above the standard, then we can classify every d in D who is below 1.70 meters in [[¬tall]]c, and every d in D who is above 2 meters in [[tall]]c. Other individuals remain in the gap. In c, the (super) truth value of statements like [[Sam is tall]]c would still be unknown (given that Sam is 1.86 meters). If in some partial extension more will become known about the standard of tall this truth value may become known.

If in a context c the positive denotation of tall, [[tall]]c, consists of very tall items, and the negative denotation of tall, [[¬tall]]c, consists of very short items, then in c, we do not yet know if anything else is tall or not. Similarly, if in c we do not yet know whether some entities, d1 and d2 stand in some relation (say, love, healthier, etc., perhaps because they do not possess much love to one another, or perhaps because in c we do not yet know what love is – that is, quantities of what the degree function of this relation represents), then in c the pair <d1,d2> is neither in the positive super-denotation that relation (e.g., in [[love]]c), nor in its negative super-denotation, [[¬love]]c. This pair falls in the gap of the two place predicate love. The super-denotations of tautological predicates (for instance, predicates of the form λx. P(x) ∨ ¬P(x)) are complete already in the zero-information context c0 (because for every d, d is either in the positive or negative extension of P in any t). For the same reason, the negative super-denotations of contradictory predicates (for instance, predicates of the form λx. P(x) ∧ ¬P(x)) are completely known already in c0.

For any one place concept P, [[the(P)]] denotes an element d∈D iff d is known to be the unique element in P's positive denotation in any t in Tc, but this is rarely if ever the case. [[the(P)]] is unknown iff P's positive super-denotation is still empty but some elements in D may still be the unique P. [[the(P)]] is undefined (denotes ⊥) iff more than two elements in D are already known to be P (P's positive super-denotation has more than one element), or all the elements in D are already known not to be P (P's negative super-denotation is known to be D).

Similarly, [[deg(α,P)]]c,g is ⊥ if α is a definite term that is already known to be a term that fails to denote. [[deg(α,P)]]c,g is unknown if α's referent's degree is not yet known (there is no unique number such that in any total context P maps the referent of α to that number). For example, we never (in no actual context) know directly the interpretation of deg(tall,Sam) (Sam's degree in tall), because we never know who Sam is, and so we cannot directly say whether the individual in D which is Sam's actual referent in the real world is tall, taller than Dan, etc. But if in any total extension of the context c we are actually in (any total extension that, according to our beliefs, may still adequately represent facts about the real world) Sam's degree is known to be above the standard (say, in any t in Tc the referent of Sam is tall or taller than Dan, whatever Sam's referant is), then these facts are known. This happens when we have access to Sam's height and we have the knowledge that entities with this height are considered tall (this height is already known to be above the standard, whatever the standard is).

The super-denotations represent any knowledge in c, be it knowledge that we intuitively feel to be given directly or knowledge that we intuitively feel to be computed by inference. For example, if in any total context above c and g Sam (who is known to be 1.90 meter tall in c) is tall (say, only entities who are 1.90 meter tall may still form Sam's reference in c, and all these entities are already part of [[tall]]c), then, intuitively, in c we can infer that Dan (who is known to be taller than Sam) "must be tall" (if Sam reaches standard, then so does Dan). Crucially, in every
total context \( t \) extending \( c \), the reference of Dan will end up in \([\text{tall}]^+_t\), \( \forall t \in T_c \), \( \forall g \in G: [[\text{Dan}]]^+_{t,g} \in [[\text{tall}]]^+_t\). In the given example, we know that Dan is tall in \( c \) by inference. So there are various ways of learning about the denotation, some of which we intuitively feel to be more direct than others (the given example is felt to be less direct than a case in which one is directly taught that Dan is tall).

Epistemic operators like must are often used (as in Sam must be hungry) when knowledge is given by inference (Veltman 1986). Veltman (1986) defines truth in a partial context (e.g., \([[\text{Sam is hungry}}]]^+_c = 1 \text{ iff ..} \) and he represents epistemic necessity as super-truth (truth in every total context), e.g. [[Sam must be hungry]]^+_c = 1 iff for any extension \( t \) of \( c \), [[Sam is hungry]]^+_c = 1). I do not represent truth in a partial context (the notion \([[]]^+_c\)). The analysis of epistemic modals like must is beyond the scope of this dissertation. However, note that, intuitively, they involve some restriction on the set of possible total contexts. One says that Sam must be hungry iff one is almost, but not completely, sure that Sam is hungry; This may happen because no one but Sam has access to Sam's state of hunger, whose extent determines Sam's degree in hungry; One can only inductively infer that this degree is above the standard of hungry, by using indirect, inconclusive symptoms, such as Sam's being pale or nervous.

6.2.7 The additional elements in the interpretation of predicates, \( I \)

On the present proposal, the interpretation of predicates \( P \) in a total context \( t \) and assignment \( g \) includes additional notions except for the denotation \([P]^+_t,g\).

\[
\begin{align*}
(24) \quad \text{Let } I &= \langle f^+, \text{ Standard}^+, \text{ Domain}^+, \text{ Tran}^+, F^+, \text{ Weight}^+, \text{ Value}^+ \rangle \text{ s.t.} \\
&\forall n \in \mathbb{N}, \forall P \in \text{CONCEPT}^n, \forall t \in T, \forall g \in G: \\
&\text{a. The "additive-function"-assigning function, } f^+, \text{ is a total function from a triple consisting of a total context } t \in T, \text{ an assignment } g \in G, \text{ and a predicate } P \in \text{CONCEPT}^n \text{ (for some } n \in \mathbb{N}) \text{, to an additive degree function } f^+(P,t,g) \in D_f \text{ (the additive degree function of } P \text{ in } t \text{ and } g). \\
&\text{The function } f^+(P,t,g) \text{ is a total function from any } n \text{ tuple of entities } \langle d_1, \ldots, d_n \rangle \in D^n \text{, to a unique real number } f^+(\langle d_1, \ldots, d_n \rangle, P,t,g) \in R \text{ (the additive degree of } \langle d_1, \ldots, d_n \rangle \text{ in } P \text{ in } t \text{ and } g). \text{ If the function } f^+(P,t,g) \text{ receives as an input a tuple with the undefined element } \bot \text{ in one of its positions (i.e. a tuple which is not part of } D^n \text{) it returns } \bot\text{, as it is only defined for elements in } D^n. \\
&\text{b. The "transformation-value"-assigning function, } \text{Tran}^+, \text{ is a total function from a triple consisting of a total context } t \in T, \text{ an assignment } g \in G, \text{ and a predicate } P \in \text{CONCEPT}, \text{ to a unique real number } \text{Tran}^+(P,t,g) \in R \text{, } P \text{'s transformation value in } t \text{ and } g \text{ (the role of this notion is clarified and demonstrated in chapter 9).} \\
&\text{Briefly, the interpretation of } \text{deg}(P) \text{ is sometimes given directly by the function } f^+(P,t,g) \text{ (for instance, } [[\text{deg(long)}]]^+_t,g = f^+(\text{long},t,g) \text{ for any } t \text{ and } g), \text{ but sometimes, the values of } f^+ \text{ are transformed by a constant } \text{Tran}^+(P,t,g) \text{ (for instance, on my proposal, for any } t, g, \text{ and } d, [[\text{deg(x,short)}]]^+_t,g(x/d) = \text{Tran}^+(\text{short},t,g) - f^+(d,\text{short},t,g), \text{ such that in many contexts } t, \text{ for any } g, 
\end{align*}
\]
Tran\(^{\dagger}\)(short,t,g) is a real number other than zero, as stated in 7.3, and discussed at length in chapter 9).

c. The standard-assigning function, Standard\(^{\dagger}\), is a total function from a triple consisting of a total context \(t \in T\), an assignment \(g \in G\), and a predicate \(P \in \text{CONCEPT}\), to a unique real number \(\text{Standard}^{\dagger}(P,t,g) \in R\), P's standard of membership in \(t\) and \(g\) (a number such that entity tuples whose degrees are below this number are not \(P\) in \(t\) and \(g\), and entity tuples whose degrees are at least as big as this number are \(P\) in \(t\) and \(g\), as explained in 7.1).

d. The domain-assigning function, Domain\(^{\dagger}\), is a total function from a triple consisting of a total context \(t \in T\), an assignment \(g \in G\), and a predicate \(P \in \text{CONCEPT}\)\(^n\) (for some \(n \in \mathbb{N}\)), to a set of \(n\)-tuples of entities in \(D\), \(\text{Domain}^{\dagger}(P,t,g) \subseteq D^n\), P's domain in \(t\) and \(g\) (a set of entities based on which P's standard can be determined). As explained in 7.2, P's domain does not include entities with zero quantity of the quality \(\text{deg}(P)\) measures (for instance, stories cannot be part of the domain of tall).

As already noted in 5.6, I propose that dimensions are normal concept-names (predicates) of the same arity as the predicate they are dimensions of (predicates like small, feathered, singing and flying for bird, and like has height for tall). Thus, dimensions are associated with degree functions \(f^{\dagger}\) just like predicates, and their interpretation is context dependent just like the interpretation of predicates. Degree functions of dimensions (the values that they assign to entities) constrain the degree functions of the predicates they are dimensions of. As explained in 7.4-7.5, the type of constraint depends on the type of (degree function that is associated with the) predicate.

e. The dimension-set-assigning function, \(\mathbf{F}^{\dagger}\), is a total function from a triple consisting of a total context \(t \in T\), an assignment \(g \in G\), and a predicate \(P \in \text{CONCEPT}\)\(^n\) (for some \(n \in \mathbb{N}\)), to a set of \(n\)-place predicates, \(\mathbf{F}^{\dagger}(P,t,g) \subseteq \text{CONCEPT}\)\(^n\), P's dimension-set in \(t\) and \(g\).

f. The weight-assigning function, Weight\(^{\dagger}\), is a total function from a quadruple consisting of a total context \(t \in T\), an assignment \(g \in G\), and two predicates \(P, F \in \text{CONCEPT}\)\(^n\) (for some \(n \in \mathbb{N}\)), to a unique real number between zero and one, \(\text{Weight}^{\dagger}(F,P,t,g) \in [0,1]\), the weight of the dimension \(F\) for the category \(P\) in \(t\) and \(g\). For example, the weight of the dimension size for the category bird, \(\text{Weight}^{\dagger}(\text{bird},\text{size},c,g)\), tells us how important size is in discriminating good examples of birds from bad examples (or non-birds) in \(t\) and \(g\).

g. The selected-value-assigning function, Value\(^{\dagger}\), is a total function from a quadruple consisting of a total context \(t \in T\), an assignment \(g \in G\), and two predicates \(P, F \in \text{CONCEPT}\)\(^n\) (for some \(n \in \mathbb{N}\)), to a unique real number, \(\text{Value}^{\dagger}(P,F,t,g) \in R\), the selected value of \(F\) in \(P\) in \(t\) and \(g\). For example, the selected value of the dimension size in the category bird, \(\text{Value}^{\dagger}(\text{bird},\text{size},t,g)\), represents the ideal size for birds in \(t\) and \(g\). In birds, this selected size is relatively small.
Technically, I associate all the concepts with dimension weights and selected values but they play no role in the interpretation of one-dimensional concepts; their role is explained and demonstrated in 7.4-7.5. In fact, dimensions of multi-dimensional concepts, unlike dimensions of one-dimensional adjectives, can be said to always have a maximum or ideal point – their selected value. For example, the one-dimensional adjective tall has no maximum (the larger one's height, the taller one is), but the multi-dimensional modified noun tall person does seem to have a maximum in that exceeding a certain height, Value⁺(tall-person,tall,t) (say, 2.5 meters), only reduces one's typicality in tall-person.

I remove from the interpretation of predicates several notions that are used in other theories (reviewed in 3.2), including the notion of an ordering on the degrees, <P (Rullmann 1995; Kennedy 1999; Rotstein and Winter 2005, and many others), the unit parameter and the supremum and difference operations (Landman 2005). I also assume that degrees are numbers not tuples of a number, a unit, etc. In these respects, my theory is more economic than other theories. In ongoing chapters, I show that in spite of this economy, my theory can predict the facts for which previous theories have postulated these notions as part of predicate interpretation, as well as additional facts.

6.2.8 The super-elements of the interpretation of predicates, I, relative to c and g

Crucially, our knowledge about the degree function that is linked with a given predicate may be partial. Though in each total context each predicate P is linked (relative to an assignment g) with but one total additive degree function, f⁺(P,t,g) ∈ D, in a partial context c, we may not be sure precisely which function it is that P is linked with (relative to any given g). For instance, different possible dimension weights correspond to different possible additive degree functions for bird (cf. 7.4).

In addition, if, in a given context c and a given g, the standard, standard⁺(P,c,g), is yet unknown, we do not have direct evidence concerning whether an entity reaches the standard or not. But we can still consider all the possible ways of extending our information (specifying the standard), and if an entity d reaches the standard in all these cases (for instance, because d's degree is greater than the degree of some known member), then we can indirectly infer that d is P. If the standard is known to be a certain number in c and g, then that standard stays the same through all extensions of c (relative to g).

Similarly, our knowledge about the domain of P may be partial. For example, we may know that stories are not part of the domain of tall or healthy wrt blood pressure (they do not have height or blood pressure, so their degree in these predicates is zero in any actual context c, and, in accordance, they are excluded from the domain in c), but are animals part of the relevant domains in an actual context c? Are children or old people part of it? The answer to these questions is more context-dependent (and our specification of a standard is dependent on context specific answers to them).

Similarly, recall (cf. 5.5) that a representation of transformation values is crucial for deriving the polarity effects, and that a crucial part of the explanation has to do with the fact that knowledge about the transformation value may be partial.
The knowledge about dimension sets may be partial, too. In partial contexts, we may not know
that, say, size is a dimension of birds, or we may not be sure how important it is, etc. Thus, in
partial contexts these notions should be partially specified.

All this means that we have to represent all these notions (the value that P’s degree function
assigns to each entity, the transformation value, the value that forms P’s cutoff point, P’s domain,
and P’s dimension-sets, weights and selected values) using positive and negative super-sets per
each context c and assignment g, on a par with the representation of partial knowledge about
super-denotations. Super-sets are defined for all the parts of the interpretation of predicates, as
follows.

(25) The parts of P’s interpretation in a partial context c:
\[ \forall n \in N, \forall P \in \text{CONCEPT}^n, \forall c \in C, \forall g \in G, \forall <d_1, ..., d_n> \in D^n: \]

The positive additive-degree of \(<d_1, ..., d_n>\> in P in c and g, \(f(<d_1, ..., d_n>,P,c,g)\), is a unique real
number \(r \in \mathbb{R}\), unless this degree is not yet known in c and g (in which case \(f(<d_1, ..., d_n>,P,c,g)\) is
unknown). The negative additive-degree-set, \(\neg f(<d_1, ..., d_n>,P,c,g) \subseteq \mathbb{R}\), is the set of reals that are
known not to be \(<d_1, ..., d_n>\>’s additive-degree in P in c and g.

a. Super degrees:
\[ f(<d_1, ..., d_n>,P,c,g) = r \]  
iff  \[ \forall t \in T_c, f^+ (d_1, ..., d_n>,P,t,g) = r \]
\[ f(<d_1, ..., d_n>,P,c,g) = \text{unknown} \]  
otherwise
\[ \neg f(d,P,c,g) = \cap \{ \mathbb{R} – \{f^+(d,P,t,g)} \mid t \geq c \} \]

The positive transformation-value of P in c and g, \(\text{Tran}(P,c,g)\), is a unique real number, unless
this value is not yet known in c and g, in which case \(\text{Tran}(P,c,g)\) is unknown. P’s negative
transformation-set, \(\neg \text{Tran}(P,c,g) \subseteq \mathbb{R}\), is the set of real numbers that are already known not to be
P’s transformation constant in c and g.

b. Super transformation-value:
\[ \text{Tran}(P,c,g) = r \]  
iff  \[ \forall t \in T_c, \text{Tran}^+(P,t,g) = r \]
\[ \text{Tran}(P,c,g) = \text{unknown} \]  
otherwise
\[ \neg \text{Tran}(P,c,g) = \cap \{ \mathbb{R} – \{\text{Tran}^+(P,t,g)} \mid t \geq c \} \]

The positive standard of membership of P in c and g is a unique real number \(\text{Standard}(P,c,g) \in \mathbb{R}\), unless
the standard is not yet known in c and g, in which case \(\text{Standard}(P,c,g)\) is unknown. P is
also linked with a negative standard-set, \(\neg \text{Standard}(P,c,g) \subseteq \mathbb{R}\), the set of real numbers that are
already known not to be P’s standard in c and g.

c. Super Standard:
\[ \text{Standard}(P,c,g) = r \]  
iff  \[ \forall t \in T_c, \text{Standard}^+(P,t,g) = r \]
\[ \text{Standard}(P,c,g) = \text{unknown} \]  
otherwise
\[ \neg \text{Standard}(P,c,g) = \cap \{ \mathbb{R} – \{\text{Standard}^+(P,t,g)} \mid t \geq c \} \]
The positive domain of $P$ in $c$ and $g$, $\text{Domain}(P,c,g)$, is a set of $n$-tuples of entities in $D$ which are already known to be in $P$'s domain in $c$ and $g$. The negative domain of $P$ in $c$ and $g$, $\neg\text{Domain}(P,c,g)$, is a set of $n$-tuples of entities in $D$ which are already known not to be in $P$'s domain in $c$ and $g$.

d. Super Domain:
\[
\begin{align*}
\text{Domain}(P,c,g) &= \cap \{ \text{Domain}^+(P,t,g) \mid t \geq c \} \\
\neg\text{Domain}(P,c,g) &= \cap \{ D^p - \text{Domain}^+(P,t,g) \mid t \geq c \}
\end{align*}
\]

The positive dimension-set of $P$ in $c$ and $g$, $F(P,c,g)$, is a set of $n$-place predicates which are already known to be dimensions of $P$ in $c$ and $g$. The negative dimension-set of $P$ in $c$ and $g$, $\neg F(P,c,g)$, is a set of $n$-place predicates which are already known not to be dimensions of $P$ in $c$ and $g$.

e. Super dimension-sets:
\[
\begin{align*}
F(P,c,g) &= \cap \{ F^+(P,t,g) \mid t \geq c \} \\
\neg F(P,c,g) &= \cap \{ \text{CONCEPT}^p - F^+(P,t,g) \mid t \geq c \}
\end{align*}
\]

For each dimension $F$ of $P$, the positive weight of $F$ in $P$ in $c$ and $g$, $\text{Weight}(F,P,c,g)$, is a unique real number between zero and one, unless this weight is not yet known in $c$ and $g$. The negative weight, $\neg\text{Weight}(F,P,c,g) \subseteq [0,1]$, is the set of all the real numbers in $[0,1]$ which are already known in $c$ and $g$ not to be $F$'s weight in $P$.

\[
\forall F \in F^+(P,t,g):
\]

f. Super Weights:
\[
\begin{align*}
\text{Weight}(F,P,c,g) &= r \quad \text{iff} \quad \forall t \in T_c, \text{Weight}^+(F,P,t,g) = r \\
\text{Weight}(F,P,c,g) &= \text{unknown} \quad \text{otherwise} \\
\neg\text{Weight}(F,P,c,g) &= \cap \{ [0,1] - \{ \text{Weight}^+(F,P,t,g) \} \mid t \geq c \}
\end{align*}
\]

The positive selected-value of $F$ in $P$ in $c$ and $g$, $\text{Value}(P,F,c,g)$, is a unique real number, unless $P$'s selected $F$ value is still unknown in $c$ and $g$. The negative selected-value, $\neg\text{Value}(P,F,c,g) \subseteq \mathbb{R}$, is the set of reals which are already known in $c$ and $g$ not to be $P$'s selected $F$ value.

g. Super Values:
\[
\begin{align*}
\text{Value}(P,F,c,g) &= r \quad \text{iff} \quad \forall t \in T_c, \text{Value}^+(P,F,t,g) = r \\
\text{Value}(P,F,c,g) &= \text{unknown} \quad \text{otherwise} \\
\neg\text{Value}(P,F,c,g) &= \cap \{ \mathbb{R} - \{ \text{Value}^+(P,F,t,g) \} \mid t \geq c \}
\end{align*}
\]

For example, if in a context $c$ the positive dimension set of $chair$, $F(chair,c,g)$, (for an arbitrary $g$) consists of $\{ \text{used as a seat}; 4 \text{ legs}; \text{a back} \}$, and the negative dimension set, $\neg F(chair,c,g)$, consists of $\{ \text{white} \}$, then in $c$ and $g$, we do not yet know if anything else is typical...
of chairs or not. But \( c \) is extended by a set of total extensions, \( T_c = \{ t : T : c \leq t \} \), which represent all the possibilities compatible with \( c \) and \( g \) of specifying the complete dimension sets of \textit{chair} and all the other predicates. For instance, in each \( t \) in \( T \), \textit{wooden} is either in the positive or the negative dimension set of \textit{chair}. Things work in the same way with all the other notions.

Note that the above definitions are also \textbf{consistency} constraints. They require that the positive and negative sets in any part of interpretation of any predicate or proper name in any \( c \) and \( g \) will be disjoint.

Each intermediate context \( c \) in \( C \) is extended by a set of other contexts in which more entities are added to the (super) denotations of terms and concept names, and more is known about (super) parts of the interpretation of predicates. The lines in Figure 15 represents this type of monotonic information extension (the relation \( \leq \)). Given that the information in a context \( c_1 \) is \textit{monotonically extended} by the information in another context \( c_2 \), \( c_1 \leq c_2 \), iff \( T_{c_1} \) is a superset of \( T_{c_2} \), it follows that for any assignment \( g \), all the positive and negative denotations, domains, degree functions, standard-sets, and dimension-sets (including weights and selected-values) in \( c_1 \) and \( g \) are extended by (i.e. are subsets of) the corresponding positive and negative sets in \( c_2 \) and \( g \) (except when they are unique elements, and then they are either empty in \( c_1 \) or identical in \( c_1 \) and \( c_2 \)).

In any total context \( t \) in \( T \) (\textit{supervaluation} in van Fraassen 1969), all the extensions, and all the parts of interpretation of all the predicates relative to all the assignments are completely known, \textbf{there are no gaps} (because there is no context other then \( t \) itself in \( T_t \)).
7 A DEGREE-FUNCTION BASED TYPOLOGY OF PREDICATES

The core of my theory is a set of compatibility constraints that require of different parts of the interpretation of a predicate in a context and assignment to be compatible. These principles constrain the interpretation of predicates, although they do not necessarily say anything about the structure of phrases with predicates or about the morphological structure of predicates. These constraints are formulated for contexts of total information. Since information extension is monotonic, it follows that the information in partial contexts is compatible with these constraints too, for otherwise they could not be extended into total contexts, as required by the totality constraint. Throughout the rest of the dissertation I discuss these constraints in detail.

I begin with clarifying the role of the membership standard and the domain; continue with presenting my proposal for the difference between negative and positive predicates (though most of the consequences of this proposal are discussed in chapter 9); and then present my proposals concerning the degree functions of different predicate types, nominal predicates, conjunctive and disjunctive adjectival predicates, and distance predicates. These proposals account for the linguistic contrasts between nominal and adjectival predicates.

For convenience, throughout the rest of this dissertation, I will use the following notations:

The notation $\text{deg}^+(P,t,g)$ refers to $\text{[[deg}(P)]+_{t,g} \text{]}$, (‘the final degree function of P in t,g’)

The notation $\text{deg}(P,c,g)$ refers to $\text{[[deg}(P)]_{c,g} \text{]}$, (‘the final degree function of P in t,g’)

The notation $\text{deg}^+([[[\alpha]]+_{t,g}^P,t,g])$ refers to $\text{[[deg}(\alpha,P)]+_{t,g} \text{]}$, (‘the degree of $[[\alpha]]+_{t,g}^P$ in P relative to t,g’)

Accordingly, I will usually refer to $\text{[[deg}(x,P)]+_{t,g(x/d)} \text{]}$ as $\text{deg}^+(d,P,t,g)$ (“d’s degree in P relative to t,g”).

7.1 Categorization is standard-based

7.1.1 My proposal

Like most standard semantic and psychological gradability theories, I postulate that n place predicates are associated with positive and negative denotations such that the value of the membership standard represents the cutoff point between them. So here is my first compatibility constraint:

(1) **Categorization is standard-based:**

$$\forall x_1 \ldots x_n \in \text{TERM}, \forall n \in \text{N}, \forall P \in \text{CONCEPT}^n, \forall g \in \text{G}, \forall t \in \text{T}:$$

a. $<[[\alpha_1]]+_{t,g} \ldots [[\alpha_n]]+_{t,g} >\in [[P]]+_{t,g}$ iff $\text{deg}^+([[[\alpha_1]]+_{t,g} \ldots [[\alpha_n]]+_{t,g} >_{t,g}^P,t,g]) \geq \text{Standard}^+(P,t,g)$

b. $<[[\alpha_1]]+_{t,g} \ldots [[\alpha_n]]+_{t,g} >\in [[P]]^-_{t,g}$ iff $\text{deg}^+([[[\alpha_1]]+_{t,g} \ldots [[\alpha_n]]+_{t,g} >_{t,g}^P,t,g]) < \text{Standard}^+(P,t,g)$

For example, $[[\text{Dan}]]+_{t,g} \in [[\text{tall}]]+_{t,g} = 1$ iff $\text{deg}^+(\text{Dan},\text{tall})+_{t,g} \geq \text{Standard}^+(\text{tall},t,g)$.

The fact that constraint in (1) is formulated for total contexts in a full vagueness model is crucial. Let us see its consequences.
7.1.2 Direct consequences:
Partial and context-dependent knowledge about comparative relations

I said earlier that using a full vagueness model allows for an improved account of the interpretation of comparative statements. Given the present analysis, we can see that that is indeed the case.

Standard vagueness-based gradability theories use simplified models $M_c$ with but one partial context $c$, and they assume that degrees (or ordering relations) in predicates are determined by the proportion of total contexts in which entities are members in the positive extension (cf. 3.2.1; Kamp and Partee 1995; Kamp 1975, and many others since). These theories have the undesired consequence that, for any gradable predicate $P$, it is predicted that all denotation members in the unique partial context $c$ should be equally $P$ in $c$ (or $M_c$). In addition, these theories fail to represent the fact that degrees or ordering relations may vary through contexts, and may be partially unknown (cf. 3.2.1.2, problems 3 and 4). Conversely, the current analysis does not use the principles just mentioned. Thus, it is perfectly consistent now to consider Dan and Sam as two denotation members but not equally tall (perhaps one of them is mapped to a higher degree). In addition, since the degree function of predicate may vary between total contexts, the degrees of some of the entities may well be unknown in a partial context. Thus, the ordering between entities (and accordingly, the truth of a comparative statement like Dan is healthier than Sam) may also be unknown.

Recall also that some predicates, which are called absolute, are regarded as predicates whose standard is fixed by their semantics to be the maximum or minimum degree on the predicate’s scale. The fact that absolute predicates are gradable is problematic for standard vagueness based gradability theories (cf. 3.2.1.2, problem 5), because these predicates have a fixed standard. For example, entities are classified as full iff the degree function of ’full’ maps them to the maximal degree (they are maximally full). If the only source for vagueness that is actually represented is the value that forms the standard of membership (cf. 3.2.1), that leaves no space for vagueness in these cases. The denotations of absolute predicates end up identical in all the total contexts. And if gradability is reduced to vagueness, this leaves no space for gradability in these predicates (in these theories no two denotation members can have different degrees).

I propose that the standard of absolute predicates is not fixed once per model. While we never treat long as measuring happiness, we sometimes do treat the standards of absolute predicates as non-maximal / non-minimal (as in empty for a Hollywood film theatre, close for a door of a two month old baby, etc.; Kennedy 2001). The default standard of any absolute predicate can be represented by some context $c$ in $C$ (and the total contexts above it). In this context the standards of absolute predicates may indeed be fixed. But the context structure starts out with an empty context, $c_0$, in which no standard is known, and our knowledge about the standard may not be complete in many of the contexts between $c_0$ and $c$. In addition, the degree functions may grow gradually from $c_0$ up to $c$. Thus, the denotations may grow gradually from $c_0$ up to $c$, either directly, or based on the partial knowledge about degrees and standards. For

---

1 Entities’ degrees in $P$ represent the proportion of total contexts in which they are $P$: 
   a. $\forall d \in D: \deg(d, P, c) = \frac{m(\{t \in T_c: d \in [P]\})}{m(T_c)}$
   b. $\forall d_1, d_2 \in D: <d_1, d_2> \in [\text{is more } P], \text{ iff } \{t \in T_c: d_2 \in [P]\} \subset \{t \in T_c: d_1 \in [P]\}$

2 The referent of Dan and the referent of Sam (whomever they are) are both tall but not equally tall.
example, some context $c_1$ below $c$ may be such that it is already known that instances which are empty are not full. In $c_2$ above $c_1$ and below $c$, it may be known that entities that are almost empty are not full, and so on. Thus, even if speakers know that the standard requires being maximally full, the denotations of full may have been learnt gradually. Unlike previous theories, on the present analysis the partial contexts under the actual context $c$ are represented (if a speaker learns what the default standard is right away, this means that the speaker 'jumps' to $c$, but contexts that are more partial than $c$ are still part of the structure). Finally, on this analysis, degrees in predicates are not determined by the proportion of total contexts in which entities are members in the positive extension. Thus, even in contexts in which the standard has already been fixed and many facts about membership have been inferred, two members may be mapped to different degrees. Gradability in the current analysis reflects the order in which entities (elements of $D$) are added to the denotations, and the denotation can be learnt gradually even in absolute predicates or nouns (the learning constraint is given in chapter 8). In sum, the fact that absolute predicates (or any predicates whose denotations are [relatively] known) are gradable is no longer a problem. Since a standard-based categorization rule is employed, if, for instance, a certain degree of full-ness is considered full, then any bigger degree of full-ness is also so considered. And if a certain degree of open-ness is not considered open, then no smaller degree of open-ness is so considered. So the connections between gradability and (partial knowledge regarding) membership in the denotation are adequately described, despite (or in virtue of) the removal of the rules that are employed by previous vagueness-based gradability theories.

7.2 Membership standards are always domain-based

7.2.1 My proposal

Like most standard semantic gradability theories, I postulate that the domain of $P$ in $t$, $\text{Domain}^+(P,t,g)$, helps determine the cutoff point between the negative and positive denotations. We have seen in 3.2.1.2 that there are three important predicate types (with maximum-, minimum- or relative-standards). Previous theories (Winter and Rotstein 2005; Kennedy and Mcnally 2004) take maximum- and minimum-standards to be the maximum and minimum degrees, respectively, on bound predicate-scales. With that, these theories derive the fact that for-phrases are usually not good with absolute (maximum- and minimum-standard) predicates, because the domain plays no role in determining the standard. But these theories do not explain how the domain affects the standards in statements like *empty for a Hollywood film theatre* (Kennedy 2001). On the present proposal, the standards are in principle dependent on a domain. My proposal is otherwise similar to previous theories about membership standards. I propose that the domain determines the cutoff point in one of the following three ways.

\[
\forall g \in G, \forall t \in T, \forall n \in N, \forall P \in \text{CONCEPT}^n:\n\]

(2) **The method for determining a maximum standard for a domain $X$:**

a. If $P$ is a maximum standard predicate:

\[
\text{Standard}^+(P,g,t) = \text{Max}(P,t,g, \text{Domain}^+(P,t,g))
\]

b. For any entity set $X \subseteq D^n$, $\text{Max}(P,g,t,X)$ is the biggest $P$ degree in $X$:

\[
\text{Max}(P,t,g,X) = r_1 \quad \text{iff} \quad r_1 \in \{\deg^+(d,P,t,g) \mid d \in X\} \quad \text{and} \quad \forall r_2 \in \{\deg^+(d,P,t,g) \mid d \in X\}: \quad r_2 \leq r_1
\]
(3) The method for determining a minimum standard for a domain X:
c. If P is a minimum standard predicate:
   \[ \text{Standard}^+(P,t,g) = \text{Min}(P,t,g, \text{Domain}^+(P,t,g)) \]
d. For any entity set X \( \subseteq D^n \), \( \text{Min}(P,t,g,X) \) is the smallest P degree in X:
   \[ \text{Min}(P,t,g,X) = r_2 \text{ iff } r_2 \in \{ \deg^+(d,P,t,g) \mid d \in X \} \text{ and: } \forall r_2 \in \{ \deg^+(d,P,t,g) \mid d \in X \}: r_2 \leq r_1 \]

(4) The method for determining a relative standard for a domain X:
e. If P is a relative-standard predicate:
   \[ \text{Standard}^+(P,t,g) = \text{Mean}(P,t,g, \text{Domain}^+(P,t,g)) \]
f. For any entity-set X \( \subseteq D^n \), \( \text{Mean}(P,t,g,X) \) is the mean P degree in X:
   \[ \text{Mean}(P,t,g,X) = \left( \sum_{d \in X} \deg^+(d,P,t,g) \right) / |X| \]

I propose that adjectival domains are usually rather under-specified in partial contexts (there is a large variance between the domains of a given adjective in different total extensions of actual contexts). Only entities with no P-hood at all are semantically excluded (in the sense that in any extension t of an \textit{actual} context c they are not part of \text{Domain}^+(P,t,g)).

Given the compatibility constraint in (2)-(4), my analysis of \textit{for}-phrases is given in (5b) below. By the definition of super-truth, the sentence \textit{Dan is tall} is supertrue in c (which means that it is considered true in c) iff it’s true in every t above c, which means true relative to any possible domain which is compatible with what is known in c (5a). A \textit{for}-phrase \textit{for-}Q tells you that if you jump to an extended context t in which the domain is known to be \textit{[[Q]]}_t,g the statement will turn out true. For example, the statement \textit{Dan is tall for an eight year old} is true in a context c iff in any total context above c where the domain is the set of eight year olds, Dan’s degree is above the mean degree in the domain (5b). The standard of \textit{tall} given that the domain is restricted by \textit{for an eight year old} is the mean height in \textit{[[eight year old]]}_t,g. If we do not know the complete set of eight year olds and their heights, we do not know the exact standard value.

(5) \textit{for}-phrases:
a. \textit{[[Dan is tall]]}_c,g = 1 \text{ iff } \forall t \in T, \ t \geq c, \quad [[\text{Dan is tall}]]^+_t,g = 1
b. \text{[[Dan is tall for an eight year old]]}_c,g = 1 \text{ iff }
   \forall t \in T, \ t \geq c, \text{ s.t. } \quad \text{Domain}^+(\text{tall},t,g) = [[\text{eight year old}]]^+_t,g;
   \quad [[\text{Dan is tall}]]^+_t,g = 1 \text{ iff: }
   \forall t \in T, \ t \geq c, \text{ s.t. } \quad \text{Domain}^+(\text{tall},t,g) = [[\text{eight year old}]]^+_t,g;
   \quad [[\deg(\text{Dan,tall})]]^+_t,g \geq \text{Mean}(\text{tall},t,g, [[\text{eight year old}]]^+_t,g) =
   \quad = \left( \sum_{d \in [[\text{eight year old}]]^+_t,g} \deg^+(d,P,t,g) \right) / |[[\text{eight year old}]]^+_t,g |

I also propose that statements with \textit{for}-phrases presuppose that the domain can be restricted by the predicate in the \textit{for}-phrase. For example, the statement \textit{Dan is tall for an eight year old} has the presupposition:

c. \exists t \in T, \ t \geq c, \text{ s.t. } \quad \text{Domain}^+(\text{tall},t,g) = [[\text{eight year old}]]^+_t,g

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Sometimes we know in c a lot about the domain, as in a situation of having to sort into sizes a given set of 6 glasses. It is given in c that the domain for big is precisely these 6 glasses (i.e., in all total contexts above c, the domain is six individuals in D that have all the properties that in c we know these 6 glasses to have). The domain in c is presumably empty, since we do not know which individuals d in D these glasses are, and yet we do know something about all the total extensions. For instance, in c we may know the volume or height of these glasses (we may be seeing four glasses of 300 milliliter and two of 400 milliliter), so we may be able to say what the mean volume or height of a glass in the domain is. So if, within a context, the domain is set to a very small set of entities (say the two glasses on the table), one can immediately call the bigger glass (or anything bigger) big and the smaller one (or anything smaller) not big (for that domain). Otherwise, the standard of relative predicates is often unknown, as the complete set of entities that forms the domain and their degrees is unknown.

Similarly, in Dan is tall for an eight year old we know very little in c about who the actual individuals are in the domain of tall, but we still know quite a bit about the domain, because we know it is all the individuals with the property an 8 year old (which may be given either explicitly by a for-phrase, or implicitly). In all the total extensions t of c, where the domain is the set of eight year olds in that t (whoever they might be), Dan is tall is true. Having seen many (heights of) eight year olds, we may be able to say that Sam is taller than (what we estimate to be) the mean height for an eight year old.

If I do not know Dan's age, I can say that Dan is tall for an eight year old, but not for a nine year old, but it is inappropriate to set the domain to the set of eight year olds if you are about to rank only adult basketball players. This would leave no relevant entity (basketball player) in the negative denotation. Hence, it would render tall completely uninformative with respect to the relevant entities (it will be impossible to refer to the set of taller players as the tall players and to the set of shorter players as the ones which are not-tall, because all of them will be tall). So if a context c is incompatible with a for-phrase (another domain is set already), the presupposition is not satisfied, and the statement is regarded as infelicitous.

On this proposal, even if you have used a for-phrase like for a child it does not mean that in the context c representing your knowledge the domain is restricted to [child]^{c,g}, only that c can be extended in this way. So you hardly ever add information about the domain, except locally. Why don't I want to say that the for-phrase simply sets the domain in (all the total extensions t of) c to children? Because in c we may also want to evaluate Dan is short for an adult, so we cannot fix the domain of tall once and for all.

7.2.2 Immediate consequences: The standards of absolute predicates are domain dependent

On the present proposal maximum- and minimum-standards too are dependent on the domain. This explains how for-phrases might affect the standard in absolute predicates. For example, the standard of empty given that the domain is restricted by for a Hollywood film theatre is the maximal degree of empty in [[Hollywood film theatre]]^{t,g}, (it is known that no Hollywood film theatre is emptier). A theatre with several full lines normally does not count as empty, but as Hollywood film theatres are never completely empty (usually they are completely full) such a theatre may count as empty for that special domain.

d. [[The room is empty for a Hollywood film theatre]]_{c,g} = 1 \text{ iff } \\
\forall t \in T, t \geq c, \text{s.t. } \text{Domain}^t(\text{empty},t,g) = [[\text{Hollywood film theatre}}]]^{t,g}$
[[the room is empty]]_{t,g}^+ = 1 \iff \forall t \in T, t \geq c, \text{s.t.} \text{Domain}^+ (\text{empty}, t, g) = [[\text{Hollywood film theatre}]]_{t,g}^+;
[[\deg(\text{the room, empty})]]_{t,g}^+ \geq \text{Max}(\text{empty}, t, g, [[\text{Hollywood film theatre}]]_{t,g}) \iff \forall t \in T, t \geq c, \text{s.t.} \text{Domain}^+ (\text{empty}, t, g) = [[\text{Hollywood film theatre}]]_{t,g}^+;

Similarly, the standard of open given that the domain is restricted by for a door of a boy his age is the minimal degree of open in [[door of a boy his age]]_{t,g}^+. A door which is only almost closed normally does not count as open, but if we think of doors of boys at his age as systematically not completely closed (if, for instance, it is a newborn), a door which is almost closed may not count as open for that special domain. Conversely, if we think of boys his age as caring much for their privacy (if, for instance, it is a teenager) such a door may count as wide-open for that special domain.

e. [[The door is open for a door of a boy his age]]_{t,g} = 1 \iff \forall t \in T, t \geq c, \text{s.t.} \text{Domain}^+ (\text{open}, t, g) = [[\text{door of a boy his age}]]_{t,g}^+:
[[\deg(\text{the door, open})]]_{t,g}^+ \geq \text{Min}(\text{open}, t, g, [[\text{door of a boy his age}]]_{t,g}) \iff \forall t \in T, t \geq c, \text{s.t.} \text{Domain}^+ (\text{open}, t, g) = [[\text{door of a boy his age}]]_{t,g}^+:
\exists r \in \{\deg^+(d, \text{open}, g, t) | d \in [[\text{door of a boy his age}]]_{t,g}^+\};
[[\deg(\text{the door, open})]]_{t,g}^+ \geq r

Why are for-phrases often bad with absolute predicates (as in # this door is open for a supermarket door or #this glass is full for a big glass)? If the domain is set to the set of entities in D that actually exist, the standards of absolute predicates are the minimum and maximum degrees that their function can assign (as all the possible degrees are exemplified by actual entities in large domains). We can view this as the default case, in the lack of a for-phrase. Normally, maximum and minimum standard predicates do not accept for-phrases because, e.g., we know that there are (actual) super-market doors that exemplify the smallest non-zero degree of open (almost completely closed doors), and (actual) big glasses that exemplify the biggest degree of full (completely full glasses), so the mention of a for-phrase does not affect the meaning. It does not shift the standard to a less obvious value compared to the value given a less restricted domain, or compared to the domain in this door is open (for a door) (a domain that is fixed if the domain restriction is retrieved from the noun phrase).

The dependency of the standard on the domain explains why the use of a certain method for determining the standard varies in different uses of a predicate (Kennedy and Mcnally 2004). When we have no knowledge about the maximum or minimum in a domain (and no easy way to gain this knowledge), the felicity of an absolute predicate reduces, or else it is interpreted as relative. For example, a glass of 300 milliliter may be known to be full (full for a glass that may contain 300 milliliter liquid), but we can hardly ever tell that an ocean is full, because we do not know of specific upper bounds for the number of milliliters oceans might contain (oceans are not containers such that you can tell their maximal volume). In such a case, we cannot
determine the cutoff, and hence the denotations, for this domain, and the use of full in the ocean is full is odd. Predicates like dry are maximum-standard predicates when applied to tables or other solid surfaces, but they are regarded as relative when applied to skins of living people (Kennedy and McNally 2004), because speakers do not have the means of telling what the maximum amount of moist in the domain of skins of living people is. But given a big enough sample, speakers can suppose that the mean 'moist level' in their sample is representative of the population and they can use this mean as a cutoff point for dry when applied to skins (there is no such conventional relative interpretation for full in the case of an ocean, perhaps because most speakers usually do not know the extents to which an ocean changes its volume).

7.2.3 The domain affects the denotation, not the degree function

On the present proposal, the domain does not affect the degree function. The degree function is total on $D^n$ (for any n place predicate). It maps everything in $D^n$ to a degree in R. For example, entities like prime numbers, which have no height, are mapped to 0 by $\text{deg}(\text{tall})$ in any actual context $c$. Thus, the degree function is the same for any setting of domain and standard. This proposal predicts that an explicit mention of for-phrases (setting of the domain value) is superfluous, and therefore ungrammatical, in comparative statements and statements with numerical degree modifiers, which only make use of the degree function that is associated with the predicate, and need not use any information about its standard.

f. # Dan is taller than Sam for an eight year old

g. # Dan is taller than one meter for an eight year old

In contrast, the cutoff point (between the infinite sets of entities that form the complete positive and negative denotations of a predicate) varies with the domain. Only by chance, the denotation may remain the same for different domains. For-phrases can also be accommodated. This assumption is supported by the variability of the standard in adjectives like tall. It is well known that the interpretation of gradable predicates relies heavily on context (Kamp and Partee 1995). A very small snowman can be regarded as huge for a snowman built by a four year old. On the present proposal, when a for-phrase like for a snowman built by a four year old is accommodated, the standard is locally set to the mean in [[snowmen built by four year olds]]$_{t,g}^1$. This assumption explains the fact that intersection-inferences fail in modified nouns where the modifier is a gradable adjective. For example, given the sentence Dan is a mature child, we will not infer that Dan is mature, because the contextual denotation of mature in the former sentence, but not in the latter sentence, is set to entities whose degree is above the mean in [[children]]$_{t,g}^1$. Dan may reach the standard relative to this restricted domain, but not reach the standard relative to a less-restricted domain. In the lack of contextual cues regarding the domain (or an explicit mention of a for-phrase), a statement must be true for any possible domain. But, since, in the case of tall nothing we actually speak about will end up tall for any possible domain, the domain is usually locally restricted (e.g. to people).

Note that in some predicates the standard is set by conventions that are independent of grammar. For example, the standard for healthy wrt blood pressure is set by doctors, and other speakers accept their norms simply because they are experts (the domain is set in accordance with the doctors' standards).
7.2.4 The standards of nominal concepts

On the present proposal nominal concepts have the same parts of interpretation as adjectives, including a domain and a standard, though nouns like *furniture* or *bird* do not combine with *for*-phrases. Why do I set the theory in this way?

First, some nouns (nominalizations like *success* or *health*) can be modified by *for*-phrases (as in *success for a student conference* or *health for an eighty year old*), and the role of the *for* phrase seems to be similar to its role in the corresponding adjectives (for instance, in the given example, the standard for *success* is set to be the mean degree of *success* in the set of success instantiations in student conferences).

Second, even in nouns like *bird*, standard-based categorization rules are useful in predicting membership in nominal concepts (Hampton 1998). In addition, when experimental stimuli include non-birds – say, bats – this improves the ratings of typicality and categorization-likelihood of borderlines or bad examples of birds, like ostriches (Murphy 2002). The assumption that the nominal standards depend on a domain directly explains the fact that borderline cases exist, whose categorization is context dependent (cf. 2.2.1).

The nominal standard (for nouns like *bird*, not nominalizations of adjectives) is relative (it is certainly not a maximum or minimum standard). However, intuitively, in nouns like *bird*, the domain is significantly less variable than that of relative adjectives. Experiments show that categorization in nominal concepts like *bird* is usually affected by the (diss-)similarity of entities to other animals or animal types (Rosch and Mervis 1975; cf. 2.2 and 4.1-4.2). This can be represented by postulating that in all the context structures of competent speakers the domain is a subset of the animal domain. Everything outside this set is regarded as having zero (or just too little) of whatever it is quantities of which the degree function of *bird* represents. Asserting that plants or non-animate things are *not birds* is highly superfluous (and hence may sound odd), and I propose that this is always the case for things that are not part of the domain (like stories for *tall*). Semantically, on the current proposal, such statements are true (stories are not tall, prime numbers are not birds, etc.), but, not surprisingly, we don’t usually say these things. Thus, nouns like *bird* and their dimensions seem to be interpreted relative to a highly restricted domain. Similarly, asserting that abstract or animate things (dogs, cats and stories) are *not tables* (or *pieces of furniture*) is superfluous and funny.

Nominal concepts like *bird* or *table* cannot combine with *for*-phrases, even when they apply to an irregular domain. In that, they are different from absolute predicates. For instance, in chocolate shops, we naturally refer to chocolate animals as *birds* (as in *mummy, I want the bird*), but sentences like *for a chocolate animal, this is a bird* are highly unnatural. This may result from a presupposition failure. The domain is already set to the set of animals, so accommodating an inanimate domain is difficult. For that reason, given an actual context, a shift in interpretation (a change in the domain) is felt to be a shift between different senses (literal and metaphoric, so to speak), not a natural extension of the partial contextual interpretation. At any rate, it may be rather easy to create new senses. For example, within a chocolate shop, we can speak about birds as opposed to non-birds, where the domain is fixed to pieces of chocolate in the shop. Some of them definitely score better in bird-dimensions (they are similar to birds in shape – have chocolate wings, chocolate beaks, etc.), than others who are similar to other animals or things. We can use the noun *bird* in discussing stuffed animals, glasses, paintings, etc., when these have different quantities of F-hood in them, for some bird-dimension F.
Let us repeat. Why is it so easy to say *mummy I want the bird* but difficult to use a *for*-phrase? What is the difference? If in all the actual contexts $c$, nominal domains, unlike adjectival domains, are always highly restricted, then *for*-phrases (which presuppose that there is variance in the domain across total extensions of $c$, for otherwise explicitly restricting the domain makes no sense) are naturally bad with nominal concepts. If the domain is highly restricted, so is the standard, and the denotation. This explains the fact that in nominal concepts we find much fewer borderline cases compared to adjectives. In metaphoric senses (say, when we jump to some counterfactual context where the dimension-set of and denotation of *bird* is unusual, when we are going from talking about ‘real’ things in the actual world to talking about the ‘as if’ world of chocolate figures), the nominal domain is highly restricted too, so we cannot use a *for*-phrase for that new sense. I do not know why that is so, but I suspect that this fact, like the former fact (the fact that nominal domains and denotations are relatively highly restricted in literal senses), has to do with the fact nominal degree functions are mean functions. Let me explain.

Why can't we use *for*-phrases to restrict the literal sense of *bird* to certain types of birds, or to other types of animals (to more conservative potential local domains)? The degrees of most animals that are not birds (with the exception of bats) are so low that the mean degree in the *bird* domain can hardly be affected by adding or excluding non-birds. This may explain why saying that something is a *bird for a desert animal / for a water animal* is bad. In addition, the degrees of non-birds may be so close to one another\(^3\) that saying that something is a *bird for a reptile / mammal / insect* may be rather unhelpful for distinguishing between the presumed birds and non birds. Different sub-categories (say, species like reptiles, birds, mammals, etc.) are known to be mutually exclusive, which further reduces the acceptability of setting the standard for them. Altogether, the result is that the domain, and hence denotation, of nouns like *bird* is steady once learnt, except for few borderline cases. So in nouns (unlike adjectives), the domain and other interpretation-parts (for instance, the dimensions) cannot be realized overtly (with *for*-phrases or wrt-phrases), but they do exist and affect meaning (as psychological findings testify).

An alternative way of contextually restricting nominal denotations is by accommodating (implicit) modifiers. These modifiers may be free variables, whose values are set by the assignment function, like $X$ in $\lambda x_1. X(x_1) \land \text{bird}(x_1)$. Such variables are used in von Fintel's (1994) analysis of quantifier-domain restrictions (for restricting nominal arguments of quantifiers). But these modifiers may also be normal predicates, and they may be accommodated also in the absence of a quantifier. For instance, when discussing a farm we can say that *the bird walked across the barn-yard*. Roth and Shoben (1983) show that in such contexts people judge chicken to be better examples of birds than robins (which are normally judged to be better examples), and they also show that in such contexts categorization time is faster for chicken than for robin. I propose that in such circumstances people accommodate a constituent. They judge goodness of example in *farm bird* or *barn bird*, not in *bird* alone. Accommodation may occur often in large categories like negated nominal categories (for instance, *not a bird or non-bird*), which are usually grasped as restricted, for instance, to *animals*, but sometimes even to narrower sets. Complex predicates like *bird that lives on the farm* (unlike variables $X$) are associated with dimension-sets. Sassoon (2002) has shown that

---

\(^3\) Recall that similarity to a prototype (to the selected values on the dimensions) is related to distance from the prototype by an inversed exponential function (cf. 4.1), i.e. small distances reduce similarity considerably, but big distances that differ from one another significantly turn into very similar similarity degrees.
these dimensions are useful in accounting for facts concerning domain restrictions in statements with quantifying expressions like any, every and generic a.

Finally, intersection failures are likely to occur when implicit constituents, whether predicates or variables, are accommodated. Intersection failures ("non-intersective effects") occur in modified nouns even where the modifier is a noun. Experiments show that, for instance, blackboards may be classified as school furniture, but not as furniture. The intersection rule appears to be violated as some elements of [(school furniture)] appear not to be elements of the intersection of [(in the) school] and [(furniture)]. However, if we assume that nouns, just like gradable adjectives, have restricted domains that in some nouns may vary with context, and that implicit constituents may be accommodated into any noun phrase even in the absence of quantifiers, these non-intersective effects no more count as counterexamples to the intersection rule, as explained below.

In fact, due to certain norms of language use, nominal concepts tend to have restricted interpretations. In naming atypical examples of basic predicates (predicates like fruit, bird, book, etc.), the use of a subordinate predicate (olive, ostrich, paperback novel, etc.) is preferred (Cruse 1977), probably because the relevant examples are typical of the subordinate predicate, so it's use facilitates their activation (we have seen in 2.2.2.1 that typicality judgments are coupled with ease of retrieval from memory). As a consequence, the use of basic predicates is, by default, restricted to (relatively) typical instances (if you say bird, by default, your listeners suppose that you do not intend to speak about ostriches), and for this reason too failures of intersection inferences are more than expected. For example, the interpretation of furniture is, by default, restricted to things in the home. Blackboards are usually not found in the home. Hence, on the restricted interpretation, they may not be regarded as furniture. But modification by (in the) school (as in school furniture or furniture which are found in the school) triggers widening of the interpretation, so as to include also things which are not found in the home but are nonetheless furniture. On this wide interpretation, blackboards are pieces of furniture. This type of widening occurs systematically in all the "non-intersective" cases. For example, the denotation of sports tends to narrow down to physical activities, but in sports which are games, it widens, so as to include also non-physical activities (like chess). The denotation of dwellings tends to narrow down to things made of stone or concrete, but in dwellings which are not buildings, it widens so as to include also things which are made of other materials (like tree houses), etc. Domain restrictions like in the home may have the status of conversational implicatures (by default, you assume that they restrict the domain, or by default you accommodate them into sentences with the corresponding nominal concepts, and you cancel this assumption on convenience). Speakers may systematically switch between default restricted-contexts (that represent pragmatic enrichments) by restricting the domain in the interpretation of the nominal concepts, and non-default contexts (that represent only semantic knowledge with no pragmatic enrichments).

In sum, associating nominal phrases with a domain and standard in each context may help representing the fact that standard-based categorization rules are useful in predicting membership in nominal concepts (Hampton 1998) and the variability that is allowed by the concept (that is not felt as a shift of sense – for instance, the fact that some borderline cases exist whose categorization is context dependent). The assumption that the nominal (positive and negative) domains, unlike the adjectival domains, are relatively fixed in actual contexts, explains why nominal concepts can apply to entities in irregular domains, though these uses are felt as related to a different sense.
Given the wide range of applications of the intersection rule, and given that we know that interpretation shifts and accommodation of constituents occur systematically when they are appropriately triggered (Kadmon 2001: 17-21; Murphy 2002: 404-422), it seems best to assume that they occur in nouns too, allowing us to retain the intersection rule – instead of rejecting the intersection rule altogether, as is usually done in the psychological literature.

7.3 Negative versus positive adjectives

7.3.1 My proposal

The present analysis postulates that, in addition to associating predicates with their additive degree function, \( f^+(P,t,g) \), the model also associates each predicate \( P \), in any \( t \) and \( g \), with a transformation value (a real number), \( \text{Tran}^+(P,t,g) \). It is \( f^+(P,t,g) \) and \( \text{Tran}^+(P,t,g) \) together that determine \( \text{deg}^+(P,t,g) \), i.e., the semantic value of the term \( \text{deg}(P) \), called the final degree function of \( P \) in \( t,g \). (Recall that \( \text{deg}^+(P,t,g) \) is an alternative notation equivalent to \( [[\text{deg}(P)]]_{t,g}^+ \).) The semantic values that degree-terms (expressions of the form \( \text{deg}(\alpha,P) \)) denote, \( \text{deg}^+([[[\alpha]]]_{t,g}^+,t,g) \), is of course determined in turn by \( \text{deg}^+(P,t,g) \), via the semantic rule: \( [[\text{deg}(\alpha,P)]]_{t,g}^+ = [[\text{deg}(P)]]_{t,g}^+([[[\alpha]]]_{t,g}^+,t,g) \).

I propose that CONCEPT is divided into two sub-categories positive and negative predicates, and that positive and negative predicates differ wrt the way their degree function is defined. I propose that the interpretation of degree terms is given by the following rules.

(6) **Positive and negative (reversed) degree functions:**

\[ \forall g \in G, \forall t \in T, \forall n \in N, \forall P \in \text{CONCEPT}^n, \forall \alpha \in \text{TERM}: \]

a. If \( P \) is a **positive** predicate: \( [[\text{deg}(\alpha,P)]]_{t,g}^+ = f^+([[[\alpha]]]_{t,g}^+,P,t,g) - \text{Tran}^+(P,t,g) \)

b. If \( P \) is a **negative** predicate: \( [[\text{deg}(\alpha,P)]]_{t,g}^+ = \text{Tran}^+(P,t,g) - f^+([[[\alpha]]]_{t,g}^+,P,t,g) \)

7.3.2 Immediate consequences:

**Reversed orderings in negative and positive predicates that have the same additive degree function**

It follows from this proposal that for a positive predicate \( P \), its additive degree function determines the ordering between entities as to how good a \( P \) each entity is – for any \( t \) and \( g \), the ordering of entities in \( P \) imposed by the final degree function \( \text{deg}^+(P,t,g) \) is identical to the ordering produced by \( f^+(P,t,g) \).\(^4\) And it also follows that for the negation \( \neg P \) of a positive predicate \( P \) and for its negative antonyms \( P_{\text{ant}} \), the ordering of entities in the predicate is the reverse of that of \( P \). (I assume that predicate pairs are considered antonyms iff they have the same additive degree function – that is, iff for any \( t, g \), \( f^+(P,t,g) = f^+(P_{\text{ant}},t,g) \). I defend my assumption in ongoing sections where I discuss apparent counterexamples.) For instance, in any \( t \) and \( g \), the two predicates long and short are associated with the same additive degree function (in this case, some length function, \( f_{\text{length}} \in D_l \)), and their ordering, as determined by their final degree functions, is reversed.

\(^4\) The additive degree function also determines the entities’ degrees in \( P \), up to a shift by a numerical transformation constant.
(7) Reversed-ordering:
   a. Prediction:
      \[
      \forall P, P_{\text{ant}} \in \text{CONCEPT}, \quad \text{s.t. } P \text{ is positive and } P_{\text{ant}} \text{ is negative: }
      \forall t \in T, \forall g \in G: \quad f^*(P,t,g) = f^*(P_{\text{ant}},t,g) \quad \text{iff}
      \]
      \[
      (\forall \alpha, \beta \in \text{TERM}: \quad [\deg(\alpha,P)]^+_tg > [\deg(\beta,P)]^+_tg \quad \text{iff}
      \]
      \[
      [\deg(\alpha,P_{\text{ant}})]^+_tg < [\deg(\beta,P_{\text{ant}})]^+_tg
      \]
   b. Proof:
      Assumption: \[ f^*(P,t,g) = f^*(P_{\text{ant}},t,g) \]
      Proof:
      \[
      \forall \alpha, \beta \in \text{TERM}: \quad [\deg(\alpha,P)]^+_tg > [\deg(\beta,P)]^+_tg \quad \text{iff}
      \]
      \[
      f^*([\alpha]_{tg}, P,t,g) - \text{Tran}^*(P,g,t) > 0
      \]
      \[
      f^*([\beta]_{tg}, P,t,g) - \text{Tran}^*(P,g,t)
      \]
      \[
      f^*([\alpha]_{tg}, P,t,g) > f^*([\beta]_{tg}, P,t,g) \quad \text{iff}
      \]
      \[
      - f^*([\alpha]_{tg}, P,t,g) < - f^*([\beta]_{tg}, P,t,g) \quad \text{iff}
      \]
      \[
      \text{Tran}^*(P_{\text{ant}},g,t) - f^*([\alpha]_{tg}, P_{\text{ant}},t,g) < 0
      \]
      \[
      \text{Tran}^*(P_{\text{ant}},g,t) - f^*([\beta]_{tg}, P_{\text{ant}},t,g)
      \]
      \[
      [\deg(\alpha,P_{\text{ant}})]^+_tg < [\deg(\beta,P_{\text{ant}})]^+_tg
      \]

Winter (2001) observes that usually total predicates are positive (normal, clean, closed) and partial ones are negative (abnormal, dirty, open), but he also notes that exceptions exist, like the partial positive predicates necessary and probable, and the corresponding total negative ones, unnecessary and improbable. On my proposal, it is the degree function, not the standard of the predicate, that determines whether the predicate is conceived of as negative or positive, so the exceptions (e.g., necessary, probable, unnecessary and improbable) are not problematic (but the general tendency is not predicted by my proposal).

7.3.3 The transformation value

Recall that in any t and g, long and short are associated with the same additive function, \( f_{\text{length}} \in D_l \) (any function that is additive wrt length, i.e. whose values represent the quantity of length in any individual). The function \( f_{\text{length}} \) completely determines the individuals’ degrees in long, in the sense that for any individual, its degree in long equals its degree in \( f_{\text{length}} \). We can tell that the transformation value is zero, because entities with zero length are regarded as zero long (we can say that they are 0 meters long, 0 centimeters long, 0 inches long, etc., and for any number n other than zero, we can say that they are not n meters long, n inches long, etc.). The situation is different with short.

For the predicate short, the values are reversed (in the sense of (7a) above)\(^5\). We can tell that the reversing function (\( \text{deg}^*(\text{short},t,g) \), i.e., that function in \( R^D \) which reverses in the sense of (7a) \( \text{deg}^*(\text{long},t,g) \)) is linear (that it is based on a minus operation and not on another operation, such as a division operation). The interpretation rule for negative predicates cannot be something like: \( [\deg(x,P)]^+_tg = 1 / f^*([x]_{tg}, P,t,g) \), because we know that the differences between degrees are preserved. For example, \( d_1 \) is two meters longer than \( d_2 \) iff \( d_2 \) is two meters long.

\(^5\) In 8.7.1, I show that for negated predicates like not-tall, the fact that the values are reversed (compared to tall) actually follows from the learning principle, a principle that I present and defend throughout chapter 8.

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shorter. The interpretation rule proposed here preserves the differences between degrees while a division rule does not (the difference between 4 and 2 is 2, the difference between (Tran – 2) and (Tran – 4) is still 2, but the difference between 1/2 and 1/4 is not 2).

Yet, among all the linear reversing functions we cannot tell which one is the function that is linked with short. We do not even know the degree in short of entities with zero length. We cannot tell whether they are mapped to zero (i.e. whether the transformation value is zero) or to infinity (an infinite transformation value, as in von Stechow's 1984 theory, and Kennedy's 1991 theory), or to anything in between. Thus, for no entity d and number n can we say that d is n meters short, n inches short, etc. This explains why numerical degree predicates like n meters are odd with negative predicates.

In chapter 9, I systematically show that we can derive from this lack of knowledge concerning the transformation value many differences between negative and positive predicates (the polarity effects). But before showing this, I need to present my proposals concerning the characteristics of the additive degree functions of different predicate types.

7.4 Nominal concepts: A prototype theory

On the present proposal, dimensions are normal predicates (of the same arity as the predicate they are dimensions of), whose final degree functions (and therefore the degrees that they assign to entities) constrain the final degree functions of the predicates they are dimensions of. The type of constraint depends on the type of predicate.

Intuitively, predicates are multi-dimensional iff they may have in their dimension set several different dimensions (i.e. predicates whose (final) degree functions are not identical). For any dimension Q of a predicate P we can find another dimension Q' with an identical degree function (for example for red we have not-not-red). Different speakers may use different dimension sets, but these dimension sets will produce the same judgments as long as the dimensions in them have identical degree functions. Thus, a predicate is one-dimensional in a context t relative to an assignment g iff all the predicates in its dimension set in t and g, F*(P,t,g), have the same degree function. A predicate is one dimensional in a model M iff it is one-dimensional in any t and g.

(8) An n-place predicate P is one-dimensional iff
no two dimensions in P's dimension set ever have different (final) degree functions:
∀t∈T, ∀g∈G, F*(P,t,g) ≠ ∅ and ∀P,Q∈F*(P,t,g), [[deg(P)]] t,g = [[deg(Q)]] t,g
Otherwise, P is multi-dimensional.

Nominal concepts like bird are inherently multi-dimensional. They are known to be associated with many dimensions. But concepts like long are one-dimensional both in its temporal interpretation (where it is known to measure temporal duration, and nothing but that) and in its spatial interpretation (where it is known to measure spatial length and nothing but that). In chapter 9, I propose a stricter condition for one-dimensionality, such that some predicates may not have two dimensions with different degree functions, but still be multi-dimensional (if, say, their unique dimension is multi-dimensional).

In the case of one-dimensional predicates P, associating them with a single dimension Q is almost completely redundant. f*(P,t,g) is identical to f*(Q,t,g) in any t and g (cf. 7.5). That is
not the case with multi-dimensional predicates. We have to state how their degree functions are constrained by the degree functions of their dimensions.

### 7.4.1 My proposal

For nominal concepts, I propose to adopt the prototype theory (cf. 4.1), where a prototype is a set of dimensions, including weights and selected values for the dimensions (Murphy 2002). Thus, I propose that the degree of an entity \( d \) in a nominal concept \( P \) is inversely related to \( d \)'s mean distance from \( P \), namely to the mean of the distances between \( d \)'s values and \( P \)'s selected values on \( P \)'s dimensions. Let me explain this with a detailed example.

Consider the degree of Tweety in \( bird \) in a context \( t \) and an assignment \( g \). This degree depends on the extent to which Tweety matches the selected values (ideal values) of \( bird \) in the dimensions (predicates) in the dimensions set \( F^+(bird,t,g) \), including dimensions like \( flying, singing, has feathers, small, \) etc.

The selected values of \( bird \) in its various dimensions in \( t \) and \( g \) are the real numbers \( \text{Value}^+(bird,small,t,g) \), \( \text{Value}^+(bird,singing,t,g) \), \( \text{Value}^+(bird,flying,t,g) \), etc. The distance between Tweety's degree in \( small \) and the selected value of \( bird \) in the dimension \( small \) (such that \( f^+(small,t,g) \) is a function that represents size in any actual \( t \) and \( g \)) is this number:

\[
| f^+([Tweety]_{t,g},small,t,g) – \text{Value}^+(bird, small,t,g) |
\]

Each dimension is weighed (the weight represents its importance in determining similarity to the \( bird \) prototype). The weights of all the dimensions sum up to 1 (this is required by the definition of mean, cf. 4.1). So the weighted distance of Tweety from the \( bird \) prototype wrt \( small \) is the following number:

\[
\text{Weight}^+( small,bird,t,g) \times | f^+([Tweety]_{t,g},small,t,g) – \text{Value}^+(bird, small,t,g) |
\]

Finally, the mean distance between Tweety’s values and the selected values in all the dimensions is the following weighted mean:

\[
\text{Weight}^+( small,bird,t,g) \times | f^+([Tweety]_{t,g},small,t,g) – \text{Value}^+(bird, small,t,g) | + \text{Weight}^+(feathers,bird,t,g) \times | f^+([Tweety]_{t,g},feathers,t,g) – \text{Value}^+(bird, feathers,t,g) | + ... + \text{Weight}^+(flying,bird,t,g) \times | f^+([Tweety]_{t,g},flying,t,g) – \text{Value}^+(bird,flying,t,g) |
\]

However, for the purpose of averaging, the degree functions of all the \( bird \) dimensions (the predicates in \( F^+(bird,t,g) \)) need to be normalized so as to have the same positive range. For example, if we compute and compare averages of students in math and literature, the result is meaningless if the marks in math range between 1 and 6 and the marks in literature range between 10 and 100. Note that the distance between Tweety's size and the ideal size for birds, \( | f^+([Tweety]_{t,g},small,t,g) – \text{Value}^+(bird, small,t,g)| \), can in fact be anything in the range between zero (if Tweety's size is the ideal size for birds) and infinity (as entities can be infinitely large). The distance between any entity \( d \) and the selected value of \( bird \) in a dimension \( P \) needs to be normalized. I will write "\( \text{norm}([ f^+(d,F,t,g) – \text{Value}^+(bird, F,t,g)] \)" for values that are already normalized (that are within a given required bound range \([n,m]\)).
mean distance between Tweety's values and the selected values in all the dimensions that we are really after, then, is the following weighted mean:

$$ \text{Weight}^+(\text{small, bird, t, g}) \times \text{norm}( | f^+([(\text{Tweety}])^+_{t,g}, \text{small, t, g}) - \text{Value}^+(	ext{bird, small, t, g}) |) +$$

$$ \text{Weight}^+(\text{feathers, bird, t, g}) \times \text{norm}( | f^+([(\text{Tweety}])^+_{t,g}, \text{feathers, t, g}) - \text{Value}^+(	ext{bird, feathers, t, g}) |) +$$

$$ \cdots +$$

$$ \text{Weight}^+(\text{flying, bird, t, g}) \times \text{norm}( | f^+([(\text{Tweety}])^+_{t,g}, \text{flying, t, g}) - \text{Value}^+(	ext{bird, flying, t, g}) |).$$

In virtue of this normalization, nominal concepts always end up having bound scales. The range of a nominal additive degree function lies between the normalization bounds. (If I average across marks that lie between 1 and 100, the resulting means can only be numbers between 1 and 100). The bounds can vary between total contexts (they depend on an arbitrary choice of a normalization function).

Finally, similarity is inversely related to mean distance. The smaller the mean distance between Tweety's values and the bird selected values, the larger Tweety's degree in bird (Tweety's similarity to the bird prototype). The inverse relation is exponential: For any mean distance \(n\), similarity is \(1/e^n\) (Shepherd 1987; for a detailed discussion see 4.1 and the appendix of chapter 4).

Having demonstrated how similarity functions work, we can now postulate that in any context \(t\) and assignment \(g\), the degree function of a nominal concept \(P\) (that is associated with a non-empty prototype in \(t\) and \(g\)), maps entities to values that are inversely related to their distance form \(P\)'s selected values on \(P\)'s dimensions:

---

6 If Sam has 100 in every subject and the subjects' weights sum up to 1, then Sam's mean is 100, if Sam has 0 in every subject her mean is 0, etc.

7 We cannot assume that dimensions are normalized in the first place, because some dimensions may be adjectival (like small or long), namely predicates whose functions assign values that approximate infinity. An unbounded set of values (say, the interval \([0,\infty]\)) can only be transformed into a bound one (say, the interval \([0,1]\)), using a non-linear transformation (normalization procedure) – one that does not preserve the ratios between differences between values. For example, if the normalization procedure \(f(x) = 1 - 1/e^x\) is applied to \([0,\infty]\), we get \(f(0) = 1\), and for any \(n\) that approximates infinity, \(f(n)\) approximates zero, so the degrees range between zero and one (these numbers can be treated as bounds). But \(f\) is such that there are relatively large differences between \(f\) values of small numbers and almost no differences between \(f\) values of very big numbers. Conversely, linear normalization procedures can only transform bound intervals \([m,n]\) into other bound intervals like \([0,1]\). For example, \(f(x) = (x - m) / (n - m)\) is such that \(f(m) = 0\) and \(f(n) = 1\) (the differences are meaningful, unless \(n\) approximates infinity). Since adjectival functions adequately represent differences (we can say that 4 meters tall is 2 meters shorter than 6 meters), functions of adjectival dimensions (that may have an unbounded range) are not normalized in the first place.

8 I use an arithmetic mean function – a simple sum operation – but there are other mean functions. As explained in 4.1-4.2 and in the appendix of chapter 4, whether similarity is exponential or linear and whether the mean function is this or that, may well be a matter of context of use. These parameters can be made to vary through total contexts (by adding mean\(^+(P,c,g)\) and inverse\(^+(P,c,g)\) to the interpretation of nominal predicates). They can also be made to vary within one and the same total context, by representing them as implicit nominal parameters (semantic arguments which do not realize syntactically).

9 I allow for concepts with empty dimension-sets (as explained in 4.1.1.4 non-linearly separable concepts cannot be determined by average in a unique set of dimensions). In my proposal, in these concepts, categorization is based on the dimension-sets of their sub-categories, as explained in the section about exemplar-effects in 5.4.

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(9) Nominal concepts
\[ \forall t \in T, \forall g \in G, \forall P \in \text{CONCEPT} \; \text{s.t.} \; P \text{ is nominal and } F^t(P,t,g) \neq \emptyset: \]

a. \[ \forall d \in D, \text{d's degree in } P: \quad f^t(d,P,t,g) = \frac{1}{e^n}, \quad \text{where: } \]
\[ n = \sum_{F \in F^t(P,t,g)} (\text{Weight}^t(F,P,t,g) \times \text{norm}(|f^t(d,F,t,g) - \text{Value}^t(P,F,t,g)|)) \]

We have to ensure that the weights sum up to one in t and g:
\[ (\sum_{F \in F^t(P,t,g)} \text{Weight}(F,P,t,g)) = 1 \]

And we also have to ensure that for each set of dimensions that share the degree function, at least one receives a non-zero weight. Otherwise we cannot call them dimensions, as they do not affect the ordering (it is true that they are typical of birds iff at least one of them has a non-zero weight):
\[ \forall F_1 \in F^t(P,t,g): \text{Either (Weight}(F_1,P,t,g) > 0) \text{ Or (Weight}(F_1,P,t) = 0 \text{ and } \exists F_2 \in F^t(P,t,g) \text{s.t. } f^t(F_1,t,g) = f^t(F_2,t,g) \text{ and } \text{Weight}(F_1,P,t,g) > 0). \]

This proposal helps explaining many linguistic facts, but as I have not yet presented my analysis of adjectives, at the moment I will only state its most immediate consequences.

7.4.2 Immediate consequences:
The representation of partial and context-dependent knowledge about prototypes

My version of the prototype theory explicitly represents the context dependency of prototypes. On this theory, a 'prototype' is a positive and negative dimension set with positive and negative sets of selected values and weights for the dimension). Each prototype is represented with a context parameter c. Prototypes (and degrees that are based on them) vary across total contexts, and are partially unknown in some partial contexts. So different uses of a concept or a dimension, as well as different stages of their learning, are represented.

By adopting a prototype theory, we account for the basic typicality effects – the fact that speakers order entities by typicality along sets of ordering dimensions (cf. 2.2.1-2.2.3). We account for the fact that the instances that are regarded as more typical of a category are the instances which are more similar to its prototype (on average, their values in the dimensions are closer to the prototypical values, compared to the atypical instances). Examples which completely match the prototype always have zero mean distance between their values and the selected values in all the dimensions and hence the maximum degree in P.

The model represents the way world knowledge and contextual background may affect meaning. For example, in each context c, for each predicate P, the degree of similarity of some entities to the prototype of P (or, equivalently, their degree in P) may be directly given. The degrees of other items may be inferred from their mean-similarity in the dimension-set. The degrees of yet other items may be unknown (if, say, their degrees in the dimensions are unknown, or the dimension-set is not completely known). Similarly, in each context, some dimensions may be learnt (taught) directly, and some others may be inferred (from the knowledge of other dimensions or of the similarity degrees and category members, as explained above). Dimensions need not obey any specific constraint, probabilistic or other, apart from the
constraint which is given by the degree function of the predicate they are dimensions of. Some
dimensions may turn out to be common in the category; others may be based on cultural
conventions or scientific observations of experts (for instance, doctors study the symptoms of
different kinds of diseases; biologist characterize birds as typically *feathered*, etc.) If
dimensions are selected based on a scientific or cultural convention, the degree function is set in
accordance. In chapter 8, I present a method for dimension selection which children and adults
use, I propose, in the lack of knowledge about the dimensions or degree function.

Some properties are a relatively steady part of the linguistic definition of P (they are part of
speakers' world knowledge). Yet other dimensions are completely episodic restrictions on
relevant Ps in a particular context. For instance, my daughter strongly prefers the color red.
Thus, when we use the word *fruit* while making fruit-salad for her, dimensions like *red* and
*sweet* characterize it. They help to order the set of entities about which we speak by relevance in
the context.

### 7.4.3 Psychological reality: The representation of the nominal intension

In semantic theories, intensions of adjectives are given by a standard-based categorization
rule, i.e. by an algorithm for computing membership in any context. This accounts for the fact
that we can classify any of infinitely many novel entities we may encounter under any
adjectival concepts (provided that they reach threshold). However, semantic theories do not
have such an algorithm for nominal concepts, as nominal concepts are not represented as
gradable. On the present proposal, nominal concepts are gradable and categorization is based on
membership standards (cf. 7.1). Thus, our proposal considerably improves computability. It is
not necessary to directly encode each and every instance of a nominal concept. It is sufficient to
encode a finite set of dimensions, and a finite set of denotation members (or a standard for
membership), and the classification status of newly encountered entities whose mean on the
dimensions is higher than the standard follows automatically.\(^{10,11}\)

Given our standard-based categorization rule (cf. 7.1), it follows that in any \(t\) and \(g\), the
mean distance from P's selected values in the dimensions in \(F^\ast(P,t,g)\) of all and only the entities
in \([P]^{t,g}\) is small enough (the similarity degree, which is inversely related to distance, reaches

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\(^{10}\) On my proposal, in principle denotations include possible (not necessarily existing) entities, though existence
may be a dimension of this or that predicate. Perhaps there are systematic linguistic contexts in which denotations
are by (default) accommodation, restricted to existing entities. I leave these issues to future research.

\(^{11}\) In fact, on my analysis, super-denotations can never be represented in our minds as an enumeration of entities \(d\)
in \(D\). There are infinitely many entities \(d\) in \(D\), and we do not have direct access to any of them. (When we see an
object in front of us or use a name or definite description, we never know which \(d\) in \(D\) it corresponds to, as we
never know everything about it – we don’t know its degrees in all predicates.) We can only store information about
the super-denotation in our minds by description. For example, if a child is directly taught that the object in front of
her is an anvil, say, she cannot add anything to the positive super-denotation of *anvil*, but she can add to the
negative super-denotation all the entities \(d\) in \(D\) that fail to fit a description such as “I see it in front of me, it
appears to be made of metal, it has such and such a shape, it appears to be very heavy, etc.”. Given the above, a
child’s positive super-denotation of a concept will always be identified in the child’s mind via a description – a
certain requirement that all its members must meet. Similarly, if we know that Sam is tall, then the positive super-
denotation of *tall* includes all the entities \(d\) in \(D\) that fit the description “at least as tall as Sam is (i.e., as Sam’s
referent in all total contexts above us is)”. If we know what the threshold for being tall is – say, at least 1.90
meters in height – then the positive super-denotation of *tall* includes all the entities \(d\) in \(D\) that fit the description
“at least 1.90 meters in height”.

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the threshold, i.e. it is above the standard\(^*(P,t,g)\). Thus, obviously, we account for the fact that typicality is coupled with membership (cf. 2.2.5.3).

The set of dimensions that are actually linked with a concept like *bird* in each context is finite and partial. But these dimensions may be assigned relatively high weights, so speakers can regard items that are good in these dimensions (say certain robins, or the specie robin) as reaching the standard. Not all the values of a given robin (in all the possible predicate functions) are accessible to a speaker that watches it. Yet, the speaker can assume that most of the values that are not accessible to her are irrelevant (the predicates denoting them are in the negative dimension set of *bird* – they are already known not to affect the similarity of entities to birds), and those which may be relevant (potential dimensions) may be such that their potential weights are too low to reduce the robins' degrees below the standard. Yet in the presence of atypical instances the speaker may not be able to infer this (because its degrees in known dimensions are low). Finally, if some entities have different values in predicates which a speaker cannot positively say whether they are dimensions or not, or what precisely their weight is, the speaker cannot say which entity has a higher degree in *bird* (is more similar to the bird prototype). So different dimensions, weights, and entity-orderings may be assigned to *bird* in specific contexts of use.

For further discussion of the basic predictions of the prototype theory see chapter 4.

### 7.4.4 The felicity of nominal concepts in between predicate comparisons

An interesting, highly systematic, fact is that nouns can occur bare in between predicate comparisons, where the degrees in two different predicates are compared.

(10)  
   a. Bling Bling says "tweet" (I'm convinced he's more a bird than a cat).  
   b. … giving me three bits of furniture which she didn't want anymore (a coat rack, chair, and stool thing which is really more a table than anything else  
   c. The "wall" was rolling backward until it came to a horizontal position, now being more a table than a wall  
   d. ? A bat is no more a bird than a whale is a fish [invented]

The distribution of adjectives in the comparative is more or less complementary (they freely occur in one-dimensional comparisons, and they are hardly compatible with between-predicate comparisons).

(11)  
   a. # The table is longer than it is heavy  
   b. ? Dan is more hungry than tired  
   c. ? Dan is happier than Sue is funny

According to Kennedy (1999) the infelicity of examples like *The table is longer than it is heavy* is due to the fact that *long* and *heavy* have two different ordering dimensions. This makes them incommensurable. This account cannot be maintained given that comparisons between different dimensions freely occur in nouns, as we just saw. The present theory allows for a different explanation.

The idea is very simple. Intuitively, comparing degrees of functions that have two different ranges makes no sense, but comparisons of normalized functions, which share the range, make
sense. If (and only if) my capacity in math and in literature are both measured on a shared (say, 1 to 100 point) scale, we can we meaningfully say that I am better in math than in literature. In the present analysis nominal concepts are always readily normalized. Their range is a bound interval such as [0,1]. Thus, the nominal degrees can all be seen as representing the relative positions of entities on a scale (say, the one ranging from 0 to 1)\(^\text{12}\). It is no wonder that these degrees are easily comparable. Conversely, adjectival concepts do not denote mean functions and their scales are not readily normalized (and most often not bound). The degree of an entity on the scale of *long* or *heavy* tells you very little about its relative position on a one to zero scale. Thus, comparing degrees in two different adjectives makes no sense. Comparisons on the adjectival domain involve the use of units. As explained in detail in chapter 9, I can tell that Dan is taller than Sam when the difference between Dan's (degree of) height and Sam's (degree of) height equals the (degree of) height in a certain (non-zero) number of meter units (objects one meter long), or centimeter units, millimeter units etc. (the choice of a specific unit makes no real difference). But the difference between Dan's (degree of) height and Dan's (degree of) weight cannot be said to be equal to the (degree of) height in a certain (non-zero) number of meter units (some objects one meter long are heavy, others are light, etc.) For a discussion of conditions for the licensing of adjectives in unit-based between predicate comparisons (as in *the sofa is (one meter) longer than the table*) see chapter 9.

Comparison by normalization seems to be licensed in adjectives also when they have contrast categories. For example *sweet* has a set of contrast categories, \(K_{\text{sweet}}\), that includes predicates like *sweet, salty, bitter, sour*, etc. Similarly, *red* has a set that includes *blue, pink, yellow* etc.

(12) a. This [purple] shirt is more red than blue  
b. This [sour-sweet] Thai dish is more sweet than it is sour

According to psychological theories, in the presence of contrast categories categorization may be based on normalized degrees. The degree of an entity in a predicate \(P\) relative to a contrast set \(K_P\) is its degree in \(P\) divided by the sum of its degrees in the categories in \(K_P\).

\[
\begin{align*}
c. \quad \text{norm}^+(d, \text{sweet}, K_{\text{sweet}}, t, g) &= \frac{\text{deg}^+(d, \text{sweet}, t, g)}{\Sigma_{Q \in K_{\text{sweet}}} \text{deg}^+(d, Q, t, g)} \\
d. \quad [\text{x is more sweet than sour}]_{K_{\text{sweet}}} &= 1 \text{ iff} \\
& \quad \text{norm}^+(g(x), \text{sweet}, K_{\text{sweet}}, t, g) > \text{norm}^+(g(x), \text{sour}, K_{\text{sweet}}, t, g)
\end{align*}
\]

For a discussion of contrast based categorization, I refer the reader to chapter 4.1.1.3. Future research should determine the importance of contrast based categorization in adjectives and nouns, and its role in linguistic models.

The felicity of comparisons by normalization with adjectives improves when world knowledge, context or the adjectives’ semantics associate them with bound scales. For example, typicality adjectives are comparable (as in *Dan is more typical of a leo than of a virgo*), probably because typical makes use of the dimensions of it nominal argument, which are normalized. In addition, bound-scale predicate like *full* and *empty* are good (or at least better than predicates like *heavy* and *long*) in between predicate comparisons (as in *this glass is more full than that glass is empty*). The degrees in *full* represent the difference between the volume of a container and the volume of its containment (so we have both a maximum and a minimum

\(^{12}\) For further formal explanations see note 7.
degree), and so we can compare the ratio between (i) the volume of the glass's containment and the volume of the glass, and (ii) the volume of the empty part of the glass and the volume of the glass.

Comparison by normalization may be available even in other adjectives, given a very rich context, in which minimum and maximum points are specified for them. For example, imagine a factory that exports *long tables* by plane, and *heavy tables* by ship. Imagine also that the weights of tables produced in this factory range between 5 and 50 kilograms, and the length of tables produced in this factory ranges between 50 and 400 centimeters. A comparison by normalization requires that we map the intervals \([5,50]\) and \([50,400]\), systematically, into one interval (say, \([0,1]\)). Each \(n \in [5,50]\) can be replaced by \((n – 5) / (50 – 5)\), and each \(n \in [50,400]\) can be replaced by \((n – 50) / (400 – 50)\). In our current context, we may consider a table that weighs 10kg (i.e. its normalized degree, that represents its relative position on the weight-scale, is \((10 – 5)/ (50 – 5) = 1/9\)) and that is 150cm long (i.e. its relative position on the length scale, is \((150 – 50)/ (400 – 50) = 1/3.5\)). In the current context the felicity of the *table is longer than it is heavy* improves, because we can reason that 1/3.5 is bigger than 1/9 (but it is still odd; it is hard to force on these adjectives normalized scales; The reasons for this will be fully appreciated when the semantics of comparison statements will be discussed in chapter 9).

### 7.4.5 The dimension reading of 'typical'

The predicates *typical* or *characteristic* (and their equivalents in other languages) are often used to refer to a set of dimensions. For example, the statements *flying is typical of a bird*, conveys roughly that the dimension *flying* is part of the dimension set of *bird*:

\[(13)\]

- a. Wings, feathers, flying and nesting are typical /characteristic of birds.
- b. Smoking Nobles is typical of Kibbutz members
- c. Beautiful flowers are typical /characteristic of summer in Zermatt
- d. Despair is typical /characteristic of those who do not understand the causes of evil, see no way out, and are incapable of struggle.

And we also find corresponding statements with adverbs like *typically*:

\[(14)\]

- a. Typically, birds fly, nest…
- b. Typically, Kibbutz members smoke Nobles

We can now give a straightforward analysis to typicality statements with dimension-readings. For any n place predicate \(P\), *typical of \(P\)* is a one-place predicate that denotes sets of predicates, \(P\)'s dimensions (e.g. *typical of a bird* denotes the set of *bird* dimensions, as in *flying and singing is typical of a bird*). So I add to the language the following expressions and I add the following semantic and syntactic rules. *Typically* denotes the same operation as *typical* except that the arguments' order is reversed (the arguments "flip-flop", as they do in passive sentences compared to active ones). My analysis is presented in (15) below. I am abstracting away here from several issues concerning the semantic analysis of *typical* and *typically*, which I discuss later on throughout a variety of sections of chapter 8 (these issues include the internal structure of constituents like *typical of a \(P\)*; the fact that there are typicality statements with proper names
like *Dan is typically late*, and the fact that typicality statements can usually be interpreted in several possible, but systematic, ways).

(15) \( \forall n \in \mathbb{N}, \forall P, Q \in \text{CONCEPT}^n \), s.t. P is nominal, \( \forall t \in T, \forall g \in G \):
   
a. Let \([\text{typical of } P]\) (for example, \text{typical of a bird, typical of sisters of, etc.}) and \([\text{Typically } Q]\) (for example, \text{typically flying, typically worried, etc.}) be expressions of the language.
   
b. \([\text{Typical of } P](Q) \in \text{SENTENCE}\)
   
c. \([[[\text{typical of } P]]]^{t,g} = F(P, t, g)\)
   
\([[[\text{typical of } P]]]^{t,g} = \text{CONCEPT}^n - F(P, t, g)\)
   
d. \([[[\text{typical of } P](Q)]]^{t,g} = 1 \text{ iff } Q \in [[[\text{typical of } P]]]^{t,t}\)
   
\([[[\text{typical of } P](Q)]]^{t,g} = 0 \text{ otherwise}\)
   
\(Q \text{ is typical of } P\) is true iff \(Q\) is in \(P\)'s dimension set
   
e. \([[[\text{typically } Q]]]^{t,g} = \{P \in \text{CONCEPT}^n : Q \in F(P, t, g)\}\)
   
\([[[\text{typically } Q]]]^{t,g} = \{P \in \text{CONCEPT}^n : Q \not\in F(P, t, g)\}\)
   
f. \([[[\text{typically } Q](P)]]^{t,g} = 1 \text{ iff } P \in [[[\text{typically } Q]]]^{t,g}\)
   
\([[[\text{typically } Q](P)]]^{t,g} = 0 \text{ otherwise}\)
   
\(A \text{ P is typically } Q\) is true iff \(Q\) is in \(P\)'s dimension set

For example, *Flying is typical of a bird* is true in \(t\) and \(g\) \((([[\text{typical of a bird}(flying)])]^{t,g} = 1)\) iff \(Flying \in [[[\text{typical of a bird}]]]^{t,g}\). \(A \text{ bird typically flies}\) is true in \(t, g\) \((([[\text{typically flies(a bird)}]]^{t,g} = 1)\) iff the very same condition holds (iff \(Flies \in F^{*}(bird, t, g)\)).\(^{13}\)

Adjectives like *typical* have an additional use, in which they refer to a set of entities ("the typical-example reading"):

(16) a. Moshe is a typical kibbutz member
   
b. Robins are typical/characteristic birds
   
c. This year summer was typical of Zermatt

Other adjectives like *fun* can take as arguments, in addition to individuals, also properties. For example, Chierchia and Turner 1988 note that we can say that *Dan is fun* but also that *Reading Principia is fun*. However, we have no evidence for two separate readings of *fun*, while the dimension-reading and example-reading of *typical* are different. An individual is considered typical in a predicate iff its degrees in the relevant typicality dimensions are sufficiently high (I discuss this reading in the section on multi-dimensional adjectives), while a dimension is considered typical of a predicate under completely different conditions. It should be part of the calculation of similarity to the prototype of \(P\). I.e. there ought to be a ceteris paribus correlation

\(^{13}\) Note that the word *flying* in this context refers to the word itself. The same sort of thing seems to happen in examples like the following.

\([[\text{Everything typical of girls is typical of women}]]^{t,g} = 1 \text{ iff }\)

\(\forall x (x \text{ is typical of girls, x is typical of women})[[[\text{typical of girls}]]^{t,g} = 1 \text{ iff}\)

For all \(d \in D\) s.t. \([[\text{typical of girls}]]^{t,g}(d) = 1, [[[\text{typical of women}]]^{t,g}(d) = 1 \text{ iff}\)

For all \(d \in D\) s.t. de \([[\text{typical of girls}]]^{t,g}, de [[[\text{typical of women}]]^{t,g} \text{ iff}\)

(\(Words \text{ are treated as things, elements of } D\)).

For all \(F \in \text{CONCEPT} \subseteq D\), s.t. \(F \in F^{*}(\text{girls}, t, g), F \in F^{*}(\text{women}, t, g)\)
between the degrees of entities in Q and their degrees in P: All other things being equal, if \(d_1\)'s degree in a dimension Q is closer to P's selected value than \(d_1\)'s degree is, then the degree of \(d_1\) in P is bigger than \(d_2\)'s degree (if all their other dimensional degrees are equal, then it must be the case that \(d_1\)'s mean distance from P's prototype is smaller and hence its degree in P is bigger). Hence, it would be wrong to represent the meaning of the two kinds of uses of typical in a unified way.

In fact, dimensions can occur in statements where the of-argument of typical (e.g. bird in typical of a bird) occurs within a genitive-case position (as in Birds' typical singing), but typical examples cannot (as in *Birds' typical robin). In addition, the oddness of a coordination of an argument bearing the example role and an argument bearing the dimension role further illustrates that the two roles are distinct (as in ?? Moshe and smoking Nobles are typical of kibbutz members.) Finally, typicality adjectives (like characteristic and 'meafyen' 'typical' in Hebrew) are morphologically connected to nouns that denote sets of features, such as the noun characteristic, the Hebrew noun 'meafyen' ('a feature'), the nouns prototype and stereotype (that are usually regarded as denoting a set of features), and the noun character (which may mean type of personality, and as such it is related to sets of features).

7.5 Multi-dimensional adjectives and distance predicates

We saw in 2.2 that there are two main types of dimensions. One is the type of typicality-dimensions, which are combined together to form a categorization rule by means of a mean function. The psychological theories claim that all the concepts are linked with typicality dimensions (cf. 2.2 and 4, and Murphy 2002). The other type of dimensions is the type of necessary conditions for membership in the denotation, which collectively form a sufficient condition for membership. These dimensions are combined together to form a categorization rule, by means of a Boolean set-theoretic operation, namely, the intersection of the denotations of the dimensions forms the denotation of the predicate. These dimensions are often called 'rules'. The classical concept theory claimed that all the concepts are linked with such rules, but this theory was rejected by cognitive psychologists.

Formally, a dimension Q functions as a a necessary condition for membership in a predicate P in a context c and assignment g, iff in c and g, the standard of Q functions as a necessary criterion for categorization under P, that is, iff, in any extension t of c and g, every P is Q (P's denotation is a subset of F's).

(17) \(\forall c \in C, \forall g \in G, \forall P, Q \in CONCEPT:
\)
a. The predicate Q is a necessary condition for membership in the denotation of P in a context c (relative to g) iff:
\[\forall t \in T, t \geq c: [[P]]^+_{t,g} \subseteq [[Q]]^+_{t,g} \]

A dimension Q is a sufficient condition for P-hood in c and g iff the standard of Q functions as sufficient criterion for P-hood in c and g, that is, iff in any extension t of c and g, only Qs are Ps (P's denotation is a superset of Q's in t).

b. The predicate Q is a sufficient condition for P-hood in a context c iff:
\[\forall t \in T, t \geq c: [[Q]]^+_{t,g} \subseteq [[P]]^+_{t,g} \]
Let us refer to the set of known necessary conditions of \( P \) in a context \( t \) by the symbol \( N^+(P,t,g) \). In a partial context \( c \), the (super) set of known necessary conditions (for an arbitrary \( g \)), \( N(P,c,g) = \cap \{ N^+(P,t,g) : t \geq c \} \), need not be complete. But when information is complete (namely, in total contexts, where all the entities which are not in \( P \)'s positive denotation are in \( P \)'s negative denotation), the conjunction of the necessary conditions functions as a necessary and sufficient condition for \( P \)-hood in \( t \) (a categorization rule). This means that the denotation of \( P \) in \( t \) and \( g \) is given by the intersection of the denotations of the predicates in \( N^+(P,t,g) \).

c. \( \forall t \in C, \forall g \in G, \forall P \in \text{CONCEPT}, N^+(P,t,g) \subseteq \text{CONCEPT} \) is a set of dimensions which add categorization criteria to \( P \) and together form a sufficient condition for \( P \)-hood:

\[
[[P]]^+_{t,g} = \cap \{ [[Q]]^+_{t,g} : Q \in N^+(P,t,g) \}
\]

I propose that in principle, we have two ways in which we can process the dimension-set of a predicate \( P \): we can process the dimension-set either as a set of typicality dimensions (meaning that an entity’s degree in \( P \) is computed based on its mean similarity to the prototype in these dimensions), or as a set of rules – that is, a set of necessary conditions, which in total contexts are combined by a Boolean operation to form a sufficient condition for membership.

7.5.1 My proposal

I propose that the noun-adjective distinction functions as a cue that tells us how to process the dimensions. If a predicate is morphologically marked as a noun, the dimension set is, by default, processed as a typicality set – the dimensions are combined by a non-Boolean mean function (cf. 7.4). If a predicate is morphologically marked as an adjective, the dimension set is, by default, processed as a set of rules. The dimensions are combined by a non-unary Boolean operation which performs union or intersection of sets (disjunction or conjunction, respectively).

The categorization rule for conjunctive adjectives (like healthy) is the requirement to reach threshold in \( \land F^+(P,t,g) \) (= the conjunction of the predicates in \( F^+(P,t,g) \)), namely, to reach the standard degree in every dimension in \( F^+(P,t,g) \). The categorization rule for disjunctive adjectives (like different) is the requirement to reach threshold in \( \lor F^+(P,t,g) \) (= the disjunction of the predicates in \( F^+(P,t,g) \)), namely, to reach the standard degree in some dimension in \( F^+(P,t,g) \).

(18) **Multi-dimensional adjectives:**

\[
\forall d \in D, \forall t \in T, \forall g \in G, \forall P \in \text{CONCEPT}, \text{such that } F^+(P,t,g) = \{ F_1, \ldots, F_n \} \neq \emptyset : \\
a. \text{If } P \text{ is a conjunctive-adjectival concept: } \\
\quad d \in [[P]]^+_{t,g} \iff \forall F \in F^+(P,t,g): d \in [[F]]^+_{t,g} \iff d \in [[F_1 \land \ldots \land F_n]]^+_{t,g} \\
b. \text{If } P \text{ is a disjunctive-adjectival concept: } \\
\quad d \in [[P]]^+_{t,g} \iff \exists F \in F^+(P,t,g): d \in [[F]]^+_{t,g} \iff d \in [[F_1 \lor \ldots \lor F_n]]^+_{t,g}
\]

McCready and Ogata (2007) propose a conjunctive analysis specifically for the interpretation of typicality modifiers in Japanese. The present theory gives a conjunctive or disjunctive analysis to adjectives in general.
In accordance with that, I propose that the degree of an entity \(d\) in an *adjectival multi-dimensional* concept \(P\) is not given by averaging, but by \(d\)'s degree in the conjunction of \(P\)'s dimensions (if \(P\) is *conjunctive*) or in the disjunction of \(P\)'s dimensions (if \(P\) is *disjunctive*).

c. If \(P\) is a *conjunctive*-adjectival concept: 
\[
[\text{deg}(P)]_{t,g}^* = [[\text{deg}(F_1 \land \ldots \land F_n)]]_{t,g}^*
\]
d. If \(P\) is a *disjunctive*-adjectival concept: 
\[
[\text{deg}(P)]_{t,g}^* = [[\text{deg}(F_1 \lor \ldots \lor F_n)]]_{t,g}^*
\]

I delay the discussion concerning the way we calculate degrees in conjunctions and disjunctions to chapter 8, where I discuss productive strategies for the acquisition of degree functions in simple and complex predicates.

Note that one-dimensional adjectives can be seen as either conjunctive or disjunctive (they are both).

Finally, I propose that each dimension \(F\) in the set \(F(P,t,g)\) of a multi-dimensional adjective is of the form \(P \text{ wrt } Q\) for some concept \(Q\). For instance, the dimensions of *healthy* are not simply dimensions like (has) blood-pressure or (has) pulse. Rather, they have the form *healthy wrt bp, healthy wrt pulse*, etc. So before examining the consequences of my proposal, we have to give an interpretation rule for the degree functions of such predicates. I call them distance predicates because they are linked with distance-functions. By a distance-function I mean a function such that the value it gives is calculated based on the distance from some given value.

When a *with-respect-to* (wrt) phrase applies to a predicate pair, \(P\) and \(F\), (as in "\(P\) wrt \(F\)"), it is explicitly specified that \(F\) is the unique dimension of \(P\). Thus, wrt phrases are means of turning multi-dimensional predicates into one-dimensional ones. I propose that a predicate of the form \(P \text{ wrt } F\) maps each entity \(d\) to a degree that depends on the distance between \(d\)'s value in \(F\) and the selected value in \(F\) of "\(P\) wrt \(F\)".

Let us suppose that there is a certain \(bp\) value which is the best \(bp\) value one can wish to have– the healthiest \(bp\) possible. Let us informally call it \(bp☺\). Obviously, the smaller the distance between your actual \(bp\) value and \(bp☺\), the healthier-wrt-bp you are, and the bigger the distance – the sicker-wrt-bp you are. For any \(d \in D\), the distance in question is the following value.

\[
| bp☺_{t,g} – f^*(d,\text{bp},t,g) |
\]

What we want to predict, then, is that sick wrt \(bp\) is directly related to the above distance, while healthy wrt \(vp\) is inversely related to it. How?

Let us make the natural assumptions that *healthy wrt bp* is positive, just like *healthy* is, and that for any \(d\), \(t\) and \(g\), Value\(^*\)(healthy wrt bp,\(bp\),\(t\),\(g\)) intuitively represents \(bp☺\) in \(t,g\). I propose that the additive degree function of *healthy wrt bp* is the distance function such that \(\forall d \in D\), \(t \in T\), \(g \in G\):

\[
f^*(d,\text{healthy wrt bp},t,g) = – | \text{Value}^*(\text{healthy wrt bp, bp},t,g) – f^*(d,\text{bp},t,g) |
\]

It follows from our rule for positive predicates (cf. 7.3.1 above) that \(\forall d \in D\), \(t \in T\), \(g \in G\):

\[
\text{deg}^*(d,\text{healthy wrt bp},t,g) = f^*(d,\text{healthy wrt bp},t,g) – \text{Tran}^*(\text{healthy wrt bp},g,t)
\]

Hence, \(\forall d \in D\), \(t \in T\), \(g \in G\):
\[ \deg^+(d, \text{healthy wrt bp}, t, g) = -|\Value^+(\text{healthy wrt bp}, bp, t, g) - f^+(d, bp, t, g)| - \text{Tran}^+(\text{healthy wrt bp}, g, t) \]

So in the given example:

\[ \deg^+(d, \text{healthy wrt bp}, t, g) = -|bp \otimes_{t, g} - f^+(d, bp, t, g)| - \text{Tran}^+(\text{healthy wrt bp}, g, t) \]

Which means that the final degree of an entity d in \text{healthy wrt bp} is inversely related to the distance between d’s actual bp and bp\otimes, as desired.

As for \text{sick wrt bp}, let us make the natural assumptions that \text{sick wrt bp} is the negative antonym of \text{healthy wrt bp}. Meaning that they share the same additive degree function – that is, \(f^+(d, \text{sick wrt bp}, t, g) = f^+(d, \text{ healthy wrt bp}, t, g)\). It follows from our rule for negative predicates (cf. 7.3.1) that \(\forall d \in D, t \in T, g \in G:\)

\[ \deg^+(d, \text{sick wrt bp}, t, g) = \text{Tran}^+(\text{sick wrt bp}, t, g) - f^+(d, \text{sick wrt bp}, t, g) \]
\[ = \text{Tran}^+(\text{sick wrt bp}, t, g) - f^+(d, \text{ healthy wrt bp}, t, g) \]

But we know what \(f^+(d, \text{ healthy wrt bp}, t, g)\) is – it’s been defined above; so we may continue thus:

\[ = \text{Tran}^+(\text{sick wrt bp}, t, g) - (-|\Value^+(\text{ healthy wrt bp}, bp, t, g) - f^+(d, bp, t, g)|) \]
\[ = \text{Tran}^+(\text{sick wrt bp}, t, g) + |\Value^+(\text{ healthy wrt bp}, bp, t, g) - f^+(d, bp, t, g)| \]

So in the given example:

\[ \deg^+(d, \text{sick wrt bp}, t, g) = \text{Tran}^+(\text{sick wrt bp}, t, g) + |bp \otimes_{t, g} - f^+(d, bp, t, g)| \]

And this means that the final degree of an entity d in \text{sick wrt bp} is directly related to the distance between d’s actual bp and bp\otimes, as desired.

Note that given our assumption that positive and negative antonym pairs share the additive degree function, it follows that they also share the selected value. In fact, the additive degree function of \text{sick} represents quantities of health wrt bp, so to speak. This means that the selected value of \text{sick wrt bp}, which is part of the definition of its additive function, does not represent the ideal bp value for sick entities, but rather the ideal value for healthy entities, i.e.:

\[ \deg^+(d, \text{sick wrt bp}, t, g) = \text{Tran}^+(\text{sick wrt bp}, t, g) + |\Value^+(\text{sick wrt bp}, bp, t, g) - f^+(d, bp, t, g)| \]

Similarly, the additive degree function of the negative predicate \text{atypical (of a) mammal with respect to body-temperature} maps entities to degrees that represent the distance between their body-temperature and the body temperature typical of mammals (the selected value of \text{mammal} on any dimension that measures body temperature). The bigger this distance is, the more \text{atypical wrt body-temperature} the entities are. The smaller the distance – the more \text{typical wrt body-temperature} they are.

In general:
Distance predicates:
\[
\forall t \in T, \forall g \in G, \forall n \in N, \forall P \in \text{CONCEPT}^n, \forall F \in F^+(P,t,g):
\]

a. \(P \text{ wrt } F \in \text{CONCEPT}^n\)

b. \(F^+(P \text{ wrt } F,t,g) = \{F\}\)

c. \(\forall d \in D^n:\ f^+(d,P \text{ wrt } F,t,g) = (- | \text{Value}^+(P \text{ wrt } F,F,t,g) - f^+(d,F,t,g)|)\)

As a consequence, if P wrt F is positive, \(\text{deg}^+(d,P \text{ wrt } F,t,g) = (- | \text{Value}^+(P \text{ wrt } F,F,t,g) - f^+(d,F,t,g)| - \text{Tran}^+(P,t,g)\)

And if P wrt F is negative, \(\text{deg}^+(d,P \text{ wrt } F,t,g) = \text{Tran}^+(P,t,g) + | \text{Value}^+(P \text{ wrt } F,F,t,g) - f^+(d,F,t,g)|\)

Note also that, intuitively, the selected value of typical of a bird wrt size in the dimension 'size' (the selected value for bird in this dimension) is also the selected value of typical of a bird in the dimensions typical of a bird wrt size. In general:

d. \(\text{Value}^+(P \text{ wrt } F,F,t,g) = \text{Value}^+(P,P \text{ wrt } F,t,g)\)

Note that in order to define a distance function (a function of the form \(\lambda d. |m - f(d)|\)), we must use a reversal operation like \(\lambda n. m - n\) (or \(\lambda n. m/n\), or so on). Had we applied a 'plus' instead of a 'minus' operation (say, \(\lambda n. m + n\), which is not a reversal operation), we would not have arrived at a distance function (e.g., if \(m=100\), then if \(f(d_1)=80\) and \(if(d_2)=120\), the value of entities \(d_1\) and \(d_2\) come out different, failing to represent their (equal) distance from \(m\)). Now, empirically, the final degree functions of negative distance-predicates are distance-functions, while the final degree functions of positive distance-predicates are reversed distance-functions (distance-functions which, by further applying the reversal operation, are reversed again, i.e. functions of the form: \(\lambda d. - |m - f(d)|\)). We see an empirical generalization which can be described as follows. Grammar includes a linear reversal operation (a function that takes as an input the value of an entity in some degree function and returns as an output this value with a negative sign). The final degree functions of negative predicates can only be produced by applying this reversal operation an odd number of times (otherwise we would get the wrong ordering), while the final degree functions of positive predicates can only be produced by applying this reversal operation an even number of times (otherwise we would get the wrong ordering).\(^\text{15}\) In particular, distance functions are reversed functions, so they form the basis for negative predicates like different, sick, atypical, abnormal, etc., while reversed distance-functions form the basis for positive predicates, like similar, healthy, beautiful, etc.

The standard of membership in distance predicates is typically relative. There is variance in the value of standard\(^*\)(P wrt F,t,g) even among pairs of total-contexts and assignments that share the selected value. For example, a bird is considered typical (of a bird) wrt size iff its size is sufficiently close to the ideal size for typical (of a bird) wrt size (the selected value of bird for any dimension whose function represents size, like small). But what is the maximal distance that is sufficiently close to this ideal value? We cannot tell. This distance may vary between contexts. Similarly, a bird is considered atypical (of a bird) wrt size iff its size is sufficiently

\(^{15}\) The final degree of an entity in a positive predicate is calculated by subtracting the transformation value from the entities’ additive degree. But the fact that the transformation value is subtracted, rather than being added, has no importance at all (I have arbitrarily decided to do it one way and not another). For that reason, I do not count it as one of the applications of the reversal operation.
different from the selected (ideal) size of \textit{atypical wrt size} (the ideal size for birds), but the minimal difference that is considered sufficiently different may vary between contexts. In relative adjectives often some entities fall neither under them nor under their antonym. And indeed, birds may be neither \textit{typical wrt size} nor \textit{atypical wrt size} (iff their size is neither sufficiently close nor sufficiently different from the selected size).

Let us consider a detailed example. The dimension-set and selected values of \textit{typical of a bird} are given by those of \textit{bird}. For simplicity, let us assume that \(\text{Tran}^+(\text{typical of a bird}, t, g) = 0\), that the prototypical size for birds is 15 cm and the maximal distance from 15 allowed for typical birds is 35 (i.e. the standard of \textit{typical wrt size} is \(-35\)). The membership standard of \textit{atypical of a bird wrt size} may be either 35 or more, say 100.\(^{17}\)

\[\text{(20)}\]

\begin{enumerate}
\item \text{[Tweety is typical of a bird wrt to size]}|_{g}= 1 \text{ iff } \text{deg}'(\text{Tweety, typical of a bird wrt to size}, t, g) \geq \text{Standard}'(\text{typical of a bird wrt to size}, t, g) \text{ iff } (-|\text{Value}'(\text{typical of a bird wrt to size}, size, t) - f'(\text{Tweety, size}, t, g)|) \geq \text{Standard}'(\text{typical of a bird wrt to size}, t, g) \text{ iff } -|15 - f'(\text{Tweety, size}, t, g)| \leq -35 \text{ iff } |15 - f'(\text{Tweety, size}, t, g)| \leq 35
\end{enumerate}

This means that the distance between the prototypical size for birds and Tweety's size is sufficiently small (smaller than 35) iff \(-20 \leq f'(\text{Tweety, size}, t, g) \leq 50\).

\begin{enumerate}
\item \text{[Tweety is atypical of a bird wrt to size]}|_{g}= 1 \text{ iff } \text{deg}'(\text{Tweety, atypical of a bird wrt to size}, t, g) \geq \text{Standard}'(\text{atypical of a bird wrt to size}, t, g) \text{ iff } 0 - (-|\text{Value}'(\text{atypical of a bird wrt to size}, size, t) - f'(\text{Tweety, size}, t, g)|) \geq \text{Standard}'(\text{atypical of a bird wrt to size}, t, g) \text{ iff } |15 - f'(\text{Tweety, size}, t, g)| \geq 100
\end{enumerate}

This means that the distance between the prototypical size for birds and Tweety's size is sufficiently large (larger than 100) iff \(f'(\text{Tweety, size}, t, g) \geq 115 \text{ or } -85 \geq f'(\text{Tweety, size}, t, g)\).

If Tweety's size in \(t\) is 55, \(|15 - 55| = 40\). It is not the case that 40 \(\leq 35\), so Tweety is not typical wrt size, and it is not the case that 40 \(\geq 100\), so Tweety is not atypical wrt size.

As a consequence, some birds may be neither \textit{typical} (namely, \textit{typical in every respect}) nor \textit{atypical} (namely, \textit{atypical in some respect}). Thus, if some dimension of a multi-dimensional adjective is vague wrt the standard (has a relative standard), so is the multi-dimensional adjective (so, in principle, entities can be neither \textit{sick} nor \textit{healthy}, neither \textit{happy} nor \textit{unhappy}, neither \textit{normal} nor \textit{abnormal}, etc.)

Recall that, on the present proposal, in its dimension-reading the interpretation of any predicate of the form \textit{typical of }\(P\) is the set of \(P\)'s dimensions:

\[\text{16}\] The smaller your distance, the more typical wrt size you are, so the degree is 'minus the distance'. If you are 36 cm distant, your degree is \(-36\), and this degree does not reach threshold.

\[\text{17}\] If your distance is 36, your degree is \(0 - (-36) = 36\). So if the standard for \textit{atypical} is 35, you do reach threshold.
The dimension reading:
\( \forall t \in T, \forall P \in \text{CONCEPT}, \text{s.t. } P \text{ is nominal}, \lambda F. F \text{ is typical of } P \text{ is an adjective } \text{s.t.:} \)
\[ \exists t,g = \lambda F. F \text{ is typical of } P \]
\[ \lambda x. x \text{ is typical of } P \text{ is a conjunctive adjective:} \]
\[ \{ d \in D: \forall F \in F^{+}(P,t,g), d \in \lambda x. x \text{ is typical of } P \}^{t,g} \].

Before we move to consequences of this analysis, I need to make another remark about negative distance predicates, and the presupposition that entities which are more P are P.

When it is asserted that an entity is sicker than another, it follows that it cannot be healthy, as testified by the infelicity of sentences like # Dan and Sam are healthy, but Dan is more sick than Sam. The same phenomenon occurs also with other predicates (like ugly, stupid and dangerous). Interestingly, it seems to occur mostly with multi-dimensional predicates (those predicates that license wrt phrases). The phenomenon can be represented by assuming that, in any t and g, \( f^{t}(\text{sick-wrt-bp},t,g) \) is not identical to \( f^{t}(\text{healthy-wrt-bp},t,g) \) (it does not represent health wrt bp) but rather, it measures malady wrt bp (which is something that only entities which are not healthy wrt bp have), i.e. it maps to zero any entity d that reaches the standard of healthy wrt bp, and it maps to \( f^{t}(\text{d,health-wrt-bp},t,g) \) any other entity:
\[
f^{t}(\text{d, sick wrt bp},t,g) = \{ 0 \text{ iff } \deg^{t}(\text{d, healthy wrt bp},t,g) \geq \text{Standard}^{t}(\text{healthy wrt bp},t,g); \]
\[ - | \text{Value}^{t}(\text{healthy wrt bp,bp},t,g) - f^{t}(\text{d,bp},t,g) | \text{ otherwise} \}
\]

This would mean that we may regard predicate-pairs as antonyms even if their ordering is not completely reversed\(^18\) (it is reversed only where entities are not mapped to zero), because some antonym pairs may not be related with the same \( f^{t} \) function. It would also mean that the degree function of sick wrt bp is highly context dependent (it changes every time the standard of healthy wrt bp changes). But, intuitively, entities whose bp is not ideal are not perfectly healthy (wrt bp). They have some quantity of something which is not health (wrt bp). So perhaps the infelicity of # Dan and Sam are healthy, but Dan is more sick than Sam does not derive from properties of the degree function of sick wrt bp, but from constraints on use. I leave this issue for future research, but I would like to give some arguments in favor of this other possibility.

First, note that (perhaps due to the fact that arguments of sicker cannot have healthy entities as references), the use of healthier with arguments that denote sick entities is degraded too (as in ? Dan and Sam are sick, but Dan is healthier than Sam). When it is asserted that an entity is healthier than another, it is implied that the entities in question are not-sick, because had they been sick, the use of the comparative more sick would have been more appropriate, as it provides more information with equal effort. As a result the felicity of sentences like Dan and

\(^18\) Representatives of such a view are Winter and Rotstein (2005) and Kennedy and McNally (2005).
Sam are sick, but Dan is healthier than Sam is degraded, and some speakers seem to rule them out completely.

Second, and more importantly, the results of my general judgments questionnaire give evidence for the existence of a presupposition that entities ordered by more P are P. While 94% of the participants agreed that a sparrow (dror) is more typical of a bird than an ostrich (questions 5.1-5.2), only 65% agreed that a sparrow is less typical of a non-bird than an ostrich (5.8). Why? Some participants may have been confused by the presence of two negative operators, less and non. But some participants were unwilling to order birds (or entities which are not typical of non-birds) by typicality relative to non-bird. Some participants have directly mentioned this in the slot for notes which followed each question. If so, the answer "no" to the question whether a sparrow is less typical of a non-bird than an ostrich is to be interpreted as a metalinguistic no, which is meant to imply that one cannot felicitously apply the predicate typical of a non-bird, or the relation more / less typical of a non-bird, to arguments that denote bird kinds. The use of more, under this logic, is expected to be worse than the use of less. Indeed, fewer participants (only 56%) agreed that an ostrich is more typical of a non-bird than a sparrow (5.6), despite of the fact that this sentence is simpler in terms of the number of negative operators. Similarly, 74% of the participants agreed that a sparrow is less of a non-bird than an ostrich (5.7), but only 65% agreed that an ostrich is more of a non-bird than a sparrow (5.5).

Thus, I propose that the phenomena we are dealing with do not form exceptions to the generalization that antonym pairs are related with the same $f^+$ function (and hence that their ordering is completely reversed). Rather, they stem from violations of the presupposition that entities ordered by more P are P, which is less easy to cancel in negative predicates, and mostly in negative multi-dimensional predicates (for reasons that are not clear to me).19

Let us now see the positive consequences of our analysis of adjectives and distance-predicates.

7.5.2 Economy

On the analysis proposed here, the interpretation of a predicate becomes more economic in that the degrees assigned by all the predicates' final degree functions are ordered by the standard "bigger than" relation for the real numbers. We do not need to stipulate a degree-relation $\geq_P$ for any predicate (cf. 5.4), and at the same time, more facts are explained, such as the connections between P and F in predicates of the form $P \text{ wrt } F$. In addition, the semantics of the comparative morphemes is simplified. It does not use the predicate-specific relation $\geq_P$:

$$(23) \quad \forall c \in C: [\text{[more } P]\}_{c} = \{d_1,d_2\in D^2: \deg^+(d_1,P,c,g) \geq \deg^+(d_2,P,c,g)\}$$

19 We see that in many adjectives non-members are not ordered at all. The comparative ordering is constrained to the positive denotation (as testified by examples like, e.g., "Dan and Sam are healthy. # Dan is more / less sick than Sam" or like "Dan and Sam are intelligent. # Dan is more/ less stupid than Sam"). This data show that the comparative forms of many adjectives presuppose that the denotation is predetermined. In contrast, according to the present analysis, the positive forms of many adjectives do not presuppose that the ordering is pre-determined, because the separate conjuncts are defined to be categorization criteria. In practice, the denotations of healthy and sick may be determined by dimension intersection or disjunction, not based on knowledge of the entities' degrees (in the predicate itself) being above a threshold degree. This may directly explain the fact that the morphological form of the comparative relation is (cross-linguistically always) more complex than the positive form.
Compositional semantics for comparison statements is given in 9.3.

7.5.3 The differences between nominal and adjectival concepts

In any context t and assignment g, the additive degree function of a nominal concept P (whose dimensions are $F_1$...$F_n$ in t and g) maps entities to values that are inversely related to their distance form P's selected values on P's dimensions, i.e. $f^+(d,P,t,g) = 1/e^n$, where n is:

$$\text{Weight}^+(F_1,P,t,g) \times \text{deg}^+(d,\text{atypical wrt } F_1,t,g) + \ldots + \text{Weight}^+(F_n,P,t,g) \times \text{deg}^+(d,\text{atypical wrt } F_n,t,g).$$

Given our analysis of distance predicates, n, the distance from P's prototype, is, roughly, a weighted mean on values of degree functions of distance predicates atypical wrt $F_1$... atypical wrt $F_n$ (let us assume for the sake of this discussion that the transformation values are zero).

$$\text{Weight}^+(F_1,P,t,g) \times \text{deg}^+(d,\text{typical wrt } F_1,t,g) + \ldots + \text{Weight}^+(F_n,P,t,g) \times \text{deg}^+(d,\text{typical wrt } F_n,t,g).$$

As explained in 4.1, an exponentially reversed mean distance function, $1/e^n$, is similar to a multiplicative mean similarity function. This means that for any nominal concept P, d, t, and g, $f^+(d,P,t,g)$ can be seen as a multiplicative weighted mean on values of functions of reversed-distance predicates typical wrt $F_1$... typical wrt $F_n$ (for further discussion see 4.1).

The additive degree-function of a nominal concept P can be approximated as follows:

$$f^+(d,P,t,g) \approx \text{Weight}^+(F_1,P,t,g) \times \text{deg}^+(d,\text{typical wrt } F_1,t,g) \times \ldots \times \text{Weight}^+(F_n,P,t,g) \times \text{deg}^+(d,\text{typical wrt } F_n,t,g).$$

Conversely, in the present analysis, for any d,P,t, and g, $\text{deg}^+(d,\text{typical of } P,t,g)$ is d's value in a conjunction of reversed distance predicates typical wrt $F_1$...typical wrt $F_n$.

The additive degree-function of the adjective typical of P:

$$\text{deg}^+(d,\text{typical of } P,t,g) = \text{deg}^+(d, \text{typical of } P \wedge \ldots \wedge \text{typical of } P \wedge F_n,t,g).$$

In accordance, in order to fall under the nominal concept P, d's weighted mean in \{typical of P wrt $F_1$, ..., typical of P wrt $F_n$\} has to be sufficiently large, while in order to fall under the multi-dimensional adjective typical of P, d has to fall in the conjunction typical of P wrt $F_1$... typical of P wrt $F_n$, i.e. d has to reach threshold in each one of these predicates.

This demonstrates the basic difference between nominal and adjectival concepts, according to the present analysis. For example, imagine that the noun bird is associated with the dimensions flying and singing and that the adjective healthy is associated with the dimensions blood pressure (bp) and pulse. The denotation of bird is indicated by the mean degrees of entities on the two dimensions (roughly, the triangle in figure 1620), not by dimension-

---

20 The triangle represents the denotation given an additive mean function. For a multiplicative mean we need to replace the diagonal line with a curve, but this does not affect the argument. In addition, I use the terms triangle and square for ease of presentation. In effect, maximal degrees in the two dimensions need not exist. The square and triangle may only be sub-parts of open ended areas.

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intersection (the dotted square in figure 16; this square represents the denotation of the adjective 
*typical of a bird*). Unlike *bird*, and like *typical of a bird*, the set of healthy entities is given by the 
intersection of the two dimensions, as demonstrated in Figure 17. According to this 
proposal, the denotation of *healthy* is not indicated by averaging on the *healthy* dimensions. An 
averaging categorization rule would wrongly select a form more like the triangle in figure 16, 
while the set of healthy entities is the dotted square. For example, an entity $d_1$ which scores well 
in *bp* but has a very low health level wrt *pulse*, may not reach the threshold level in *pulse* 
and hence fall outside [[*blood pressure and pulse*]] (and outside [[*healthy*]]). At the same time, an 
exterior $d_2$ with a *lower overall mean* on these dimensions but a higher level in *pulse* might reach 
both standards and enter the denotation. We see that the mean in the necessary dimensions 
cannot indicate the denotation. Figures 16-17 demonstrate a two dimensional interpretation and 
an additive (arithmetic) mean function, but the argument applies to n dimensional cases and 
multiplicative means (or all the other generalized mean types).

**Figure 16: Nominal concepts**  
**Figure 17: Multi-dimensional adjectives**

Similarly, the denotation of *sick* (the negative antonyms of *healthy*) is fixed by dimension 
disjunction, not conjunction. For example, intuitively, it is sufficient to violate but one property 
which functions as a *healthy* dimension in a context c in order to be considered *sick* in c. Thus, 
if the standards for *sick wrt bp* and *sick wrt pulse* equal the standards of *healthy wrt bp* and 
*healthy wrt pulse*, the denotation of sick is the complement of the denotation of *healthy* in 
Figure 17. If the standards of *sick wrt bp* and *sick wrt pulse* are stricter (say, 4 not 5 on the scale 
of *healthy wrt bp* and *healthy wrt pulse*), there is a gap.

My analysis predicts the intuition that *typical of* *P* has more categorization criteria than *P*, 
although it is hard to put a finger on the exact dimensions which add criteria, if the wrt role is 
not explicitly specified and a determiner like *generally* or *some* is accommodated:

(24)  
Quantification over respects:  
\[ ([\text{Tweety is typical of a bird (in every respect)}])_{t,g} = 1 \text{ iff } \forall F \in F^*(\text{bird}, t, g): ([\text{Tweety is typical wrt to } F])_{t,g} = 1 \]  
(iff Tweety is typical wrt every bird dimension).

b. \([\text{Tweety is a bird}]_{t,g} = 1 \text{ iff } \exp(- (W(\text{flying}, \text{bird}, t) \times \text{norm(Tweety, bird wrt flying, t}) + \ldots + W(\text{singing}, \text{bird}, t) \times \text{norm(Tweety, bird wrt singing, t)}) > \text{standard(bird, t, g)})]

c. \([\text{Tweety is typical in some respect}]_{t} = 1 \text{ iff }

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\[ \exists F \in F^+(\text{bird}, t, g): \llbracket \text{Tweety is typical wrt to } F \rrbracket^+_{t, g} = 1 \]

(iff Tweety is typical wrt some bird dimensions).

The syntactic and semantic rules that I use to derive the truth conditions in (24a) and (24c) are presented at the end of this section.

We see that the interpretation of *typical of a bird* is in principle stricter than that of *bird*, though it can be relaxed by constraining its dimension set (using a wrt-phrase), or by weakening the dimensions' standard. Thus, according to my analysis, neither the degree functions nor the denotations of these two predicates (*bird* and *typical of a bird*) are necessarily always coupled. This proposal correctly predicts that, say – non biological fathers, may be judged more *typical fathers* than biological fathers. None is, strictly speaking, a typical father (none falls under the conjunction of all the dimensions), but among the things which are not *typical-fathers*, the real father may be much less typical, or typical in fewer respects. So the ordering in *typical of P* need not be identical to the ordering in P, given their different categorization criteria (the degree function of the former is not a mean function, but the degree function of a conjunction). We will see in chapter 8 that the two ordering relations may be rather similar given that there are relatively liberal compositional constraints on the ordering in the dimension-conjunction. So in judging whether entities score better in the dimension-conjunction, speakers may often resort to averaging, but the use of averaging need not be completely systematic, and the weights for the dimensions may change. E.g. definitional features like *biological* are very important for *fathers*, and as a consequence they practically function as (almost) necessary and sufficient conditions for membership in [[father]⁺]. But when judging whether two things that violate a dimension (whether important or not) are more *typical of fathers*, this dimension may well lose its central status (cf. Hampton 1979).

A similar conjunctive analysis for nouns with a morpheme like 'y' ('nouny', 'primy', 'birdy') predicts the ease with which a noun can turn adjectival.

I add to the language the following expressions and rules in order to derive the truth conditions of statements that involve quantification over respects.

(25) \[ \forall n \in \mathbb{N}, \forall Q, P \in \text{CONCEPT}^n: \]

a. Let [respect of P] be an expression of the language (for example: *respect of healthy*, *respect of healthier*, *respect of love*, etc.).

b. [respect of P](Q) ∈ SENTENCE
c. \[ \forall \phi, \psi \in \text{SENTENCE}, \forall F \in P^n\text{-VARIABLE}: \exists F(\phi, \psi), \forall F(\phi, \psi) \in \text{SENTENCE} \]

For any n place predicate P, *respect of P* is a one-place predicate which expresses properties of P – it denotes a set of entity sets, the denotations of dimensions of P, e.g. in each t and g, *respect of 'healthy'* (as in *Dan is healthy in every respect* which can be paraphrased as: *for every respect F of 'healthy', Dan is F*) denotes the set of denotations of dimensions of *healthy* in t and g).

\[
\begin{align*}
d. \quad [\text{[respect of P]}]_{t, g}^+ & = \{ A \subseteq D^n : \exists F \in F^+(P, t, g): [\llbracket F \rrbracket]_{t, g}^+ = A \} \\
& \quad [\text{[respect of P]}]_{t, g}^- = \{ A \subseteq D^n : \neg \exists F \in F^+(P, t, g): [\llbracket F \rrbracket]_{t, g}^+ = A \} \\
e. \quad [\text{[respect of P](Q)}]_{t, g}^+ = 1 \text{ iff } [\text{[Q]}]_{t, g}^+ \in [\text{[respect of P]}]_{t, g}^+, 0 \text{ otherwise} \\
f. \quad \forall \phi, \psi \in \text{SENTENCE}, \forall F \in P^n\text{-VARIABLE}: \\
\quad [\text{[\exists F(\phi, \psi)]}]_{t, g}^+ = 1 \text{ iff} \\
\quad \text{For some } A \subseteq D^n \text{ s.t. } [\llbracket \phi \rrbracket]_{t, g}(F/A) = 1, [\llbracket \psi \rrbracket]_{t, g}(F/A) = 1, 0 \text{ otherwise} 
\end{align*}
\]
\[\forall F(\varphi, \psi)\] \[+_{t,g} = 1 \text{ iff}\]

For all \(A \subseteq D^t\) s.t. \([\varphi]_{t,g}(F/A) = 1, [\psi]_{t,g}(F/A) = 0\) otherwise

Let us see a detailed example of a statement involving reference to respects of a one-place adjective, healthy.

\[(26)\quad [\text{Sam is healthy in every respect}]_{t,g} = 1 \text{ iff:}\]

\[\forall F(\text{is a respect of healthy}, \text{Sam is F}]_{t,g} = 1, [\text{Sam is F}]_{t,g}(F/A) = 1\]

For all \(A \subseteq D^t\) s.t. \([F \text{ is a respect of healthy}]_{t,g}(F/A) = 1, [\text{Sam is F}]_{t,g}(F/A) = 1\) iff

For all \(A \subseteq D^t\) s.t. \([F]_{t,g}(F/A) \in [\text{respect of healthy}]_{t,g}(F/A),\]

\[[\text{Sam}]_{t,g}(F/A) \in [F]_{t,g}(F/A)\]

For all \(A \subseteq D^t\) s.t. \(g_{(F/A)}(F) \in \{A \subseteq D : \exists F_2 \in F^t(\text{healthy}, t.g_(F/A)) : [F_2]_{t,g} = A\}\),

\(<\text{Extension(Sam, t), Extension(Dan, t)} >_{t,g}(F)\) iff

For all \(A \subseteq D^t\) s.t. \(A \subseteq D : \exists F_2 \in F^t(\text{healthy}, t.g_(F/A)) : [F_2]_{t,g} = A,\)

\(<\text{Extension(Sam, t), Extension(Dan, t)} >_{t,g}(A)\) iff

For all \(F_2 \in F^t(\text{healthy}, t.g_(F/A)) : \) \(<\text{Extension(Sam, t), Extension(Dan, t)} >_{t,g}(F_2)\) iff

Similarly for "Sam is typical of a bird in every respect" (note that here, typical of a bird has the entity-set reading, not the dimension-set reading discussed in 7.4.5).

Let us see also one detailed example of a statement involving reference to respects of a two-place adjective, healthier.

\[(27)\quad [\text{Sam is healthier than Dan in every respect}]_{t,g} = 1 \text{ iff:}\]

\[\forall F(\text{is a respect of healthier than, Sam is F Dan}]_{t,g} = 1 \text{ iff}\]

For all \(A \subseteq D^t\) s.t. \([F \text{ is a respect of healthier than}]_{t,g}(F/A) = 1, [\text{Sam is F Dan}]_{t,g}(F/A) = 1\) iff

For all \(A \subseteq D^t\) s.t. \([F]_{t,g}(F/A) \in [\text{respect of healthier}]_{t,g}(F/A),\]

\[<\text{[Sam]}_{t,g}(F/A), [\text{Dan}]_{t,g}(F/A) >_{t,g}(F/A)\)

For all \(A \subseteq D^t\) s.t. \(g_{(F/A)}(F) \in \{A \subseteq D : \exists F_2 \in F^t(\text{healthier than}, t.g_(F/A)) : [F_2]_{t,g} = A\}\),

\(<\text{Extension(Sam, t), Extension(Dan, t)} >_{t,g}(F)\) iff

For all \(A \subseteq D^t\) s.t. \(A \subseteq D : \exists F_2 \in F^t(\text{healthier than}, t.g_(F/A)) : [F_2]_{t,g} = A,\)

\(<\text{Extension(Sam, t), Extension(Dan, t)} >_{t,g}(A)\) iff

For all \(F_2 \in F^t(\text{healthier than}, t.g_(F/A)) : \) \(<\text{Extension(Sam, t), Extension(Dan, t)} >_{t,g}(F_2)\) iff

If we state that healthier is a conjunctive adjective such that \(F \subseteq F^t(\text{healthier than}, t.g)\) iff \([\text{More F than}] \subseteq F^t(\text{healthier than}, t.g)\) (which I find completely intuitive), we get that \textit{Sam is healthier than Dan in every respect} is true in \(t\) and \(g\) iff:
For all $F \in F^+(\text{healthy},t,g(\text{F/A}))$:
\[
<\text{Extension}(<\text{Sam},t), \text{Extension}(<\text{Dan},t)> \in [[\text{more F than}]]^t,g.
\]

(28) $\forall g \in G, \forall t \in T, \forall F,P \in \text{CONCEPT}$:
\[
F \in F^+(P,t,g) \text{ iff } [\text{More F than}] \in F^+(\text{more P than},t,g).
\]

To summarize our proposal, wrt-phrase create adjectival dimensions (like healthy wrt bp), i.e. distance-predicates whose membership-standards function as categorization criteria for the given multi-dimensional adjective (e.g. healthy). Furthermore, grammatical operations like quantifiers can access (the denotations of) these dimensions and operate on them. In the subsequent sections I further discuss and empirically support this proposal.

7.5.4 Except phrases

How can we empirically support (or refute) this analysis? Modification by an exception phrase is only licensed with universal quantifiers:

(29) a. Everyone is happy except for Dan
b. No one is happy except for Dan
c. # Someone is happy except for Dan

Thus, compatibility with exception phrases (as in "Dan is healthy except for his blood pressure") can form evidence for the hypothesis that interpretation involves universal quantification over dimensions, rather than existential quantification or averaging. Our proposal predicts that 'except' will be freely licensed as an operation on the dimension-set of conjunctive adjectives. This prediction is borne out by the facts. A simple google search for key-words like "healthy except", "typical except" and "healthier except" provides abundant examples:

(30) a. I am a 64-year-old man, quite healthy except for high blood pressure...
b. Sam's early development was considered typical except for slight articulation errors noted in kindergarten which resolved spontaneously.
c. … my Mother's family, mainly tradish, eat more of the tradish foods and they seem to be healthier except the cancer aspect
d. $[[\text{Dan is healthy except wrt blood pressure}]]^t,g = 1$ iff
\[
\forall F \in (F(\text{healthy},t,g) - \{\text{healthy wrt blood pressure}\})$: $[[\text{Dan is F}]]^t,g = 1$
(Dan is healthy wrt every dimension except blood pressure).

Conversely, we predict that this will hardly ever occur with disjunctive-adjjectives, whose default interpretation is mediated by an existential quantifier, and hence a non-default universal interpretation is likely to be explicitly marked (as in "sick in every respect"). The following example shows that indeed except cannot operate on the dimension set of a (bare) disjunctive adjective, and in fact, google searching for key-words like "sick except" and "atypical except" barely provides any examples of this sort.

(31) # They appear to be sick, except for the diarrhea
Finally, our proposal predicts that a negated disjunctive adjective like "not sick" will denote the set of entities that fall under no 'sick' dimension. For example, the sentence the children were not at all atypical, except that they were brighter than the average high-school Senior, an example of a negated use of 'atypical', is understood as conveying that the children were atypical in no respect, except for their intelligence. Crucially, 'no' is a universal quantifier. Thus, we predict that except phrases will occur as operating on the dimension-sets of negated disjunctive adjectives, and again, this turns out to be the case, as demonstrated by the following examples:

(32) a. Apparently, the children were not at all atypical, except that they were brighter than the average high-school Senior
b. They do not appear to be sick, except for the diarrhea
c. \[\lnot \exists F \in (F(\text{sick},t,g) - \{(\text{sick wrt}) \text{ blood pressure }\}): \ [[\text{Dan is } F]]_{t,g} = 1 \iff: \forall F \in (F(\text{sick},t,g) - \{(\text{sick wrt}) \text{ blood pressure }\}): \ [[\text{Dan is } F]]_{t,g} \neq 1 \]
   (Dan is sick wrt no dimension, except blood pressure).

The except-phrases adjacent to non-negated disjunctive adjectives operate on individuals, times or events, and the like, but not on the dimension-set, and they are usually accompanied with the explicit mention of an expression with a universal force like everybody, nobody, never or anything:

(33) a. We usually eat together as a family but this week everyone's been sick (except me--ha!) so they all ate variations of tea, toast, rice and the usual sick
b. I never got sick except for maybe a brief cold each fall and spring
c. He's definitely hungry and doesn't act sick except for right before he throws up and right after.

Other except-phrases function as hedges, canceling the earlier statement. Others belong to a completely separated phrase:

d. I haven't learned anything from being sick, except that I don't like being sick!!
e. On Thursday I was off sick. Except I was only half sick and the rest was tiredness.
f. … one would never know he was sick. Except for being bald, apparently I look great.

Interestingly, based on counts of entries of the form "ADJ except" as opposed to count of entries of the form "ADJ", in two search engines, google and msn, we can say that the proportion of except phrases (compared to uses of the adjective in general) tends to be greater in conjunctive than in disjunctive adjectives, mostly in the adjective pairs typical / atypical and normal / abnormal, as shown in Table 4. The search was of the form "ADJ except", which means that all the entries included the except-phrase immediately after the adjective, with no words, but possibly with a comma or a dot, in between.
TABLE 4:
COUNTS OF USES OF CONJUNCTIVE VERSUS DISJUNCTIVE ADJECTIVES WITH AND WITHOUT EXCEPT PHRASES

<table>
<thead>
<tr>
<th></th>
<th>google</th>
<th>msn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy</td>
<td>169,000,000</td>
<td>53,084,939</td>
</tr>
<tr>
<td>Healthy except</td>
<td>50,100</td>
<td>9,369</td>
</tr>
<tr>
<td>Healthy / healthy except</td>
<td>0.029 %</td>
<td>0.017 %</td>
</tr>
<tr>
<td>Sick</td>
<td>103,000,000</td>
<td>24,656,573</td>
</tr>
<tr>
<td>sick except</td>
<td>16,200</td>
<td>4,101</td>
</tr>
<tr>
<td>sick / sick except</td>
<td>0.015 %</td>
<td>0.016 %</td>
</tr>
<tr>
<td>Healthy/healthy except / sick/sick except</td>
<td>1.88</td>
<td>1.06</td>
</tr>
<tr>
<td>Normal</td>
<td>561,000,000</td>
<td>77,666,634</td>
</tr>
<tr>
<td>Normal except</td>
<td>518,000</td>
<td>47,562</td>
</tr>
<tr>
<td>Normal / normal except</td>
<td>0.092 %</td>
<td>0.061 %</td>
</tr>
<tr>
<td>Abnormal</td>
<td>44,800,000</td>
<td>10,390,078</td>
</tr>
<tr>
<td>Abnormal except</td>
<td>731</td>
<td>681</td>
</tr>
<tr>
<td>Abnormal / abnormal except</td>
<td>0.001 %</td>
<td>0.006 %</td>
</tr>
<tr>
<td>Normal/normal except / abnormal/abnormal except</td>
<td>56.59</td>
<td>9.34</td>
</tr>
<tr>
<td>Typical</td>
<td>203,000,000</td>
<td>27,305,203</td>
</tr>
<tr>
<td>typical except</td>
<td>15,000</td>
<td>2,077</td>
</tr>
<tr>
<td>typical / typical except</td>
<td>0.007 %</td>
<td>0.007 %</td>
</tr>
<tr>
<td>Atypical</td>
<td>12,300,000</td>
<td>643,281</td>
</tr>
<tr>
<td>atypical except</td>
<td>74</td>
<td>37</td>
</tr>
<tr>
<td>atypical / atypical except</td>
<td>0.0006 %</td>
<td>0.005 %</td>
</tr>
<tr>
<td>typical/typical except / atypical/atypical except</td>
<td>12.28</td>
<td>1.32</td>
</tr>
</tbody>
</table>

In order to support my proposal in a more systematic way, I have counted the different types of uses of except with the adjective typical (which I view as conjunctive), and with the adjective atypical (which I view as disjunctive), in the first 63 google entries for each of these two adjectives. I have chosen the adjectives typical and atypical, given their importance in the study of typicality. In addition, I have counted the different types of uses of except with the adjective healthy (which is assumed to be conjunctive), and with the adjective sick (which is assumed to be disjunctive), in the first 70 google entries for each adjective. I have selected the adjectives...
healthy and sick, because the proportion of except phrases (the number of entries of "ADJ except") within all uses (all the entries of "ADJ"), in google and msn, was almost the same in healthy and in sick. Thus, I wanted to see if indeed, the kind of uses of except phrases were substantially different in the two adjectives, as expected given the assumption that healthy is conjunctive and sick disjunctive. According to my hypothesis, the except-phrase should more easily lend itself to an interpretation in which it operates on the dimension-set, in conjunctive adjectives, than in disjunctive ones, because my hypothesis is that only in the former adjectives, the default interpretation involves (implicit) universal quantification over the dimension set. In disjunctive adjectives, for an except phrase to operate on the dimension-set, accommodation of an additional non-default constituent (such as in every respect) is required. Thus, my predictions are that dimension-set interpretations would be more common with typical except than with atypical except. In addition, I predict the reversed pattern to occur under the scope of negation. As shown in table 5, these predictions were borne out by the facts.

First, the dimension-set interpretation characterizes about half (50%) of the uses of typical except, but only 14% of the uses of atypical except. In addition, 24% of the uses of atypical except were under negation and have received the dimension-set interpretation, as opposed to only 1 such use of typical except. As predicted, the pattern was reversed under negation. The other uses occupy about 50% of the uses of typical except, and about 60% of the uses of atypical except. These include also all the cases in which universal quantification was realized overtly.

Second, the results are even more robust in the case of healthy and sick. The dimension-set interpretation characterizes 66% of the uses of healthy except, but only 1% of the uses of sick except. In addition, 10% of the uses of sick except were under negation and have received the dimension-set interpretation, but none of the uses of healthy except under negation received the dimension-set interpretation. The other uses occupy about 90% of the uses of sick except, and about 35% of the uses of healthy except.

<table>
<thead>
<tr>
<th>Except relates to:</th>
<th>The dimension set</th>
<th>Negation (not (a)typical except)</th>
<th>Hedging of the assertion</th>
<th>A different phrase</th>
<th>The set of times / places / objects, etc.</th>
<th>Total sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atypical except</td>
<td>9 (14%)</td>
<td>15 (24%)</td>
<td>3 (5%)</td>
<td>4 (6%)</td>
<td>32 (51%)</td>
<td>63</td>
</tr>
<tr>
<td>Typical except</td>
<td>31 (49%)</td>
<td>1 (2%)</td>
<td>4 (6%)</td>
<td>3 (5%)</td>
<td>24 (38%)</td>
<td>63</td>
</tr>
</tbody>
</table>

Types of uses of except with healthy versus sick in 70 google entries

<table>
<thead>
<tr>
<th>Except relates to:</th>
<th>The dimension set</th>
<th>Negation (not (a)typical except)</th>
<th>Hedging of the assertion</th>
<th>A different phrase</th>
<th>The set of times / places / objects, etc.</th>
<th>Total sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sick except</td>
<td>1 (1%)</td>
<td>7 (10%)</td>
<td>8 (11%)</td>
<td>4 (6%)</td>
<td>50 (71%)</td>
<td>70</td>
</tr>
<tr>
<td>Healthy except</td>
<td>46 (66%)</td>
<td>0 (0%)</td>
<td>2 (3%)</td>
<td>1 (1%)</td>
<td>21 (30%)</td>
<td>70</td>
</tr>
</tbody>
</table>
In sum, these findings support the proposal that when a predicate is classified as adjectival, its dimensions are combined using quantifiers (or Boolean operations), not averaging operations. In that, adjectival predicates sharply differ from nominal ones. In what follows, I show that, given this distinction, we can safely assume that nouns are gradable. The linguistic contrasts between nouns and (gradable) adjectives do not derive from the fact that adjectives are gradable and nouns are not (because they are gradable). They derive from the fact that the degree functions of nominal and adjectival predicates are inherently different.

7.5.5 Dimension accessibility: The licensing of wrt phrases

The present proposal explains the limited distribution of wrt-phrases (the accessibility of the adjectival dimensions, and the inaccessibility of the nominal dimensions). A wrt phrase denotes a relation between a predicate and one of its dimensions \((\lambda F. \lambda P. \lambda x. x \text{ is } P \text{ wrt } F)\). More precisely, the predicate that results from feeding this relation with \(P\) and \(F\), \(P \text{ wrt } F\), is interpretable only if it is a dimension of \(P\) \((P \text{ wrt } F \in F^+(P,t,g))\). However, this is not a sufficient condition for the licensing of a wrt-phrase. A wrt-phrase is not licensed in nominal concepts, and in one-dimensional concepts (as in # Tweety is a bird wrt flying and in # Tan is tall wrt height). The licensing of a wrt-phrase could have been explained by some inherent difference between the nominal and adjectival dimensions. For example, one could hypothesize that the adjectival dimensions are gradable and the noun dimensions binary. But this is not the case. For example, the ordering of birds depends on gradable dimensions like size and degree of ferocity, and the ordering in adjectives like healthy depends on dimensions that can be interpreted as 'binary' (yes / no dimensions) like has cancer. In addition, I do not see why wrt-modification and quantification would be available for gradable dimensions, but not for binary dimensions. Thus, this distinction fails to explain the differences between the nouns and the adjectives.

The discussion below focuses on dominant uses of respect phrases (as in Dan is healthy with respect to blood pressure and Dan is healthy in every respect). There are other uses of wrt phrases (as in predictions with respect to birth rates), where the wrt phrase can be substituted for an about phrase (as in predictions about birth rates). They are not in the scope of the current discussion.

Let us call \(P\) in \(P \text{ wrt } F\) the predicate-argument and \(F\) the dimension argument of the wrt-relation. I propose that a wrt-relation can combine with a predicate-argument \(P\) only when the predicate \(P\) has several dimensions which add categorization standards (only when, indeed, an entity can fall under a predicate in one respect and not in another respect), namely, in multi-dimensional adjectives:

\[
\forall c \in C, \forall g \in G:\nA \text{ predicates } P \text{ can form the predicate-argument of a wrt-phrase iff } P \text{ (e.g. healthy; similar) or } P\text{'s negation (e.g. sick; different) has several dimensions in } F^+(P,c,g) \text{ that function as necessary conditions (and } P \text{ wrt } F \text{ is one of them):}
\]

\[
P \text{ wrt } F \text{ is interpretable iff } P \text{ wrt } F \in F^+(P,t,g) \text{ and:}
\]

Note that we can interpret "P wrt F₁ and . . . and Fₙ" either as a predicate whose unique dimension is the conjunction "F₁ and . . . and Fₙ" or as a conjunction of predicates "P wrt F₁" and . . . and "P wrt Fₙ".
Either:  \[ \{ \{ \forall t \in T, t \geq c : [[P]]_{t,g} \subseteq [[F]]_{t,g} \} \mid > 1 \} \]
Or:  \[ \{ \{ \forall t \in T, t \geq c : [[\neg P]]_{t,g} \subseteq [[F]]_{t,g} \} \mid > 1 \} \]

Thus, a wrt-phrase can be licensed (and when licensed its dimension argument can interact with quantifiers) in multi-dimensional adjectives, in which several dimensions add categorization criteria. In fact, a determiner which quantifies over respects states how many of the dimensions form categorization criteria in the context of use:

\[(35) \]

a.  \[ [[\text{Dan is healthy in every respect}]]_{t,g} = 1 \text{ iff } \forall F \in F^+(\text{healthy},t,g): [[\text{Dan is healthy wrt to } F]]_{t,g} = 1 \]
(Dan is healthy wrt every health dimension).

b.  \[ [[\text{Dan is healthy in some respect}]]_{t,g} = 1 \text{ iff } \exists F \in F^+(\text{healthy},t,g): [[\text{Dan is healthy wrt to } F]]_{t,g} = 1 \]
(Dan is healthy wrt some health feature).

c.  \[ [[\text{Dan is generally healthy}]]_{t,g} = 1 \text{ iff, roughly: } \text{Dan is healthy wrt most of } F(\text{healthy},t,g). \]

d.  \[ [[\text{Dan is not sick except wrt bp}]]_{t,g} = 1 \text{ iff } \neg \exists F \in (F^+(\text{sick},t,g) - \{\text{sick wrt bp}\}): [[\text{Dan is } F]]_{t,g} = 1 \text{ iff } \] 
(Dan is sick wrt no dimension, except bp).

Conversely, neither the one-dimensional adjective tall, nor the noun bird, have two different necessary criteria in their dimension set, so wrt modification and quantification over dimensions is infelicitous.

Before we move on to explaining this generalization in different types of nouns, let me note that the main virtue of this proposal is that it simply forms a sub case of the general presupposition pattern of quantifiers like every, generally, and maybe even some. For example, the generalization every boy walks is normally uttered only if there are several relevant boys (cf. de Swart's 1991 plurality condition on quantification domains) and the question is whether they all walk (for some boys walk the question is whether some of the boys walk). Similarly, our proposal requires the domain in quantified statements like Dan is healthy in every respect to be neither empty nor a singleton, and the question they answer is whether all the elements in the domain (the healthy dimensions) fall into the set designated by the nuclear scope (the set of dimensions with respect to which Dan is indeed healthy).

It is possible that these are other, additional constraints on the licensing of wrt phrases. However, in the following section, I show how far we can go if we assume that (34) is the only licensing condition. I show that this proposal describes correctly the facts in basic nouns, exceptional nouns (animate nouns and nominalizations), and conjunctions.

7.5.5.1 The 'definitional' dimensions in Basic nouns

I use the term basic nouns to refer to typical nouns, namely, nouns like bird and furniture – as opposed to nominalizations like happiness, height, health, success, agreement, similarity, difference etc., and to animate nouns that have an adjectival entry like Italian. The latter noun types are discussed separately.

How does the weighted mean theory represent the fact that certain dimensions may, sometimes, be treated as definitional (necessary and / or sufficient for membership) in concepts
like *bird* or *grandmother*? Dimensions which are strictly speaking necessary for membership are hardly ever observed in nouns (cf. 2.2). Philosophers and cognitive scientists have argued that nouns do not have semantic necessary conditions for membership at all (Wittgenstein 1968 [1953]; Fodor et al 1980; Fodor 1998). Yet, as ordinary speakers, we might, for instance, count an individual as a *bird* iff it *has bird genes*, or iff it *has a bird essence* (in the lack of knowledge about genetics). Contemporary researchers treat the nominal 'definitional' dimensions (e.g. the dimension *unmarried adult* for *bachelor*), not as necessary, but as particularly important typicality dimensions. That is, the effect of the (so-called) necessary dimensions on categorization is accounted for by assuming that these dimensions weigh more heavily than others, while averaging on the dimensions (Hampton 1979). In order to predict the fact that members (instances) of a definitional dimension are always considered as more typical of the nominal concept than non-members, this definitional dimensions ought to weigh more than the non-definitional dimensions all together (Hampton 1995). For example, imagine that a dimension $F_i$ is assigned the weight 0.9 (so all the other dimensions together weigh only 0.1). Entities which do not match the prototype in $F_i$ are likely to have a very low typicality (and hence membership) degree, even if they completely match the prototype in all the other dimensions. On the other hand, entities that match the prototype in $F_i$ are likely to have a very high typicality (and hence membership) degree, even if they do not match the prototype in all the other dimensions. For example, in order to count as a *child* an entity must meet certain age limits. Yet, childish look and behavior affect categorization too. According to the above proposal, an age limit functions as (almost) necessary by being a very important typicality dimension (the weights of the other dimensions are small enough for it to be perceived as definitional). When an *important typicality dimension* $F$ is a conjunction, then all its conjuncts affect membership in $P$ (they add categorization criteria). That is, all the conjuncts can be said to be very important, though it is only their conjunction which directly participates in the averaging process.

Experiments with many different types of nouns (Hampton 1979 and 1995) show that an important typicality dimension of a noun is neither strictly speaking necessary nor strictly speaking sufficient for $P$-hood, if any other dimension is assigned a non-zero weight, as argued by Wittgenstein (1968 [1953]) and Fodor et al (1980). Even the dimensions of natural-kind concepts are often not treated as strictly necessary for membership. For example, a horse genotype is intuitively thought to be necessary for horses. Yet, experiments show that entities that possess, for instance, a zebra genotype (but due to a special diet or medical treatment become highly similar to horses in appearance and behavior) are often judged to be horses. Nor do the noun dimensions stand for sufficient conditions for membership. For example, popes and homosexuals who have been living with a partner for many years are adult males that were never married, but are they ‘bachelors’? In sum, the weight of non-important dimensions is usually sufficient to compensate for lack of an important dimension (for entities which are highly typical wrt the less important dimensions). In addition, the weight of non-important dimensions is usually sufficient to render entities that satisfy the definitional dimensions non-members (if they are atypical in all the less important dimensions).

The noun *bird* (unlike other natural kind names) was an exception in Hampton’s (1979)’s experiments in that no serious exceptions to its important features were found in this experiment. Yet, one can easily imagine a situation in which a mutation in a certain type of *bird* produces a new type, which is, say, *feather-less*. It is not clear at all that this type will be automatically classified as falling outside the denotation of *bird*. This shows that *has feathers* is
no more than a very important typicality dimension, which, in a sense, only by accident does not yet have any exceptions. Exceptions are likely to reduce its importance.

Thus, the empirical facts show that, by default, the dimensions of nouns are interpreted as typicality features (they are combined by an averaging function to form a categorization standard). The notion of strictly speaking necessary conditions (collectively sufficient for membership) is void of content when the noun dimension-set is concerned. When at least two dimensions are regarded as very important the fact that they are two (or more) reduces their weights and they do not count as necessary.

Scientific contexts might differ from the default linguistic context, though. For instance, an expert on birds might in principle characterize (all and only) birds by the possession of, say, 100 separate genes (nouns may be associated with necessary and sufficient conditions in scientific contexts). Our proposal correctly predicts that in such a context, the expert might indeed describe new species which possess only some of the bird genes as birds wrt gene A but not wrt gene B. At any rate, according to my proposal, such definitions diverge from the default, natural type of noun interpretation (or processing). These interpretations may be represented separately, as additional knowledge which is not automatic and procedural, but more open to introspection, as the rule-based (adjectival) interpretations usually are (cf. 5.7.1; and Ashby and Maddox 2005).

Nouns like *boy* seem to have two necessary conditions for membership (categorization criteria): *non-adult* and *male*. Some think that the felicity conditions for the licensing of quantifiers require that the domain will consist of more than two entities. This may explain the infelicity of wrt-phrases with nouns that have two necessary (or important) dimensions, like *boy*, *girl*, *man*, *woman*. In addition, these nouns are also associated with many unimportant typicality dimensions that may reduce the importance of these apparently necessary conditions when they form part of the dimension set.

### 7.5.5.2 The nominal 'domain-restrictors'

Some predicates (like *animate*, *solid*, *thing*, *non-abstract*, *flying* or *walking* or *moving*, etc.), despite the fact that they are in effect necessary conditions for membership in predicates like, e.g. *bird*, merely function as domain restrictors and do not help to distinguish *birds* from *non-birds* (from entities such that saying that they are *not birds* or *non-birds* is not funny or odd). These predicates are also not useful for licensing wrt-phrases. For instance, in most contexts the predicate *animate* restricts the domain of birds and non-birds. We hardly ever call prime numbers *non-birds*. Similarly, predicates like *human* or *person* are necessary conditions for membership in predicates like *boy* or *man*, but more often than not when we discuss individuals that are *not-boys* we only have human beings in mind, too.\(^{22}\)

Let us call the predicates R which are necessary for membership in P's domain domain-restrictors (for any arbitrary c and g, \(\forall t \in T, t \geq c: \text{Domain}^+(P,t,g) \subseteq [[R]]_{t,g}\)). When speakers are asked to list characteristics of nouns like *bird* they often omit these dimensions, or they treat them as characterizing both the noun and its negation (Hampton 1997).

We said that a wrt phrase, and hence quantification over dimensions, makes sense only when, indeed, one can felicitously argue that an entity falls under a predicate in one respect and that it falls under its negation in another respect. But the violation of dimensions such that it is funny to

\(^{22}\)I have suggested in 7.2.4 that these are cases that involve accommodation, e.g. utterances of predicates like *not-boys* are interpreted roughly like *human beings who are not boys*. 199
say that things that violate them are non birds (because in effect we often use them to
pragmatically restrict the set of non birds or not-birds) do not satisfy this requirement. Entities
that violate them (e.g. prime-numbers) indeed fall under not-birds, but, in most contexts, they
are considered irrelevant (precisely because they are not in the domain of bird) and stating this
fact is grasped as infelicitous or superfluous. Since, according to my proposal, by saying that
something d is a bird wrt to F₁ one implies that there is another dimension F₂ such that d is not a
bird wrt F₂, it is but natural that if (for every F₂) saying that d is not a bird wrt F₂ is infelicitous
or highly superfluous, then saying that d is a bird wrt F₁ is infelicitous or superfluous.

7.5.5.3 Nominalizations and animate nouns

Nouns that systematically allow wrt modifiers (animate nouns like idiot and property denoting
nouns like happiness, height, health, success, agreement, similarity, difference etc.) are similar
to adjectives in other respects as well (such as agreement, argument structure, etc.).
Nominalizations form exceptions to almost any generalization about nouns, whether syntactic
or semantic. For example, usually verbs or adjectives denote eventual types (categories of
events or states), while nouns denote non-eventual entity-types, and usually, the verbs or
adjectives have an elaborate argument structure, while the nouns do not (Landman 2000). The
property denoting nominalizations are atypical of their category in that they often seem to
denote eventual types (success, disaster, agreement) and they have an elaborate argument
structure. For instance, while normal nouns cannot take for arguments (as in # Tweety is a bird
for a water-bird), these exceptional nouns can (as in the conference was a success for a student
conference).Animate nouns like idiot also often resemble adjectives. For example, in languages
like Hebrew, the morphological form of either verbs or adjectives (36a), but usually not of
nouns (36b), agrees with the subject in gender. In addition, the copula can be omitted when the
predicate position is occupied by an adjective (36a), but usually not when it is a noun (36b).
The animate nouns behave like the adjectives (36c).

(36) a. Dan (hu) yarok [Dan is green]<sup>MASC</sup>]; Beth (hi) yeruka [Beth is green]<sup>FEM</sup>]
b. Dan #(hu) cipor [Dan is a bird]; Beth #(hi) cipor [Beth is a bird]
c. Dan (hu) idiot [Dan is an idiot]<sup>MASC</sup>]; Beth (hi) idiotit [Beth is an idiot]<sup>FEM</sup>]

Accordingly, these nouns allow (or they sound better with) wrt phrases (as in Dan is an idiot
wrt money / in every respect / except wrt money, and in the conference was a success wrt the
quality of the papers / in every respect / except for the papers). I would argue that these nouns
do not form counterexamples to the proposed generalization.

Concerning nominalizations, future research should focus on nominalizations and their
interpretation. Before we gain more knowledge about them it is hard to say definitive things
about them. But as things stands now, I propose that it is at least possible that their ability to
combine with wrt-phrases or for-phrases is due to the fact that adjectives that combine with a
wrt phrase can then be nominalized and the result is a nominalization with a wrt phrase

Furthermore, in nominalizations the wrt-argument functions differently. It neither adds the
categorization criterion which it usually adds, nor reduces the number of ordering criteria as it
usually does in adjectives. For instance, compare healthy wrt bp with health wrt bp. An entity
falls in the denotation of the adjective healthy wrt bp iff it possesses enough of some quality –
in fact, if it possesses enough of the quality ‘health wrt blood pressure’ (= being close enough to
the ideal blood pressure for health). So the wrt-phrase not just turns healthy into a one-dimensional adjective (with a dimension which is an easily measurable quality), but also adds a clear and usable categorization criterion.

In contrast, it does not seem to be the case that an entity falls in the denotation of the noun health wrt bp iff it possesses enough of some quality (certainly not of one that can be easily measured or even clearly grasped.) To fall under the denotation, the entity itself presumably has to be a quantity of something, e.g. of ‘health wrt blood pressure’. But it certainly does not have to possess a large enough quantity of ‘health wrt blood pressure’ (which is what it itself is). It is only in the adjective that the requirement to possess enough of the relevant type of health comes in, and in that sense the wrt-argument adds a categorization criterion to the adjective, which is not added to the noun. To fall under the denotation of the nominalization, the entity does have to have enough of ‘being health wrt bp’ – that is, to be ‘health wrt bp’ to a large enough degree. But that does not give us any useful information in terms of determining membership. In that sense, the wrt phrase does not supply a (usable) categorization criterion.

Similarly, concerning animate nouns, I propose that animate adjectives can combine with wrt phrases and then turn into animate nouns. In sum, nouns which are derived from adjectives (or are systematically connected to an adjective), namely, nominalizations and animate nouns, inherit the wrt-phrase from the adjective. Animate nouns seem to have a double entry, both as a noun and as an adjective. As adjectives they can be modified by more and wrt, and they can denote human beings, cities etc. (as in Dan is more Italian than Sam wrt their cooking and Florence is more Italian than Torino wrt food and weather). The nominal entry is more restricted. It can only denote human beings (as in Dan is an Italian wrt cooking versus # Florence is an Italian wrt food). So the noun is characterized by a richer set of dimensions, as expected from a noun. The noun phrase is interpreted more like the modified noun phrase an Italian man. So the wrt-phrase is licensed in virtue of the occurrence of an adjective, Italian.

This proposal goes well with the fact that more is incompatible with nominalizations and animate nouns. In my searches of the internet, I have found but few examples with more in within-predicate comparisons (comparisons of degrees of two entities in one predicate), where the predicate was a noun and more occurred without the morpheme of.

(37) a. That's how much more a success Torino was, compared to Athens
    b. I'm always a boy; but I'm more a boy when I perform
    c. I'm more a boy than everyone in your team

In all the other cases, when more occurred with a noun (whether a basic noun, nominalization or an animate noun), the comparisons were between degrees in two different predicates.

d. probably this is more an Italian tradition than a British one
    e. To Italians he is almost more an Italian than an English poet

23 If you know what counts as blood pressure or health it may be easy for you to say how much of it there is. But how do you decide that something counts as a (quantity of) health, this or that type of malady, happiness, success etc.? A rich set of symptomatic non-necessary dimensions characterizes disease types, and entities like success can hardly be defined by a set of necessary conditions which are jointly sufficient. Thus, nominalizations are mapped to the noun category because they are not associated with a set of respects, and they are often associated with a rich set of typicality dimensions. The denotation of some nominalizations (say, height) may be fixed in the whole context structure, but the denotations of many others (success; health) may be highly context dependent.
f. these young Japanese Americans prove their patriotism through unquestioning obedience to authority, ironically a trait more Japanese than American

g. Columbus was more a "success" for having landed in the Bahamas than in Bombay

h. He's much more a boy from Long Island than a boy from Brooklyn

i. The hero seemed more a boy than a man.

In that, the nouns are sharply distinguished from the corresponding adjectives, for which plenty of examples can be found of within-predicate comparisons:

j. the southern part of the region is far more Italian than Alto Adige

k. More charming than Boston, more romantic than Vegas and more Italian than Naples, Providence is an undiscovered gem of a city with no traffic. ...

Examples like *Dan is more Italian than Sam* become odd when an article is added (as in *# Dan is more an Italian than Sam*), unless the particle *of* is added.

Perhaps we can utter statements like *Non-Japanese who love Japan become more Japanese than the Japanese* because the noun and adjective denotations need not be completely identical, in virtue of the indeterminacy in the dimension set of the adjective (and to some extent also the noun). In the given example, the noun denotes the set of Japanese by nationality or birth, while the adjective is interpreted wrt behavior (Dan's behavior is more of a stereotypical Japanese behavior than the behavior of the Japanese by birth). That is, statements like *Dan is Japanese* need not be completely identical to ones like *Dan is a Japanese*, if, for instance, *Japanese* in the former statement is interpreted wrt stereotypical Japanese behavior.

7.5.5.4 Conjunctions

Finally, consider, conjunctions, including conjunctions of gradable adjectives, such as *bald and tall*. Conjunctions have at least two necessary dimensions – the conjuncts themselves (e.g. *bald and tall*). But conjunctions cannot combine with wrt-phrases. If a wrt-dimension is selected from the dimension-set of one of the conjuncts the use of the conjunction (say, "healthy and happy" wrt blood pressure) rather than just one conjunct (say, healthy wrt bp) becomes superfluous and hence infelicitous. And if one of the conjuncts is selected (*bald and tall wrt tall*) the use of the conjunction becomes even more puzzling.24

Given that wrt phrases are not licensed in conjunctions, conjunctions are bound to always be multi-dimensional. We will now see that this fact has interesting implications.

7.5.6 The licensing of comparative morphemes

I submit that comparative morphemes require that their predicative argument be one-dimensional. Multi-dimensional adjectives can combine with wrt phrases so they may turn one-dimensional and as such combine with *more*. But nouns cannot combine with a wrt phrase, so

---

24 If you think of the respect-argument as an adjectival thematic role, then, clearly, argument roles are associated with lexical items not conjunctions.
they remain inherently multi-dimensional and as such they cannot combine with more.25 Similarly, we saw that conjunctions of gradable adjectives cannot combine with a wrt phrase. They too are bound to remain multi-dimensional. So we predict that they should not combine with comparative morphemes (like nominal concepts). Interestingly, this prediction is supported by the facts. For instance, I found (in 35 subjects) that the preferred interpretation for more bald and tall is balder and taller. That is, and is not within the scope of more (the connective takes wide scope). This gives further evidence for the assumption that more does not take two dimensions, simultaneously.

I have found evidence that speakers are unwilling to interpret and inside the scope of more, i.e. more P and Q is interpreted as more P and more Q, not as More (P and Q). The same, though to a lesser extent, holds for or. For example, in question 4 of the general-judgment-questionnaire, the subjects read a description of two characters, Moshe and Danny. Moshe weighed 100kg as opposed to Danny’s 70 kg, but Danny was bald but Moshe was not bald. 93% said that Moshe is fatter and Danny is balder (4.1-4.2), but, interestingly, most subjects (90%) said that questions like ‘which one is more: fat and bald?’ cannot be answered. The subjects said that Moshe is not more fat and bald; that Danny is not more fat and bald; and that they are not equally fat and bald (4.3-4.6). This shows that the common interpretation of more bald and fat is: balder and fatter, and the common interpretation of equally bald and fat is: equally fat and equally bald.

Furthermore, in question 3, the two characters were described as equally tall (1.95m), but Moshe was fatter (100kg as opposed to Danny’s 70 kg). Still, most subjects refused to say that either Moshe or Danny is more fat and tall (3.1-3.8). A similar pattern occurred with respect to more fat and bald (9.1-9.8), when the characters were described as equally fat, but Moshe was balder.

In sum, a truly compositional interpretation which traces the function of the property fat and bald is hardly occurring naturally (at least in the lack of an encouraging context). In and-conjunctions more modifies each conjunct, separately (as the comparative morpheme in within-predicate comparisons is undefined in the lack of a unique dimension).

More cannot felicitously modify the property denoted by a modified noun.

(38)  a. * d1 is (a) more fat bald man than d2  
    b. * d1 is (a) more clean tall boy than d2  
    c. * d1 is (a) more midget giant than d2

When it is licensed, it either associates with the modifier alone, or has to be modified by of.

d. d1 is a fatter bald man than d2  
e. d1 is more of a midget giant than d2

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25 The conjunction (or disjunction) of the nominal dimensions cannot form a unique ordering dimension for the nouns, because the ordering given by \(\operatorname{deg}_{F(P,t,g)}\) is too different from the noun ordering, \(\operatorname{deg}_{F(P,t,g)}\) in any t and g. For example, a bird \(d_1\) which falls under all the dimensions but has a relatively low average is less good in bird, but better in the dimensions-conjunction, compared to a bird \(d_2\) which violates one (unimportant) dimension but has a higher overall average. Had we used \(\operatorname{deg}_{F(bird,t,g)}\) to define a derived comparative for bird, the former bird would have ended up more bird than the latter (\(\operatorname{deg}'(d_1, \land F'(bird,t,g),t,g) < \operatorname{deg}'(d_2, \land F'(bird,t,g),t,g)\)), though its degree is lower (\(\operatorname{deg}'(d_1,\land F'(bird,t,g),t,g)< \operatorname{deg}'(d_2,\land F'(bird,t,g),t,g)\)).
Otherwise, the noun phrase must be modified with typical, where the adjective typical forms the argument of more, and the noun phrase only fills the dimension-set argument of typical (the of argument).

f. d1 is a more typical clean boy than d2

In the use of more, the constituents in modified nouns cannot be separated the way the conjuncts in an and-conjunction do, so more cannot modify each one separately. The other interpretation (where more modifies the whole modified noun) is not available because the modified noun is multi-dimensional. Thus, more can take neither wide scope nor narrow scope relative to the modified noun. In the lack of available interpretation, the derivation crashes.

Given these data, it is likely that the same phenomena occur in the use of conjunctive multi-dimensional adjectives with more (like typical wrt flying and singing or healthy wrt blood pressure and pulse). That it, by and large, and takes wide scope, and more ends up with one dimension per conjunct. I.e. I predict that we understand healthier wrt bp and pulse to mean healthier wrt bp and healthier wrt pulse (that we are reluctant to say about two entities such that one is healthier wrt bp and the other healthier wrt pulse, that they stand in the relation healthier wrt bp and pulse).

The requirement for a unique dimension in the use of a comparative morpheme can only be abandoned in between-predicate comparisons. Conjunctive and disjunctive concepts seem to be felicitous and to receive interpretations with the connective in narrow scope in such comparisons, though more systematic future research needs to carve out the precise set of interpretations that may be assigned to such statements.

g. This is more a kitchen utensil than an electronic device.
h. This is more a piece of furniture and a game than a kitchen utensil or an electronic device.
i. Dan is more fat, bald and unhappy than good-looking, energetic and funny.

7.6 Intermediate conclusions

To summarize, the adjective-noun distinction encodes two different ways in which we can process the dimension set in the interpretation of a predicate. If the predicate is a noun, the dimensions are combined by averaging operations to form a categorization rule. If the predicate is an adjective, the dimensions are combined by Boolean set-theoretic operations to form a categorization rule (the dimensions’ denotations are intersected or joined). This proposal correctly accounts for the basic meanings of nouns and adjectives, for the restrictions on the licensing of wrt phrases, and for a variety of facts regarding restrictions on the licensing of comparative morphemes. That the current theory does not suffer from problems of classical dimension theories (the Kripke-Putnam problems; Russell's paradox, etc.) is shown in the appendix.
8 THE LEARNING PRINCIPLE AND COMPLEX CONCEPTS

8.1 My proposal

Let us add to the language the relations $\geq P$ (in adjectives this relation is identical to at least as $P$ as), $> P$ (more $P$ then in adjectives), and $= \text{ (equally } P \text{ in adjectives). In nouns, these relations represent speakers’ judgments of acceptability of examples (which in English cannot be referred to using the comparative forms more $P$, equally $P$, etc.)}

(1) $\forall t \in T, \forall g \in G, \forall P \in \text{CONCEPT}:$
   
   a. $[[\geq P]]^t_{tg} = \{ <d_1,d_2> \in D \times D \mid \text{deg}^+(d_1,P,t,g) \geq \text{deg}^+(d_2,P,t,g) \}$
   
   b. $[[> P]]^t_{tg} = \{ <d_1,d_2> \in D \times D \mid \text{deg}^+(d_1,P,t,g) > \text{deg}^+(d_2,P,t,g) \}$
   
   c. $[[= P]]^t_{tg} = \{ <d_1,d_2> \in D \times D \mid \text{deg}^+(d_1,P,t,g) = \text{deg}^+(d_2,P,t,g) \}$

Recall that we call the sets of things whose membership status in a predicate $P$ is known, $[[P]]_{c,g}$, the positive super denotation of $P$ and $[[\neg P]]_{c,g}$, the negative super denotation of $P$.

(2) Super-denotations:

   $\forall c \in C, \forall g \in G, \forall P \in \text{TERM} \cup \text{CONCEPT}:$

   \[
   [[P]]_{c,g} = \cap \{ [[P]]^+_{tg} \mid t \geq c \}
   \]

   \[
   [[\neg P]]_{c,g} = \cap \{ [[P]]^-_{tg} \mid t \geq c \}
   \]

Using a full vagueness model, Sassoon (2002) argues that the gradable structure of predicates, for instance tall, reflects the order in which entities are learnt to be denotation members (tall or not-tall), directly or by inference, through contexts and their extensions. I adopt this view.

(3) The learning constraint: $\forall g \in G, \forall t \in T \forall d_2,d_1 \in D: \quad$

   $<d_1,d_2> \in [[\geq P]]^t_{tg}$ (d_1's degree in P is at least as big as d_2's degree in t and g)
   
   iff d_1 is added to P's super-denotation at an earlier stage under t:

   In any context c leading to t, if d_2 is P, so is d_1, and if d_1 is $\neg P$, so is d_2.

   \[
   (\forall c \leq t: (d_2 \in [[P]]_{c,g} \text{ or } d_1 \in [[P]]_{c,g}) \quad \& \quad (d_1 \notin [[\neg P]]_{c,g} \text{ or } d_2 \in [[\neg P]]_{c,g}). \quad )
   \]

Imagine again that in c I know Sam's height and I know that it reaches the standard (whatever it is), but I do not know Dan's height. In such a case, Sam is already known to be tall, but Dan is not. Does the learning principle wrongly predict that Sam is regarded as taller? No. We have to be careful about whether we discuss individuals or (referents of) proper names (these can be unknown). The learning principle describes individuals. Let us work out the truth conditions of statements with $\geq P$ applied to terms.

(4) $\forall \alpha, \beta \in \text{TERM}, \forall g \in G, \forall t \in T:$

the pair of $\alpha$ and $\beta$'s referents falls under $\geq P$ in t and g) iff for g:

   In any context c leading to t:

   If in c, $\beta$'s referent in t is P, so is $\alpha$'s referent in t

   \[
   (\text{if } [[\beta]]_{tg}^+ \text{ is } P \text{ in c, then so is } [[\alpha]]_{tg}^+ \text{ in c}),
   \]

   and if $\alpha$'s referent in t is $\neg P$, so is $\beta$'s referent in t

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(if $[[\alpha]]_{tg}^t$ is not $P$ in $c$, then so is $[[\beta]]_{tg}^t$ in $c$).

$(\forall c \leq t$: $([[\beta]]_{tg}^t \not\in ([P])_{c,g} \text{ or } [[\alpha]]_{tg}^t \in ([P])_{c,g}$) \\
\& $([[\alpha]]_{tg}^t \not\in ([\neg P])_{c,g} \text{ or } [[\beta]]_{tg}^t \in ([\neg P])_{c,g}$.

) iff for $g$: In any context $c$ leading to $t$:

If \[ in any total extension $t'$ of $c$, $\beta$'s referent in $t$ is $P$ in $t'$],
also \[ in any total extension $t'$ of $c$, $\alpha$'s referent in $t$ is $P$], and
If \[ in any total extension $t'$ of $c$, $\alpha$'s referent in $t$ is not $P$ in $t'\],
also \[ in any total extension $t'$ of $c$, $\beta$'s referent in $t$ is not $P$]:

$(\forall c \leq t$: $(\neg \forall t' \geq c$: $([[\beta]]_{tg}^t \in ([P])_{t,g}$) or $(\forall t' \geq c$: $([[\alpha]]_{tg}^t \in ([P])_{t,g}$) \\
\& $(\neg \forall t' \geq c$: $[[\alpha]]_{tg}^t \in ([P])_{t,g}$) or $(\forall t' \geq c$: $[[\beta]]_{tg}^t \in ([P])_{t,g}$.

For example, Dan's height is bigger than (or equal to) Sam's height in $t$ and $g$ (and so $Dan$ is at least as tall as $Sam$ is true in $t$ and $g$) iff in $g$ and any $c$ leading to $t$, (i) either the entity that forms Sam's referent in $t$ (let us call it $d_{Sam,t}$) is not yet known to be tall in $c$, or the entity that forms Dan's referent in $t$, $d_{Dan,t}$, is already known to be tall in $c$, and (ii) either the entity that forms Dan's referent in $t$, $d_{Dan,t}$, is not yet known to be not-tall in $c$, or the entity that forms Sam's referent in $t$, $d_{Sam,t}$, is already known to be not-tall in $c$.

Given that for any $g$ and $t$, $[[=P]]_{tg}^t = ([[\geq P]]_{tg}^t \cap [[\leq P]]_{tg}^t)$, it is derived that Sam and Dan are equally tall iff in any context leading to $t$, if Sam is tall, so is Dan, and if Dan is non-tall, so is Sam, and vice versa (they are always learnt to be members or non-members in the very same contextual stage). Given that for any $g$ and $t$, $[[>P]]_{tg}^t = ([[\geq P]]_{tg}^t - [[=P]]_{tg}^t)$, it is derived that Dan is taller than Sam in $g$ and $t$ iff $[[[Dan]]]_{tg},[[Sam]]_{tg} \in ([[\geq tall]]_{tg})$ and $[[[Sam]]]_{tg},[[[Dan]]]_{tg} \not\in ([[\geq tall]]_{tg})$, i.e. iff in any context leading to $t$, if Sam is tall, so is Dan, and if Dan is non-tall, so is Sam, but not vice versa: In some context leading to $t$, either Dan (but not Sam) is (already known to be) tall, or Sam (but not Dan) is (already known to be) non-tall:

\[
[[Dan$ is taller than $Sam]]_{tg}^t = 1 \text{ iff } \forall c \leq t$: $[[[Dan]]]_{tg}^t \not\in ([[tall]])_{c,g}$ or $[[[Dan]]]_{tg}^t \in ([[tall]])_{c,g} \text{ and: } \\
\exists c \leq t$: $[[[Dan]]]_{tg}^t \in ([[\neg tall]])_{c,g}$ or $[[[Sam]]]_{tg}^t \in ([[\neg tall]])_{c,g}$.

Let us work out truth conditions for partial contexts too:

\[
\forall g \in G, \forall c \in C: \\
[[\alpha \geq P \beta]]_{tg}^t ("\alpha" \text{ is at least as P as } \beta" \text{ in } c \text{ and } g) \quad \text{iff:} \\
(\forall t \geq c$: $<[[\alpha]]_{tg}^t,[[\beta]]_{tg}^t> \in ([[\geq P]]_{tg}^t)$

(Any $t$ above $c$, $\alpha$ and $\beta$'s referents fall under $\geq P$ in $t$ and $g$)

$\forall t \geq c$: $\forall c' \leq t$: $(\neg \forall t' \geq c': [[\beta]]_{tg}^t \in ([P])_{t,g}$ or $(\forall t' \geq c': [[\alpha]]_{tg}^t \in ([P])_{t,g}$)

\& $(\neg \forall t' \geq c': [[\alpha]]_{tg}^t \in ([P])_{t,g}$ or $(\forall t' \geq c': [[\beta]]_{tg}^t \in ([P])_{t,g}$)

For example, Dan's height is bigger than Sam's height in $c$ and $g$ (and so $Dan$ is taller than $Sam$ is true in $c$ and $g$) iff in any $t$ above $c$, Dan's height is bigger than Sam's height in $t$ and $g$ (and so $Dan$ is taller than $Sam$ is true in $t$ and $g$).
Nominal predicates are usually thought of as having an additional interpretation as kind-denoting terms (type shift by a down operator, $\cap$, results in a kind reading). According to Chierchia (1998), in each world (or total context) $t$, these terms denote the plural individual that is the sum of all the $P$ instances in $t$. In any $t$ and $g$ some $d$ in $D$ is $P$'s kind. Let us call it $d_{r,t}$. For example, $[\cap robins]^+_{t,g} = d_{r,t}$ and $[\cap ostriches]^+_{t,g} = d_{o,t}$. The individuals $d_{r,t}$ and $d_{o,t}$ are bird sub-kinds, where sub-kinds are individuals in $D$, just like the referents of Tweety and Tan, except that they are plural individuals. These are the predictions of the learning principle:

$$\forall g \in G, \forall c \in C:$$

$$[[\geq bird(robins, ostriches)]^+_c]_{c,g} = 1$$  (Robins have a higher degree in $bird$ than ostriches in $c$ and $g$)  iff:

$$(\forall t \geq c): <[[[robins]]^+_t,g,[[ostriches]]^+_t,g] \in [[\geq bird]]^+_t,g$$

(Any $t$ above $c$, robins have a higher degree than ostriches in $t$ and $g$)  iff:

$$(\forall t \geq c): \forall c' \leq t: ([[ostriches]]^+_t,g \notin [[bird]]^+_c,g \text{ or } [[robins]]^+_t,g \in [[bird]]^+_c,g) \text{ & }$$

$$(d_{o,t} \notin [[bird]]^+_{c,g} \text{ or } d_{r,t} \in [[bird]]^+_{c,g})$$

(In any $t$ above $c$, in any $c'$ under $t$, either ostriches are not yet known to be birds in $c'$, or robins are already known to be birds in $c'$ and either robins are not yet known not to be birds in $c'$, or ostriches are already known not to be birds in $c'$)  iff:

$$(\forall t \geq c): \forall c' \leq t: (d_{o,t} \notin [[bird]]^+_{c,g} \text{ or } d_{r,t} \in [[bird]]^+_{c,g}) \text{ & }$$

$$(d_{r,t} \notin [[\neg bird]]^+_{c,g} \text{ or } d_{o,t} \in [[\neg bird]]^+_{c,g})$$

(In any $t$ above $c$, in any $c'$ under $t$, either (not in any $t'$ above $c'$ ostriches are birds in $t'$), or (in any $t'$ above $c'$ robins are birds in $t'$) and either (not in any $t'$ above $c'$ robins are not-birds in $t'$), or (in any $t'$ above $c'$ ostriches are not-birds in $t'$)

I.e. in any $c'$ above or below $c$, any entity pair $d_{r,t}$ and $d_{o,t}$ that forms the references of $robins$ and $ostriches$ respectively in some $t$ above $c$, is such that: (i) Either $d_{o,t}$ is not yet known to be a bird in $c'$, or $d_{r,t}$ is already known to be a bird in $c'$, and (ii) Either $d_{r,t}$ is not yet known not to be a bird in $c'$, or $d_{o,t}$ is already known not to be a bird in $c'$. If, for instance, I get to know that robins are birds, and only later on I get to know that ostriches are birds, then, be the individuals that form the referents of $robins$ and $ostriches$ what they may, for any $t$ above $c$, the former is added earlier under $t$ to the denotation of $bird$, compared to the latter.

This prediction is borne out by the facts. The learning principle is supported by robust empirical facts which in 2.2 I have called learning-order effects. In the rest of chapter 8 I explicate the positive consequences of the learning proposal, and other issues that may be related to it.

8.2 Typicality is tightly coupled with learning-order.

Interestingly, robust empirical findings show that the mapping of entities to degrees is tightly coupled with the order in which items are learnt to be denotation members, whether directly or
by inference (for a review see Mervis and Rosch 1981: 97-100, who claim that these learning-order effects are among the most robust typicality effects). Developmentally, children tend to learn the typical members of natural concepts earlier. For example, children are able to classify newly encountered typical animals like wombats and anteaters, when they are not able to classify more accessible, yet atypical animals (from the child's perspective) like butterflies or ants as being 'animals' (Anglin 1977). Children learn the good examples of, for instance, basic color concepts, before learning the poor examples (Mervis and Rosch 1975). This phenomenon is replicated in experiments with invented concepts (for instance, toy concepts; Rosch 1973; Mervis and Pani 1980).

Why do we find these learning effects? A child who possesses enough knowledge about the dimensions of a concept may be able to tell that certain items, objects that she encounters, or the referents of certain terms she hears, reach threshold, in which case she can automatically infer that they fall under that concept. (All the individuals d in D which may still be that object or the referent of that term now become members of her positive super-denotation of that concept.) Now it would often be easier for a child to tell of a given typical example of the concept that it reaches threshold in that concept, than it would be for her to tell of a given atypical exemplar that it reaches threshold. For instance, encountering a hairy, four-legged, tail-wagging, barking creature (i.e., a creature with a high average in the dimensions of dog), a child may be able to tell that that creature reaches threshold in dog, and therefore, that it is a dog (she then adds all the individuals d in D that might still be that creature to the positive super-denotation of dog). The same child, on encountering a bald, unusually small, and silent dog (with a low average in the dimensions of dog), may not be able to tell that it reaches threshold, and therefore not classify it as a dog. So the child will infer about more typical members of a concept that they are members (i.e., add them to to her super-denotation of that concept) before she can infer about less typical members that they are members. This creates a correlation between typicality and the order in which items are learned to fall under the super-denotation of a concept.

Experiments show also that the coupling between typicality and learning orders persists in adults. For example, they were found in adult learning of form concepts (in cultures that do not posses them; Rosch 1973), and of invented concepts such as dot patterns and stick figures (Rosch and Mervis 1975; Rosch, Simpson and Miller 1976; Mervis and Pani 1980).

8.3 Acquisition of predicates' interpretation is based on the early acquired entities

I propose that when the degree function is vague, speakers use the learning principle as a rule that helps reduce the space of possibilities as to which degree function the predicate denotes. This is of crucial importance in nominal concepts.

Experiments show that when children (or adults) do not possess knowledge about the degree function of a noun (they cannot calculate entities' mean degrees on the noun dimensions), they base their choice of a dimension-set on the earliest acquired items. Acquisition is delayed (is slower) with early exposure to bad examples (items with low means on the actual dimensions), or even with exposure to the whole category in random order, compared to acquisition rates with early exposure to good examples (items with high means on the actual dimensions; Mervis and Pani 1980). I propose that the early acquired items are assumed to be the best on the category dimensions, as predicted by the learning principle. Therefore, their properties are regarded as dimensions and their values on these properties are regarded as the category's selected values.
When exposure is to an atypical item, the inferred dimensions and values are wrong and need to be corrected later on. This delays acquisition. Let us demonstrate this with examples.

What happens when, Say, Sam's teacher shows her a robin and tells her that this is a bird? Sam can tell that there is an entity in front of her, but this entity is only partially accessible. Sam can see the entity's shape and colors, she can feel her texture or odor, see the manner of her movement, etc., but she may not know many other properties or property-values pertaining to the entity (its weight, what it is made of, how it behaves or functions, what its evolutionary origins are, etc.) Thus, Sam does not know which one of a whole set of possible entities is standing right next to her (for many entities, if they had been standing in front of her instead of the actual bird she sees, she would not have been able to recognize the difference). Moreover, her teacher cannot tell her all the facts about the entities she sees, because the entities are only partially accessible to her teacher, too. But in spite of her partial knowledge, Sam's teacher positively classifies the entity as a bird. For that reason, Sam can presume that classification depends on the dimensions that are accessible to her and her teacher. This means that any of the entity's accessible properties may be part of the dimension-set of bird, the entity's values may form the selected values, and the mean of these values may form the cutoff-value (standard). Thus, the membership of things that deviate from these values even a bit is still questioned. If Sam is told about several examples of robins simultaneously that these are birds, she can probably eliminate from the dimension-set any of the properties along which these robins differ from one another. They are now known not to be necessary (or almost necessary) conditions for membership. In addition, they may also be taken not to play a role at all in determining typicality, since items with different values in these dimensions are added to the super-denotation earliest, which means that they are supposed to all be the best examples (equally good examples). There is little likelihood that all these entities will end up having equal means (equal typicality degrees in bird), unless all those dimensions in which some of them have lower degrees than others are ignored.¹

If at a later stage, Sam is told that a certain pigeon that she sees is a bird too, she can tell that the standard values in many potential dimensions that she has in mind are not the tightest possible. Perhaps she can also tell that properties in which the pigeon scores better than the robin are not bird dimensions or are dimensions with low weights. How can she tell that, for instance, cooing like a pigeon is not a bird-dimension? For all she knows, most birds may coo. After all, even if the learning principle tells her that robins are more typical birds than pigeons are, it remains possible that in that particular dimension the pigeon scores better than the robin. Still, the weight for this dimension should be low enough, so as not to render pigeons more typical birds than robins. After a while, Sam can obtain a partial set of bird dimensions (and negative dimensions, i.e. properties that she positively classifies as not being bird dimensions), dimension-values (and negative values), a set of potential weights for the dimensions (and sets of negative weights, i.e. numbers that cannot form the weights for any given dimension), and negative standards (values that she already positively classifies as not forming the standard of membership in bird). These partial sets can adequately predict many facts about membership of entities in

¹ In addition, in principle, if two equally good examples of birds differ on a certain property, but they are completely alike in all other respects, this property cannot be a bird dimension (though naturally, real items usually differ in more than one property that we can detect).
bird. Sam may remain uncertain only concerning the membership of entities with low rates in the dimensions she obtains.²

What happens if, for example, Sam knows nothing about the characteristics of birds, and her initial exposure to birds is through ostriches? According to my proposal, she will classify items by their similarity to ostriches. That is, she will think that the ostrich is a representative bird, and that its known dimensions and dimension-values (running, ostrich size, etc.) are the dimensions and selected values of the concept bird. But Sam is not doomed to "remain in the cave" for ever. Mean functions are such that, all other things being equal, an increase in one of the values increases the mean. Thus, our proposal that nominal concepts are linked with mean functions (the prototype approach) predicts that, all other things being equal, the more typical an entity is of a certain dimension (say, flying), the more typical this entity is of the concept (bird). Thus, in initial exposure to ostriches, Sam will expect that (all other things being equal), entities which are, say, more typical runners (given that running is a dimension of ostriches), will be more typical birds. These inferences will be canceled later on, when non-birds (or poor examples of birds) will be discovered to be better on this dimension than (good examples of) birds. Sam is taught about many flying and non-running animals that they are birds. She may, initially, wrongly consider them atypical birds. But she is also taught about many running and non-flying animals that they are not-birds. These non-birds are more similar to the examples that she wrongly considers typical birds than to the examples that she wrongly considers atypical, in these, and maybe also other, dimensions (and consequently, in their weighted means). This eventually forces her to abandon her initial assumption that ostriches are the prototypical birds. The role of frequency in such cases is discussed below. Deference may enhance this decision too (if indeed Sam's language community considers ostriches non-representative).

But this process will slow down acquisition. In fact, in certain children, early exposure to atypical members completely blocks acquisition, at least within the experiment time (Mervis and Pani 1980). They refuse to abandon inferences that are based on the early-acquired members. Certain children in Mervis and Pani's (1980) experiment have explicitly told the experimenters that showing them the typical examples first would have been much more helpful (or less confusing).

The prototype theory has triggered the discovery of these facts. Yet, this theory's probabilistic criterion (the view that dimensions are inferred from their observed frequency within and outside the concept, cf. 4.3) does not go well with the fact that the typical examples have facilitated acquisition more than exposure to the whole concept has. Nor can the 'knowledge' criterion for dimension selection (cf. 4.3) explain these findings (the subjects in this study possessed no prior knowledge about the dimensions). Only a criterion which states that properties (and property values) of the early acquired entities are selected for the dimension set, as the learning principle predicts, directly explains the data (Sassoon 2006).

Frequency of occurrence, or likelihood of membership, does play a role in acquisition, when it comes to error corrections. Corrections occur when one is faced with two contradicting hypotheses about the facts. For example, if one has been exposed to the P-hood of an item i₁ (e.g. the bird-hood of ostriches) earlier than to the P-hood of i₂ (the birdhood of pigeons or chicken), and at the same time has discovered that i₂ scores more highly in the set which one presumes to be the set of

² Most likely, it is not always logical inferences that the child makes. She may just take guesses that are in line with what she knows, and learning orders may strongly bias these guesses (e.g., dimensions of pigeons may be eliminated simply because robins are the earliest acquired members and so their dimensions are favoured).
typicality dimensions of P (e.g. flying, perching, etc.), what would one infer? Would one eliminate the problematic dimensions from the dimension-set and infer that indeed i₁ is more typical than i₂, or would one leave the dimension-set as is, and infer that i₁ is not more typical than i₂? In such cases, considerations other than the learning order may be called for. For instance, if one encounters P members which are similar to i₂ (pigeons) exceedingly more frequently than P members which are similar to i₁ (ostriches), the latter option (that i₁ is not more typical than i₂) might seem more attractive. However, if one observes that the 'problematic' dimensions are significantly less frequent in P than in other categories (e.g. flying is more frequent in birds than in mammals, and running more frequent in mammals than in birds, but if it was the other way around), or more precisely, if one does not observe a particularly good correlation between items' known P degrees and their known degrees on what he presumes to be the category dimensions), then the first option, namely the assumption that in fact i₁ is more typical than i₂ may seem more attractive. In this way, frequency does play a role, albeit a less central one, in the determination of typicality judgments.

My hypothesis about the role of frequency is reminiscent of Markman's (1989: 215) view of the role of frequency in the acquisition of novel predicates. For example, Markman suggests that children assume a principle of mutual exclusivity, according to which each object has but one label. When encountered with a novel label but no novel object which it may label, mutual exclusivity may cause the child to infer that the label is related to a part of the object, or to one of its dimensions (color, size or so on). However, Markman does assume that mutual exclusivity may be abandoned and a second label accepted, when hearing a second label (animal) applied repeatedly to an object with a known label (dog).

Thus, the learning-order bias is the default strategy. When no learning-order cues are available (that produce consistent information structures), for instance in experiments in which all the category members are presented simultaneously or in a random order, the learning-order bias is abandoned (otherwise, acquisition is blocked).

8.4 Classification of typical entities by inference

Once the dimensions and selected values are set, they are used in order to infer facts about membership of new items. At that stage, the actual order in which items are directly learnt to be members becomes irrelevant. Which does not mean that the learning principle ceases to apply. What happens is that a newly-encountered item that is good enough in the dimensions is automatically regarded as a member, and it is regarded as an already known member.

First, studies of damaged neural network simulations and of aphasic patients (Kiran and Thompson 2003 and references therein), show that following training with sets of typicality dimensions, exposure to atypical items results in spontaneous recovery of categorization of untrained typical items, but not vice versa. Exposure to typical items does not result in recovery of categorization of untrained atypical items. The membership of the typical items is inferred from the membership of the atypical ones, but not vice versa. We see that, even when facts pertaining to the membership of atypical items are directly learnt (or taught) before facts pertaining to the membership of more typical items, knowledge of the category dimensions allows inferring the latter facts from the former, and not vice versa. Facts pertaining to the membership of typical instances do not license inferences regarding the membership of less typical instances. Again, we can say that the membership of typical instances (in the positive super-denotation) is acquired earlier. Typical items (like different types of birds that are small, that sing, perch and so on) are good in the known dimensions so they are automatically classified as birds. Other items
that a speaker encounters and whose mean degree in the dimensions is not higher than that of items that are already known to be members are not automatically classified.

Second, in healthy adults, often typical items which are seen for the first time are falsely thought to already be known (Reed 1988). For example, participants presumed to have identified criminals in a line-up, who in truth they never saw before, only because they obtained characteristic dimensions of the given category of criminals (for further discussion of learning-order effects related to inferences and memory see 2.26). Why? Given that the newly encountered items are typical, and that typical items are classified relatively early (earlier than less typical ones), once encountered, typical items are presumed to already be known (classified).

8.5 Familiarity effects

This section does not present conclusive evidence for our proposal, but it suggests that the learning principle may explain our reactions to unfamiliar items.

Sometimes speakers are unfamiliar with an item (a referent of a proper name or a kind-name). In Hampton (1998)'s study of about 500 items in about 20 categories, familiarity was one of the three main factors producing dissociations between typicality degrees and membership likelihood (number of subjects that classify a given item as a member). Instances that average highly in the dimensions of a concept may nonetheless receive low typicality ratings, if subjects are not familiar enough with them (with their values on the dimensions). For instance, Flemish is considered to be a less typical language than English, despite their similarity (Larochelle, Richard and Soulieres 2000). To take another example, when the fruit Nectarine arrived in Tel Aviv, nobody had any doubt that it was a fruit, because it has the typical properties of fruits, and is in fact very similar to the (very typical) peach. And yet it was unfamiliar, and so it was (and for some still is) considered less typical than highly similar yet more familiar fruit types (such as peach).

Ratings of familiarity of items may stem from a variety of factors, such as frequency of exposure to items with the same or with similar property values, and acquaintance with (knowledge or lack of knowledge about) the item's properties and property values. Can the learning principle shed light on how these factors might affect typicality judgments?

First, items can be unfamiliar because, unlike typical items, we usually do not get in contact with them or get reminded of them very often. For instance, because they are usually hidden, they can only be found far away, or because they are so expensive that most of us never get near them (cf. examples of unfamiliar items are exotic fruits or species, languages like Flemish, far away planets, etc.) These features may count against them and reduce their degree in their category. If with time this changes (Nectarines used to be exported from far away, once, but today Nectarines grow everywhere), our judgments concerning their typicality may change. Multi-dimensional concepts differ from one-dimensional concepts in this respect. Intuitively, in predicates like tall, we will not be willing to infer based on the learning principle that Sam is taller than Dan, based on the fact that we already know that Sam reaches threshold, but we do not yet know Dan's height. Such predicates are known to be one-dimensional. Thus, no hidden dimensions can be assumed to be part of their dimension-set, and to reduce Dan's degree. But this is not the case for multi-dimensional categories. In such categories you can always assume that additional properties form category dimensions, as long as this assumption (the new mean function that it produces) preserves the known ordering relations between items. If all the known fruit types are grown locally, then this property can be added to the dimension-set and reduce the typicality of
unfamiliar items that are only found far away. (So from their late acquisition we infer that these unfamiliar items are atypical and that their exceptional properties reduce their typicality).

Second, regarding cases in which speakers do not know the item's degrees (or selected values), in properties (dimensions of predicates like bird, fruit, language, etc.), these are cases in which the speakers cannot calculate the item's degree in these predicates (nor can they automatically infer that it is a members or a non-member). Interestingly, in such cases, even if they are directly taught that the item is a member of the positive super-denotation, speakers infer that the item is atypical of the respective predicate (the mean of whatever entity it may end up being is smaller) compared to whatever entity known items (members) may end up being. Why? These speakers have already learnt about the membership of many other items, and they can infer the membership of many items, but they cannot yet infer that the given item is in the positive super-denotation. This is the case for items whose degrees are too small to be positively classified as reaching the standard (smaller than the degree of known members). The membership of such items can only be taught directly, not by inference. Presumably, speakers wrongly generalize that the given unfamiliar item, like these atypical items, has a small degree (rather than that it still lacks a degree). From the lack of knowledge of the item's status in the category dimensions, speakers may infer that its degree is small on these dimensions, and hence that its degree in the category is small. In such a way, from its late acquisition, they infer that the unfamiliar item is a less-typical individual. It is but natural that given the learning principle, speakers presuppose that they know all the existing typical items (as once they see them they immediately recognize them as category members), so they reason that newly encountered items whose classification is not automatic must be atypical (they must come from far away and have different characteristics). This reasoning may occur in particular in classifications of sub-kinds. For instance, if speakers assume that there is a finite list of edible fruit types, and they know almost all of them, in encountering a new kind term, Nectarine, they will infer that it is atypical regardless of most of the actual characteristics of its exemplars. In fact, two Hebrew speaking informants did not know what Nectarine is. Even when I told them that it is a fruit type very similar to a peach (or a type of peach), they have insisted that it is atypical (because otherwise they would have known it; because it is an exotic fruit, etc.), though at the same time they have admitted that if I show them a specific Nectarine they will probably say that it is typical (presumably because the accessible dimensional values of the object they will see will be very close to the values of a peach).

In sum, according to the learning constraint, delays in classification (or classification at a stage in which most of the existing category members – say, fruit types – are assumed to already be known) reflect low typicality. Thus, it is inferred that the item is less typical than known members (and hence, atypical). In addition, when people rate unfamiliar items they can assume that some known dimensions reduce the items' mean typicality (e.g. grows locally). In one-dimensional adjectives with known dimensions speakers cannot make this assumption. In nouns (or other multi-dimensional predicates) they can, because they are vaguer with respect to the set of dimensions, their weights, etc.

8.6 Typicality and proper names: First impression effects

The discussion in most of this section is not directly related to the learning principle, but to gradability in proper names and other terms. This is an interesting topic in its own right, but I specifically discuss it here because it is crucial for the presentation of the first impression effect, an effect which provides further evidence for the learning principle.
Typicality effects occur not only with (expressions denoting) categories (predicates), but also with (expressions denoting) individual concepts (terms). Despite the pervasiveness of this phenomenon, it is rarely accounted for in the experimental literature. Let us look at some examples. In the following sentences, we see that the property \textit{smokes Nobles} counts as a dimension of \textit{John}.

\begin{enumerate}
\item Smoking Nobles is typical of \textit{John}.
\item Typically, \textit{John} Smokes [Nobles].
\end{enumerate}

An entity ordering is also sometimes associated with individual concepts. In the following examples we see that there are typicality orderings with respect to \textit{John} – some things are more (or less) typical of \textit{John} than others.

\begin{enumerate}
\item This is not the typical \textit{John}.
\item John right now is more typical of himself than \textit{John} an hour ago.
\item When \textit{John} is angry, he is not really himself.
\end{enumerate}

But \textit{John} is a term (it is an expression that denotes a single individual in any given situation), while typicality is something that is related to sets of entities (where some are more typical than others in given dimensions). What is it that is being ordered in (8c)-(8d)? Well, entities such as situations with \textit{John}, or temporal stages of \textit{John}. For example, (8c) can be understood as conveying that the current situation with \textit{John} is not a typical situation with \textit{John}. (8d) can be understood as conveying that the current temporal stage of \textit{John} is more typical than the stage of \textit{John} an hour ago.

Furthermore, the statement in (9a) is about the typicality relation that holds between different perspectives of \textit{Sam}, \textit{Sam as a secretary} and \textit{Sam as a doctor}.

\begin{enumerate}
\item \textit{Sam} is less typical of herself as a secretary than as a doctor.
\item Dan in the morning(s) is less typical of himself than (Dan) late at night
\end{enumerate}

Bartsch (1986) and Landman (1989) show that an individual may possess different properties when it is regarded under different perspectives. For example if \textit{John} is both a judge and a hangman than \textit{John as a Judge} is on strike might be true and \textit{John as a hangman} is on strike false at the same time. Indeed, focus on a term might express a contrast between different perspectives or restricted aspects of the referent. For example, the following is a plausible discourse between a woman soldier \textit{B} and a friend of hers \textit{A}. While \textit{A} is asking about \textit{B} as a citizen, \textit{B} wrongly answers about herself as a soldier:

\begin{enumerate}
\item A. What do you usually wear?
\item B: A B-type Uniform.
\item A: No, what do [YOU]$_F$ usually wear?
\item B: oh, I wear short skirts and….
\end{enumerate}

Our different perspectives on (or the different aspects of) the referent of a term like \textit{John} or \textit{Sam} can be seen as restricted-individuals. But what are restricted individuals? Researchers regard them as either sets of properties, i.e. of the type of generalized quantifiers (Bartsch 1986;
Landman 1989) or individuals in their own right (Landman 2000; see also Chierchia and Turner 1988 for a theory of properties as either functions or individuals). Landman (2000: 163) observes that the latter view is no more of an ontological extravagance than the first view, given that the set of properties that make up a restricted individual ought to be an ultrafilter. I will not commit myself to any specific analysis of restricted individuals. Future research has to explicate the interpretation of expressions like these. But I propose that perspectives or aspects of an individual may be regarded as sub-categories. Perhaps the category of Sam situations has sub-categories, namely categories of situations with Sam as something (a doctor, a secretary etc.), of situations with Sam in different times of the day (in the morning, in the evening, etc.), in different ages, etc. I.e. the hierarchical relations "sub-category of" or "sub-kind of" that intuitively represent relations between concepts hold of categories or kinds that are based on individual-concepts (terms).

(11) Possible interpretations for 'animal':
   a. animal
   b. bird
   c. robin
   d. the northern gray robin
   e. the northern gray robin standing in front of me

(12) Possible interpretations for 'Sam':
   a. Sam
   b. Sam as a doctor
   c. Sam as a doctor working nights
   d. Sam as a doctor working nights ten years ago
   e. Sam as a doctor working nights ten years ago on the 9th of September.

In fact, different dimensions may be typical of the category of stages with Sam, of its sub-categories (Sam in the evening, Sam in the morning, Sam when she is angry…), etc. Consider, for example, the typicality ordering of sub-individuals of an individual (i.e. of an individual when restricted in different ways). Properties which are variant across different sub-individuals (but are steady within the occurrences of any given sub-individual) can be regarded as typical or atypical of the individual. For example, if Dan has the habit of smoking Nobles at night, and we take Dan at night to be more 'himself', we can understand smoking Nobles is typical of Dan as stating that categories of situations with Dan at night (which are more typical of having the habit of smoking-Nobles compared to Dan in the morning, Dan at noon, etc.), are more typical of the category of situations with Dan in different times of the day (all other things being equal). The habit of Smoking Nobles is typical of the category (set) of categories of situations with Dan in different times of the day. In just the same way, expressions like Sam as a doctor and Sam as a secretary may be denoting different sets of situations with Sam (secretary situations and doctor situations) – sub-categories of (the large category of situations with) Sam. In sum, one benefit of looking into the gradability effects in terms is that we see that we have means of representing a phenomenon that has been dealt with before (perspectives of individuals) in a way that unifies it with phenomena observed with predicates (categories).

In general, the internal of-argument of typical can be occupied by a predicate, a proper name, a pronoun, or a quantified noun phrase:

   e. Smoking Nobles is typical of a Kibbutz member
f. It is typical of John that when you tease him, he immediately gets insulted

g. Being late is typical of him

h. Financial problems are typical of every student

But, as I said earlier, typicality is something that is related to sets of entities (where some are more typical than others in given dimensions). I propose that a relation R between entities (like "\(\lambda x \lambda s. s\) is a situation with \(x\)", "\(\lambda x \lambda s. s\) is a time interval with \(x\)", etc.) is always accommodated in such statements, and when it takes the term denotation as an argument the result is a predicate (like "\(\lambda s. s\) is a situation with Dan", etc.). This predicate forms the of argument of typical. Thus typicality statements are inherently context dependent also in the sense that different relations may be accommodated in each context of use. For example, one possible interpretation for it is typical of John that when you tease him, he immediately gets insulted or for Typically, when you tease John, he immediately gets insulted is "all other things being equal, situations in which you tease John and he immediately gets insulted, are more typical than other situations with (you teasing) John". I.e. the property situation in which you tease John and he gets insulted is typical of the predicate situation in which you tease John. Here are several other examples of such interpretations:

(13)

a. Smoking Nobles is typical of (situations with) John

b. \(\forall x, \text{student}(x):\) Financial problems are typical of (time intervals with) \(x\)

Typicality statements with predicates are also ambiguous in the same way. An analysis of the of argument as a kind-denoting term and an accommodation of a relation (like "\(\lambda x \lambda k. x\) is an instance of \(k\)" or "\(\lambda x \lambda k. x\) is a sub-kind of \(k\)", etc.), that produces a predicate like ("\(\lambda k. x\) is a sub-kind of \(k\)"), can account for this ambiguity:

c. Smoking Nobles is typical of (instances of) \(\cap \text{Kibbutz member}\)

d. Flying is typical of (sub-kinds of) \(\cap \text{bird}\)

e. Arriving late is typical of (stages of) \(\cap \text{Kibbutz members}\)

The metaphysical apparatus (ontology) assumed by different semantic theories becomes ever richer, including points in time, periods, events, situations, worlds, intervals, paths and so on and so forth. For example, statements in Event theory, (before closure by the discourse existential operator), denote sets of events. Predicates over events may induce ordering like any other predicate. Their dimensions must be event predicates themselves (by applying existential closure, they may become entailments, implicatures or presuppositions). I propose a unified analysis for typicality in different category types, and hence in this section I use the term "situation" or "stage" as a general term for indices like time points, events, periods etc., so as not to take a stand on questions such as whether statements (or other expressions) should be analyzed as sets of stages (as in Carlson 1977), or events (as in Landman 2000), or situations (as in Kratzer 1989), etc.

On my proposal, typicality represents the order in which entities are classified under super-denotations of categories, where super-denotations are entity sets. Membership may be based on inference. According to McCready and Ogata (2007) Japanese has an adjectival entry 'rashii', whose meaning corresponds roughly to the meaning of the adjectives typical or characteristic in
English, except that, to take an example, sentence (14a) below entails that the given temple belongs to the category 'Kyoto'.

(14) a. Kono tera-wa honotooni Kyooto rashii
This temple-Top really Kyoto RASHII
This temple is really typical Kyoto

Interestingly, this entry has an additional use as an inferential evidential. For example, in (14b), the use of 'rashii' indicates that the speaker bases her statement indirectly, via inference (McCready and Ogata 2007).

b. Konya Jon-ga kuru rashii
Tonight john-Nom come RASHII
John is coming tonight (it seems)

While McCready and Ogata 2007 fail to find any connection between these two uses of 'rashii', given the learning principle, the connection becomes quite obvious. A speaker uttering (14a) asserts that a given temple must belong to the category 'Kyoto' based on the fact that it satisfies the typicality dimensions to a sufficient degree and hence this categorization fact can be inferred. A speaker of (14b) asserts that John must come in the given night, based on the fact that, John-tonight satisfies the typicality dimensions of "coming tonight" to a sufficient degree and hence this categorization fact can be inferred (or the time / event that is being discussed satisfies the typicality dimensions of "λe. e is tonight and John is coming in e" to a sufficient degree and hence this categorization fact can be inferred, or… etc.) This Japanese entry provides further evidence for the connections between typicality and inferences about categorization.

An additional and important benefit of looking into the gradability of proper names is that once you realize the gradability structure of proper names, you can check and see that the predictions of the learning principle are supported in this area of data as well. The first impression that an individual makes on other people affects their opinions concerning this individual most. For example, if my earliest experiences with Dan were situations in which Dan was late, I am likely to characterize the category of Dan-stages by the stage-dimension Dan is late. Situations with Dan being late will then be regarded more typical than similar situations in which Dan is not late. On the other hand, if my earliest experiences with Dan were such that he arrived on time, the stage-dimension (times in which) Dan is late may not be regarded as typical of Dan-stages, even if Dan will later arrive late from time to time. Situations with Dan being late will then be regarded less typical than similar situations in which Dan is not late.

The learning principle predicts that the earliest experiences with an item would be regarded as typical of (experiences with) it. Given the natural assumption that terms can be interpreted as categories of situations or stages (when fed into a relation like "λxλs. s is a situation with x", as explained above), the learning principle immediately accounts for the first impression effect.

Definite noun phrases refer back to a discourse entity that was already introduced. I proposed that this is also the case for discourse entities that are sub-kinds. Thus, after utterances like "Flying is typical of ducks / a duck", one can say that, for instance, "The duck(s) move(s) the wings very quickly". In addition, if I continue with "eating insects is typical of ducks / a duck", I may signal that I do not discuss the exact same sub-kinds of ducks. That is, the definite and indefinite article has its usual semantics and pragmatics in typicality and generic statements with predication over sub-kinds.

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8.7 Complex concepts

Usually, semantic theories do not assume that complex concepts (say, conjunctions or modified nouns) are linked with a gradable structure. However, we have seen in 2.2 and 4.2 that people treat asgradable very complex noun phrases. People map entities to degrees in predicates which contain relative clauses, negated sentences, etc. (predicates like pets which are not birds or non-fan, non-entertainment, smart up-market British paper).

On my proposal, dimensions of multi-dimensional adjectives add categorization criteria. So they help to determine the denotations, but they do not directly tell us how the degree function is defined. I am not familiar with a theory that gives compositional rules that completely determine the degree function of a conjunction or disjunction (including, in particular, a conjunction or disjunction of dimensions of a multi-dimensional adjective) based on the degree function of the constituents (the conjuncts or disjuncts), and I do not think that such a theory exists. As for complex nominal concepts, we have seen in 4.2 supporting evidence for the proposal that speakers create prototypes (dimension-sets) for modified nouns, based on a special (non-Boolean) criterion for selecting and weighing dimensions of the constituents (Hampton 1997). The composition criterion is a mathematical function that for each dimension takes its weights in the constituents and returns a new weight. But first, Hampton does not provide us with a unique composition function for every concept. It is nice that we can find such a function for each concept separately, but we still cannot tell how we get to choose the specific function in each concept. Second, Hampton (1997) rejects the assumption that the intersection rule is valid, and this move turns puzzling the fact that necessary (or very important) dimensions of constituents tend to remain necessary for the modified noun. Why should they? Third, Hampton's function cannot derive the weights that are assigned to new (emergent) dimensions that do not characterize any of the constituents (cf. 2.2.7; like lives in cages and can talk for pet birds, like wooden for big-spoon versus metal for small spoon, and like hard for boiled eggs versus soft for boiled potatoes).

What I have proposed in this work is a theory that takes both the intersection rule and the prototype theory to be valid. Such a theory naturally explains the fact that necessary dimensions of constituents tend to remain necessary for the modified noun. In this section, I will show that it also provides an explanation for failures of inheritance of dimensions from constituents to modified nouns and for emergence of dimensions. Thus, this theory explains why we have to resort to functions roughly like the ones discovered by Hampton (1987-1997) for the composition of composite prototypes. In that I show that a theory which incorporates the logical rules is more explanatory than a theory that does not.

Furthermore, given that there are no compositional rules that completely determine the dimension-sets of complex nominal concepts based on the dimension-sets of simple nominal concepts, I argue that the learning principle provides a straightforward way of selecting a dimension-set and selected values (and hence a degree function) for either simple or complex nominal concepts. I also show that the learning principle correctly predicts a variety of findings about negated predicates, and it provides some explanation for findings about ordering effects in multi-dimensional predicates and conjunctions.

This section is to a large extent not directly about the learning principle, but rather about issues concerning complex concepts. However, the discussion in the different sub-sections does
show that the learning principle has an important role in explaining our knowledge concerning the interpretation of complex concepts.

8.7.1 Negated predicates

In 7.3, I have proposed a rule that states that the ordering of entities in positive and negative categories is reversed. Indeed, it is often argued in the semantic and psychological literature (see, for instance, Kamp 1975; Kennedy 1999; Smith et al 1988; Hampton 1997), that the ordering in a negative (or a at least a negated) predicate is the reverse of the ordering in the positive predicate. Smith et al (1988), Hampton (1997), and Levine (2002) have empirically supported the reversed-ordering-intuition. Smith et al (1988) have found that the ordering of instances in non-red fruit is essentially the reverse of that in red fruit (p. 511). Hampton (1997) shows that the ordering of instances in non-P is essentially the reverse of that in P in conjunctions of the forms Q which is P versus Q which is not P.

In the case of negated predicates, the learning principle predicts the reverse ordering rule. This rule follows from the fact that, by the learning principle, an entity d1 is at least as P as d2 iff whenever d1 is P, d2 is P and whenever d2 is ¬P, d1 is ¬P. Thus, an entity d1 is at least as good an example of things which are not-P as d2 iff whenever d2 is ¬P, d1 is ¬P and whenever d1 is ¬¬P (i.e. P), d2 is ¬¬P (i.e. P). The meaning of ¬¬P is identical to that of P in any model MC, because the denotation of not-P is always the complement set of the denotation of P (\([[-P]]^+_tg = [[P]]^+_tg\) and \([[-P]]^-tg = [[P]]^-tg\) in any t in T for any g in G. So \(de[[[-P]]]c,g\) iff for any t above c, \(de[[[-P]]]^+_tg\) iff for any t above c, \(de[[[P]]]^-tg\) iff for any t above c, \(de[[[P]]]^+_tg\) iff \(de[[[P]]]c,g\). Thus, d2 is at least as good an example of things which are not-P as d1 iff d1 is at least as P as d2. It follows that deg(not-P) must always be the inverse of deg(P).

\[\text{(15) The reversed ordering rule for negations: } [[\vdash P]]_t = [[\vdash P]]_t, \] 

\[\text{Proof: } \] 

\[\vdash P]_t^+ = \{ <d_1,d_2> \in D \times D \mid \forall c \leq t: \ (d_1 \in [[P]]c,g \text{ or } d_2 \in [[P]]c,g) \ (d_1 \notin [[P]]c,g \text{ or } d_2 \notin [[P]]c,g) \} \] 

\[\vdash P]_t^- = \{ <d_1,d_2> \in D \times D \mid \forall c \leq t: \ (d_1 \in [[P]]c,g \text{ or } d_2 \in [[P]]c,g) \ (d_2 \notin [[P]]c,g \text{ or } d_2 \in [[P]]c,g) \} \] 

So we see that the learning principle makes an intuitive prediction concerning negations. It is preferred to other theories that also make this prediction like Kamp (1975), because it does not suffer from the problems of these theories.4

4 In 7.1.2, I show that I resolve problems of previous theories by linking a predicate P with a degree function per context and assignment, and by abandoning Kamp and Partee’s (1995) principle, according to which an entity’s degree represents the proportion of total contexts in which it is P. Note now that the learning principle does not create the respective problems. First, according to my analysis (unlike its predecessors), it is perfectly consistent to consider two entities to be two denotation members (or non-members), but not “equally tall”, because one may be added to the positive (or negative) super-denotation earlier (in a context in which the other is not yet in it).

Second, truth values for comparative statements are assigned per total context. Thus, they may vary between total contexts and be unknown in partial contexts.

\(<d_1,d_2> \in [[P]]c,g \text{ iff } \forall t \in T_c: <d_1,d_2> \in [[P]]c,g>\]
As discussed in 7.3, my position regarding negative antonyms is as follows. Negative antonyms (like *non-bird* or *short*) are perceived as such precisely because their ordering is reversed to the ordering of the positive predicate. Yet, unlike negated predicates, the denotation of negative antonyms need not be the complement of the denotation of the positive predicate. It may be more restricted. If the standard of negative adjectives like *sick* (or *short*, *ugly*, etc.) is stricter than the standard of *not-healthy* (which must be equivalent to the standard of *tall* or *healthy*), the expression *sick* (or *short*, *ugly*, etc.) become stronger than the expression *not healthy* (or *not tall*, *not beautiful*, etc.) In order to fall under the negated positive-adjecive (*not healthy*), one's degree in the positive adjective needs to be below standard, but in order to fall under the antonym (*sick*), one's degree in the positive adjective needs to be even lower. This fact may form the basis for the use of negation to induce mitigation (Giora, Balaban and Alkabetz 2004). For example, it is likely that for this reason, people locate *not-beautiful* below the standard of *beautiful*, but not at all in the edge of the beauty scale, where *ugly* is located. I.e. the ordering relation of the negated predicate and the antonym are identical, and it is only their different standards that are responsible for the mitigation effects in tasks that compare a negated predicate and a negative antonym.

As discussed in 7.2.3 (the section concerning nominal domains and intersectivity failures), we can regard cases in which our ordering judgments in concepts and their negations are not reversed as cases in which the negated concept is contextually restricted. Since negated and negative categories (like *not a bird* and *non-bird*) are very large, speakers are likely to often interpret them as contextually restricted (and hence as more informative), by accommodating a constituent like *animate* for *not birds*. My proposal predicts reversed orderings only for predicates and their negations (*birds* and *not-birds*), not for predicate pairs like *birds* and *animals which are not birds*.

### 8.7.2 Conjunction effects and fallacies

Recall that speakers sometimes judge instances to be more typical of, or more likely members of, a modified noun than of the noun or modifier. Cognitive psychologists classify these two judgments as a conjunction effect and a conjunction fallacy, respectively (Tversky and Kahneman 1983; Smith et al 1988; Murphy 2002). Kamp and Partee's (1995) typicality theory has failed to correctly predict these judgments (cf. 3.3), and psychological theories that predict them see them as evidence against the logical rules (cf. 4.2). The learning principle, in contrast, can predict them while being based on logical rules. Let us see how.

The last item which is learnt to be an *apple* in a given context may well be the first item to be classified under *brown apple*. Since, given the learning principle, the classification order reflects

\[
<d_1,d_2> \notin [[\geq P]]_{t,g}^T \iff \forall t \in T_c: <d_1,d_2> \notin [[\geq P]]_{t,g}^{t,c}
\]

unknown otherwise

In this way my model captures the fact that in partial contexts, speakers may not know the degree function of a given predicate. The degree function, and, accordingly, also the dimension-set and dimension weights may vary between total contexts.

Finally, the main advantage of my analysis is that gradability is not related to vagueness per-se, but rather, to the *order in which vagueness is removed* (predicates' ordering relations reflect the order in which entities are added to their denotations, directly or by inference). Thus, it is predicted that non-vague predicates (for which competent speakers usually assign an interpretation which is already nearly complete) may be gradable (if the membership of some of the entities in their denotations have already been inferred, when the membership of others have still been questioned). We now see that Kamp and Partee (1995) have failed to capture these effects (3.3) precisely because they have used a model that did not represent the order of learning.
(or is reflected by) the typicality degrees, it is predicted that this item will be judged to be a good example of a brown apple and a poor example of an apple, as desired (to have a higher degree in brown apple than in apple). This judgment is an instance of the conjunction effect (cf. Smith et al 1988). We also know that likelihood judgments are often affected by typicality (cf. 2.2.5.3). Thus, it is predicted that this item may be judged more likely a brown apple than an apple (because it is the most representative brown apple, but almost a borderline case for apple). We see that this judgment, which is called a conjunction fallacy (Tversky and Kahneman 1983), is completely compatible, and even explained, by the knowledge-structure and the logical rules.

Similarly, one may classify an ostrich as a bird late (compared to other bird types), and, at the same time, classify it as an ostrich early (compared to other ostriches). Thus, the same phenomenon in lexical nouns is accounted for in just the same way.

8.7.3 Emergent dimensions and inheritance failures

My theory is an "intersective prototype theory" in the sense that I postulate both the intersection rule (cf. 6.2.5 (16a) together with (14g))5,6 and a standard-based categorization rule (cf. 7.1):

\[
\forall t \in T, \forall g \in G:
\begin{align*}
\text{a. The intersection rule:} & \quad [[P \land Q]]_{t,g}^* = [[P]]_{t,g}^* \cap [[Q]]_{t,g}^* \\
\text{b. The standard-based rule:} & \quad [[P]]_{t,g}^* = \{d \in D | \deg^+(d,P,t,g) \geq \text{Standard}^+(P,t,g)\}
\end{align*}
\]

Recall that the degree of an entity in a nominal concept is inversed to its mean distance from P's prototype. So, in practice, P denotes the set of entities whose mean distance from P's selected values on P's dimensions is small enough (i.e. their similarity to P is high enough). Let us say that in any t and g the number Distance^+(P,t,g) represents the maximal distance allowed for positive denotation members (it is the mean distance from P's prototype of any entity whose degree is precisely the standard value, Standard^+(P,t,g)). We can state that:

\[
\text{c. Given the standard-based rule, for any nominal concept P:}
\begin{align*}
[[P]]_{t,g}^* & = \{d \in D: \\
(\Sigma_{F \in F^+(P,t,g)} \text{Weight}^+(F,P,t,g) \times \text{norm}(\deg^+(d,F,t,g) - \text{Value}^+(P,F,t,g))) \\
\leq \text{Distance}^+(P,t,g)\} 
\end{align*}
\]

This intersective-prototype theory predicts that each constituent in a modified noun adds a categorization criterion. For instance, an item d is a male nurse iff d's mean distance form both the prototype of male and the prototype of nurse is sufficiently small:

\[
[[P \land Q]]_{t,g}^* = \{d \in D | (\Sigma_{F \in F^+(P,t,g)} \text{Weight}^+(F,P,t,g) \times \ldots) \leq \text{Distance}^+(P,t,g) \\
\text{and } (\Sigma_{F \in F^+(Q,t,g)} \text{Weight}^+(F,Q,t,g) \times \ldots) \leq \text{Distance}^+(Q,t,g)\}
\]

---

5 In the following, I use P \land Q as a shorthand for \(\lambda x. P(x) \land Q(x)\).
6 Given the rule in (16a), in any t in T and g in G, the predicate P \land Q ( \(\equiv \lambda x. P(x) \land Q(x)\)) denotes the set of entities d in D that verify the conjunction P(x) \land Q(x) in t and g(x/d). Given the rule in (14g), P(x) \land Q(x) is true in t and g(x/d) iff d is both in [[P]]_{t,g(x/d)}^+ and in [[Q]]_{t,g(x/d)}^+. Thus, these rules predict that a conjunctive predicate is interpreted intersectively in any t and g.
Does this theory predict that the dimension set of a modified noun should be the union of the constituents' dimension sets ($\forall g \in G, \forall t \in T: F^+(P \land Q, t, g) = F^+(P, t, g) \cup F^+(Q, t, g)$)? The answer for this question is no. To the contrary, this theory is incompatible with this union rule. I demonstrate below that it actually predicts that no context $t$ in $T$ conforms to the union rule. For any $t$ and $g$:

$$[[P \land Q]]^{+}_{t, g} \neq \{d \in D| (\Sigma_{F \in F^+(P, t, g) \cup F^+(Q, t, g)} \text{Weight}^+(F, P, t, g) \times \ldots \leq \text{Distance}^+(P \land Q, t, g)\}$$

Thus, the intersection rule for denotations actually forces effects such as failures of inheritance of dimensions from the constituents and emergence of new dimensions in modified nouns (i.e., deviations from the union rule for dimensions) to occur.

Let us demonstrate this with a simple case in which $P$ and $Q$ are ordered by the same two dimensions, $F_1$ and $F_2$. The denotations of $P$ and $Q$ can be represented as (symmetric) diamonds in the two dimensional space with axes $F_1$ and $F_2$. The standard-based rule in (15c) means that the pairs of degrees in $F_1$ and $F_2$ that correspond to members of $[[P]]^+_{t, g}$, are given by a formula of the form $w_1x + w_2y \leq n$. The set of number-pairs $<x_i, y_i>$ that solve the equation $w_1x_i + w_2y_i = n$, are points that create a diamond shape in the space whose axes are $x$ and $y$. Any two points such that $w_1x + w_2y < n$ fall inside the diamond. Thus, entities whose mean degrees are equal to or smaller than Distance$^+(P, t, g)$ ($P$ instances) fall on or inside a diamond in the space whose axes are $F_1$ and $F_2$. Similarly, entities whose mean degrees are equal to or smaller than Distance$^+(Q, t, g)$ ($Q$ instances) fall in or inside another diamond. But, the crucial point is that the intersection of two diamonds may not form a diamond. It may form a parallelogram or a kite shape, which are not both ways symmetric, as demonstrated in Figure 18.

Figure 18: The intersection of $P$'s and $Q$'s denotations has an asymmetric shape that cannot be given by equation (15e) (by mean on the union of $P$'s and $Q$'s dimension sets $\{F_1, F_2\}$).

For no possible weights for $F_1$ and $F_2$ can a mean distance equation like (15e) (an equation of the form $w_1x + w_2y \leq n$) identify such a shape (it can only identify a diamond shape). Thus, by assuming that the denotation is intersective, we have ruled out the possibility that the modified noun's dimensions would be $\{F_1, F_2\}$, contrary to the predictions of the union rule. When we construct a categorization criterion for a nominal conjunctive concept $P \land Q$ (a modified noun like male nurse, pet fish or wooden spoon), we must move to a new space of dimensions $\{F_1, F_2, F_3\}$ or $\{F_1, F_4\}$, or... etc., such that the average on them will give us the intersective denotation (in the new space, the entities which are both $P$ and $Q$ will form a diamond).\(^7\)

We see then that the intersection rule is compatible with the prototype theory, but incompatible with a union-rule for dimension-sets. What I am saying is that it is the latter rule that is wrong. Not Boolean compositionality for denotations needs to be abandon: Boolean compositionality

\(^7\) Note that for the same reason the degree function of a conjunction cannot be based on mean in the conjuncts.
for dimension-sets needs to be abandoned. But that means, in turn, that the dimension-sets of modified nouns must be arrived at in a different way. It is therefore always the case that some dimensions are not inherited from the constituents to the modified noun (for example, fish typically live in the open oceans and pets are typically warm and affectionate, but pet fish are neither; cf. 2.2 and 4.2), or some new dimensions emerge (for example, talks and lives in a cage for pet bird, and big for wooden spoon), or both, as experiments consistently show. These effects are not despite, but due, to intersectivity, so an intersective theory is more explanatory than a non-intersective theory.

Furthermore, given the intersection rule, it is but natural that dimensions which are effectively necessary (which are violated only in rare cases) for either constituent are necessary for the modified noun, and dimensions which are impossible for either constituent are impossible for the modified noun. The weights of other dimensions may be their mean weights in the constituents as proposed in Hampton (1987-1997), but their selected values may be rather different from their selected values in the constituents, as explained below.

How do we select emergent dimensions? Crucially, given the learning principle, the modified nouns' dimensions can be selected in just the same way as dimensions are selected in basic lexical items. They are the dimensions of early acquired (and hence typical) instances of the modified nouns, which correctly predict known facts about similarity and categorizations, as explained in 8.3. Otherwise, they simply do not emerge. Note that the earliest acquired male nurse may be a rather atypical male and a rather atypical nurse. Accordingly, its dimensions, or at least their selected values, may be rather different.

In sum, logical rules which are parts of the formal semantic theories, such as the intersection rule, are not only compatible with the prototype view, but in fact help us explain phenomena that have been taken to refute them such as conjunction fallacies and emergent features (regarding intersection failures [overextension-effects] see discussion in 7.2).

The argument that we have made extends to any form of the generalized mean equation. Recall, that the mean distance of an entity d from P (on P's dimensions) is represented in different psychological models by different types of weighted means. The notion of a generalized mean, Dis(d,P,r), is an abstraction of the different types of weighted means (Weidman 1993). The value of the exponent r determines the type of mean.

\[(17) \quad a. \quad \text{The generalized weighted-mean of the distance of d from P in } F(P): \]
\[
\text{Dis}(d,P,r) = \left( \sum_{F \in F(P)} (\text{Weight}(F,P) \times \text{Dis}(d,P,F))^r \right)^{1/r}
\]

(where \(r \neq 0\), the dimension weights are all positive, and they sum up to 1).

For example, for \(r = 1\), we get the arithmetic mean: The sum of d's (weighted) degrees in every dimension,

\[
b. \quad \text{Arithmetic mean-distance (for } F(P) = \{F_1, \ldots, F_n\}): \]
\[
\text{Dis}(d,P,1) = (\text{Weight}(F_1,P) \times \text{Dis}(d,P,F_1)) + \ldots + (\text{Weight}(F_n,P) \times \text{Dis}(d,P,F_n))
\]

for \(r = 2\), we get the Euclidian mean,.

\[
b. \quad \text{Euclidian mean-distance (for } F(P) = \{F_1, \ldots, F_n\}): \]
\[
\text{Dis}(d,P,2) = \left[ (\text{Weight}(F_1,P) \times \text{Dis}(d,P,F_1))^2 + \ldots + (\text{Weight}(F_n,P) \times \text{Dis}(d,P,F_n))^2 \right]^{1/2}
\]
etc. If \( r = 2 \), then, e.g., \( P \)'s and \( Q \)'s denotations are spheres in the space whose axes are \( F_1 \) and \( F_2 \). But the intersection of two spheres is not a sphere (unless one sphere is included in the other). So it is a shape that cannot be determined by a categorization rule based on mean distance with \( r = 2 \). The same holds true for any \( r \geq 2 \), and any number of dimensions. More generally, the function in (16a) with \( r=2 \) and \( n \) dimensions (like any mean function for \( r \geq 2 \)), is continuous. It has a derivative in every point (it identifies an \( n \)-dimensional surface which has a uniquely defined tangent in every point). But the intersection of two such forms need not be continuous. The points in which two spheres (such that none is included in the other) intersect do not have a uniquely defined tangent. It follows that an intersective denotation cannot be given by a categorization equation which is based on a mean function like (16a) if the dimensions are fixed by the union rule.

The argument extends also to more complex categorization criteria. For instance, Ashby and Maddox (1993) use equations with factors that represent dependencies between dimensions. These equations too are continuous, but the intersection of the forms (curves) that they produce is often not given by a continuous equation.

The argument extends also to exemplar based categorization equations (cf. 4.1). For simplicity, let us assume a context in which \( P \) and \( Q \) are not associated with a contrast set, so categorizing under them is simply standard-based. In such circumstances, for each newly encountered item, the exemplar theory predicts that it would be classified under \( P \) iff it is similar enough to at least one \( P \) exemplar (for instance, classification under \( bird \) is assumed to be based on classification under the exemplars \( robin, pigeon, chicken \), etc.) The instances which are similar enough to an exemplar are clouded around it forming a shape of, say, a sphere (for \( r=2 \)). In principle, the intersection rule together with an exemplar theory require that each entity in \([P \land Q]\) would be similar enough to at least one \( P \) exemplar and at least one \( Q \) exemplar, as demonstrated in Figure 19, where \( P_1 \ldots P_5 \) represent \( P \)'s exemplars and \( Q_1 \ldots Q_5 \) represent \( Q \)'s exemplars. Thus, \([P \land Q]\) is the union of the non-empty intersections of one \( P \) sphere and one \( Q \) sphere. As explained above, none of these forms can be given by the usual categorization equations which are based on mean similarity to an exemplar.

![Figure 19: The intersection rule with an exemplar based categorization rule](image)

The arguments extend also to the dimension sets of disjunctions and negated predicates. Here too, dimension-inheritance failures and emergent dimensions are consequences of the logical rules for the denotations.
One might expect the negation of a predicate $P$, $\neg P$, to be characterized by the negations of the dimensions of the predicate $P$:

\[(18) \quad \text{The dimension set of } \neg P \text{ is the set of negated dimensions of } P: \]

$$F^+ (\neg P, t, g) = \{ \neg Q | Q \in F^+ (P, t, g) \}$$

This rule predicts that a dimension $F$ would be typical of $P$ iff not-$F$ is typical of not-$P$. Indeed, intuitively, if $F$ orders $P$, of every two entities equal in all other respects, the less typical $F$ is the less typical $P$, and accordingly, the more typical $\neg F$ is the more typical $\neg P$. E.g. flying is diagnostic of bird iff non-flying is diagnostic of non-bird. For example, intuitively, an example is good in red bird to the extent that it is red and to the extent that it is a bird. Similarly, an example is good in not a red-bird to the extent that it is not-red and to the extent that it is a non-bird, despite the fact that it is enough for membership in non red-birds either not to be red or not to be a bird (maximal redness is not necessary of red-birds either, but it still raises their typicality). To take another example, a bird is more typical of non-robin to the extent that it is not red in the chest. A red thing is more typical of non-robin to the extent that it is typical of non-bird (say, a red car or ship is worse in non-robin than a red helicopter). Robins and other entities which are birds and red in the chest (and that do not violate other characteristics of robins) are within the worst examples of non-robins. Indeed, in my study of 35 Hebrew speakers (the general judgment-questionnaire; cf. appendix to chapter 2), for example, 88% of the subjects agreed that to fly is typical of birds, and 81% agreed that not to fly is typical of non-birds; Moreover, 92% treated the conjunction of negations of bird-properties: not to fly and not to sing and not... as typical of non-birds.

At the same time, it has been empirically established that some dimensions fail to be negated, and new dimensions (whose negation does not characterize $P$) emerge (Hampton 1997). And in fact, if we adopt the Boolean complement rule for the interpretation of negation (for any $t$ in $T$ and $g$ in $G$, $[\neg P]_{t, g}^+ = D - [P]_{t, g}^+$), then this is more than expected. For example, consider the structure in figure 20. Given the complement rule, the denotation of the negation of $P$ covers the whole surface outside the sphere which represents $P$'s denotation. But the complement of a sphere is not a sphere. Thus, if we are about to build a prototype for the negated predicate, we must move to a new space of dimensions.

Figure 20: Negation

In my view, as a basis, we do regards the negations of $P$'s dimensions as potential dimensions of not-$P$, but for the complement rule to apply, some dimensions are dropped and / or others emerge, based on the dimensions and degrees of the early acquired not-$P$s.

Similarly, if the denotation of a disjunction $P \lor Q$ is given by the Boolean union rule, then it covers the whole surface inside the union of the two spheres which represents $P$'s and $Q$'s denotations – a shape that need not be a sphere.
Finally, I would like to suggest that during acquisition, if the interpretation (dimension-sets and denotations) of negated, conjunctive or disjunctive expressions is initially extracted independently of their constituents (based on the dimensions of the early acquired members), this may provide the means to acquire the meaning of the logical connectives. When enough knowledge about simple and complex expressions is accumulated, the systematic connections between the meaning of the complex expressions and the meanings of their constituents (not, or, bird, flying ...) can be extracted. For example, the fact that the set of members of a negated concept like not-a-bird is the complement of the set of members of the (non-negated) concept bird, constitutes the core of the semantics of the functional word not in phrases like animals which are not birds. This fact can be extracted from knowledge about the meanings of pairs of negated and non-negated concepts (Chierchia and McConnel Ginnet 2002). In turn, perhaps the availability of semantic knowledge about the connectives can assist the extraction of syntactic rules. I leave these issues to future research.

8.7.4 The ordering in conjunctive predicates: Compositionality and its limits

Let us call conjunctive any predicate whose interpretation involves intersection. The notion is meant to cover and-conjunctions, modified nouns, and conjunctive multi-dimensional adjectives like healthy (assuming that these are interpreted as healthy wrt blood pressure and healthy wrt pulse and ... ; cf. 7.5). Except for Kamp and Partee 1995’s analysis of the conjunction fallacy, the predictions of vagueness-based gradability theories with regard to the ordering in conjunctive predicates were not investigated in the past. Indeed, this may be due to the fact that conjunctions cannot felicitously combine with more (except if more is within the scope of the connective). But that does not prove that they don’t have a gradable structure. In fact, the empirical facts suggest that they do, as modified nouns and disjunctions have been shown to be associated with dimension sets and entity orderings (cf. 2.2 and 4.2).

The learning principle predicts that the ordering in conjunctive and disjunctive predicates is only partially determined by the ordering in their constituents. I propose that this creates vagueness concerning the ordering relations of conjunctive and disjunctive predicates. This vagueness may be resolved by resorting to averaging. This explains the intuition that entities’ mean on the dimensions of multi-dimensional adjectives affect their ordering, and that the entities’ mean in the conjuncts or disjuncts affect their ordering in conjunctions, disjunction and modified nouns.

The prediction made by the learning principle about the ordering in conjunctions and disjunctions is as follows:

\[
\begin{align*}
(19) \quad & a. \quad \langle d_1, d_2 \rangle \in \left[ [\leq (P \land Q)] \right]_{t,g}^+ \text{iff } \forall c \in C, c \leq t: \\
& \quad (d_1 \notin [P \land Q]_{c,g} \text{ or } d_2 \in [P \land Q]_{c,g}) \& \quad (d_2 \notin [\neg(P \land Q)]_{c,g} \text{ or } d_1 \in [\neg(P \land Q)]_{c,g})
\end{align*}
\]

Whenever \( d_1 \) is already classified in the conjunction \( d_2 \) is already so classified, and whenever \( d_2 \) is already classified in a negation of a conjunct (in the disjunction of negations of conjuncts), \( d_1 \) is already so classified.

\[
\begin{align*}
(19) \quad & b. \quad \langle d_1, d_2 \rangle \in \left[ [\leq (P \lor Q)] \right]_{t,g}^+ \text{iff } \forall c \in C, c \leq t: \\
& \quad (d_1 \notin [P \lor Q]_{c,g} \text{ or } d_2 \in [P \lor Q]_{c,g}) \& \quad (d_2 \notin [\neg(P \lor Q)]_{c,g} \text{ or } d_1 \in [\neg(P \lor Q)]_{c,g})
\end{align*}
\]

Whenever \( d_1 \) is already classified in a disjunct (i.e. in the disjunction), \( d_2 \) is already so classified, and whenever \( d_2 \) is already classified in the conjunction of negations of disjuncts \( d_1 \) is already so classified.
Support for all these predictions was found in my study of the judgments of 35 Hebrew speakers (the general judgment-questionnaire; cf. appendix to chapter 2). My study shows that when subjects are forced to give a compositional interpretation (to give comparative judgments about the degrees of instances or properties with respect to conjunctions or disjunctions), the comparative judgments seem to be determined by the order of classification in the positive (or negative) denotations of the conjunction or disjunction. For example, the subjects were given a description of two characters, Moshe and Danny, where Moshe weighed 100 kg and Danny only 70 kg (Moshe was fatter), but Danny was bald and Moshe was not bald (Danny was balder). According to the few subjects that selected one character to be more: fat and/or bald than the other, (i) Danny scored much better than Moshe relative to fat or bald (questions 4.7-4.10 and 4.19-4.21), because it is easier to determine that Danny is fat or bald (Danny is bald, while Moshe is not bald and is not necessarily fat); (ii) Danny also scored much better than Moshe relative to fat bald / bald fat (4.11-4.18), because it is easier to determine that Moshe is not: fat and bald (because Moshe is not bald), while Danny might still be fat and bald. A similar pattern occurred in other scenarios as well. When the two characters were equally tall (1.95m), but Moshe was fatter (100 kg as opposed to Danny’s 70 kg), among those who answered the question, Moshe was regarded as more fat and tall than Danny (3.1-3.8). When the characters where fat, and equally fat (100 kg), but Moshe was balder, 90%-100% of the subject agreed that Moshe is more fat or bald than Danny (9.9-9.15).  

Now, the learning principle also determines that the ordering of conjunctions and disjunctions cannot be completely dependent on the ordering of the conjuncts or disjuncts. We cannot use the sets \([\leq P])^i_t \) and \([\leq Q])^i_t \) in any simple way in order to compose \([\leq (P \wedge Q)])^i_t \) or \([\leq (P \vee Q)])^i_t \). The following list of non-entailments shows that:

\[
\begin{align*}
&c. \quad <d_1,d_2> \in [\leq (P \wedge Q)]^i_t \not\Rightarrow <d_1,d_2> \in [\leq P \wedge \leq Q]^i_t \\
&<d_1,d_2> \in [\leq (P \vee Q)]^i_t \not\Rightarrow <d_1,d_2> \in [\leq P \vee \leq Q]^i_t \\
&<d_1,d_2> \in [\leq (P \wedge Q)]^i_t \not\Rightarrow <d_1,d_2> \in [\leq P \wedge \leq Q]^i_t \\
&<d_1,d_2> \in [\leq (P \vee Q)]^i_t \not\Rightarrow <d_1,d_2> \in [\leq P \vee \leq Q]^i_t \\
\end{align*}
\]

Simple connections between the ordering relations of wholes and parts exist only in trivial cases. With regard to predicates P and Q which are neither equivalent nor contradicting (for example, fly and/or sing), the learning principle only predicts the rather weak constraints on the ordering relations of the whole and the parts that are given in (20). The formal proofs are found in the appendix.

---

8 Still, for most of the subjects none of the characters was more: fat and/or bald, and nor were they equally: fat and/or bald. Why? Because conjunctions and disjunctions are inherently multi-dimensional. In the lack of a wrt-argument, no unique dimension can be accommodated into their semantics. The comparative morpheme, in within-predicate comparisons, is undefined in the lack of a unique dimensions. This requirement for a unique dimension explains the fact that the interpretation of connectives in narrow scope is usually not available in the derived comparatives of conjunctive and disjunctive predicates.

This observation may also explain the fact that given two non-flying creatures, such that the first is more typical in squeacking than the latter which is known not to squeak, only 37% of the subjects agreed that the first is more typical in flying and squeaking than the latter, and even fewer informants – 29% – agreed that the first is more typical for a squeaking flyer (7.1-7.2). However, 80% agreed that the first is more typical in flying or squeaking than the latter.
be better in singing, and the other in flying (it might be that \( \text{either as a}
\text{say, a}
\text{male
flying or}
\text{singing}\). However, two birds equally typical in "flying and singing", or in "flying or singing", might still be neither equally typical in singing nor equally typical in flying. One might be better in singing, and the other in flying (it might be that \( [[=P]]^{t,g} \cap [[=Q]]^{t,g} \subseteq [[=(P\land Q)]^{t,g} \cap [[>Q]]^{t,g} \neq \emptyset \)). The predictions made by (20c-d) are intuitive too. For example, intuitively, two birds such that the first is more typical in "flying and singing", and the second are such that the first bird is more typical in "flying and singing", and in "flying or singing" than the latter. However, if, say, the first bird is more typical in "flying and singing", or "flying or singing", the latter bird might still be either at least as typical in singing or at least as typical in flying (it might still be that \( [[<P\land Q)]^{t,g} \cap ([[<P]]^{t,g} \cup [[<Q]]^{t,g}) \neq \emptyset \)).

Constraints (20e-f) tell us something about all entity-pairs – including those entity-pairs whose members do not stand in the same relation in \( P \) and in \( Q \). The predictions made by (20a-b) are intuitive. For example, intuitively, two birds equally typical in both flying and singing are equally typical in flying and singing, and in flying or singing. However, two birds equally typical in "flying and singing", or in "flying or singing", might still be neither equally typical in singing nor equally typical in flying. One might be better in singing, and the other in flying (it might be that \( [[=P\land Q)]^{t,g} \cap [[<P]]^{t,g} \cap [[<Q]]^{t,g} \subseteq \emptyset \)).

Less typical male nurses are either less typical males or less typical nurses. I.e., atypical male-nurses cannot be very typical in both conjuncts simultaneously.

As desired, the learning principle allows the best ‘\( P \) and/or \( Q \)’ to be neither a very good \( P \) nor a very good \( Q \). For instance, the best male nurse doesn’t have to be the best in male, nor the best in nurse, because it is enough for it to be better in male than some entities, and better in nurse than others. In fact, the best males may not be nurses, and the best nurses may not be males, so the earliest acquired "\( P \) and \( Q \)" may be an atypical \( P \) and an atypical \( Q \). For that reason, entity-prototypes seem to violate compositionality, that is, not to be inherited from the parts to the whole (Osherson and Smith 1981; Partee and Kamp 1995). For the same reason, the learning principle allows for the conjunction effects. For example, chess is an atypical sport but a most typical game, so it can be typical of games which are sports.

In addition, given the learning principle, it is predicted that if the first male-nurse known to us is a typical male but an atypical nurse – the concept male-nurse will inherit the typicality properties of male (because the dimension set is composed of properties of the earliest acquired entities, as discussed in 8.3, and in the given example, these are properties typical of males); and if the first male-nurse is a typical nurse but an atypical male – the concept male-nurse will inherit the typicality properties of nurse. If it is neither typical of male nor of nurse, it will have new properties of its own which will form the emergent part of the prototype of the whole. Thus, it is predicted that one constituent may be more dominant (Hampton 1988, 1987, 1997, Costello 1998). In male-nurse, the dimension-set may contain properties such as long hair, female or...
feminine. Yet, short hair and manliness are stereotypical of male. Whether it is the dimension-set of the noun or of the modifier that gets inherited to the whole is dependent on context (Kamp and Partee 1995).

Hybrid' interpretations of disjunctions (or overextension in disjunctions) are also expected. For example, suppose that in c, balloon r is perfectly red (i.e. r is not pink), and balloon p is perfectly pink (i.e. p is not red). Then obviously both p and r are red or pink. Now, intuitively, we may perfectly well regard a third balloon rp, exactly in between these two colors, as perfectly red or pink. That this is possible is predicted, because rp is less pink but more red than p, and less red but more pink than r.

The learning principle predicts the following:

\[ g. \ (\leq (P \wedge Q))_{t,g} \leq ((P \vee Q))_{t,g} \]

But the learning principle does not predict the following intuition:

\[ i. \ (\leq (P \wedge Q))_{t,g} \leq ((P \vee Q))_{t,g} \]

In order to see this, consider a pair \(<a,b>\) \in \([=P]_{t,g} \cap \([=Q]_{t,g}\)\), such that in some context \(c_1\) under \(t\) only 'b' is added into \([=Q]_{c_1,g}\) in some context \(c_2\) above \(c_1\) under \(t\) 'a' too is added into \([=Q]_{c_2,g}\) and in some context \(c_3\) above \(c_2\) under \(t\) both 'a' and 'b' are added into \([P]_{c_3,g}\). This pair belongs in \([=P]_{t,g}\) not in \([<P \wedge Q]_{t,g}\). Thus the learning principle only predicts that \([=P]_{t,g} \cap \([<Q]_{t,g}\) \leq \([<P \wedge Q]_{t,g}\) while our intuitions support a stronger generalization, namely that \([=P]_{t,g} \cap \([<Q]_{t,g}\) \leq \([<P \wedge Q]_{t,g}\).

I propose that the stronger generalization is based on the following facts. Sometimes, we know of a pair of items \(<i_1,i_2>\) whether it stands in the relation better, worse or equally good examples for each one of the conjuncts in a certain conjunction (for example, for any extension \(t\) of \(c\), the entity pair that this item-pair is discovered to be, \(<d_1,d_2>\), is such that \(<d_1,d_2>\) is in \([=P]_{t,g} \cap \([<Q]_{t,g}\)\), but the learning principle still does not allow us to tell whether the pair stands in the relation better, worse or equally good examples for the conjunction (in the given example it is only predicted that in any \(t \in T_c\) \(<d_1,d_2>\) is in \([<P \wedge Q]_{t,g}\)). In such cases, in order to remove the vagueness in the ordering relation of the conjunction, we base our ordering decisions on averaging. Now, formally, for any \(t\) and \(g\), any pair of entities \(<d_1,d_2>\) in \([=P]_{t,g} \cap \([<Q]_{t,g}\) is such that, for any weight \(W_P\) for \(P\) and any non-zero weight \(W_Q\) for \(Q\), \(d_1\) has a lower average than \(d_2\) in \(P \wedge Q\). Hence, \(<d_1,d_2>\) is added to \([<P \wedge Q]_{t,g}\).

We see that the ordering in conjunctive predicates cannot be based on the entities' weighted mean in the conjuncts, or their dimensions (cf. 5.8; 7.5, and 8.7.3), except when the learning principle leaves open the question of what the ordering is in \(P \wedge Q\). Then ordering by mean in the conjuncts may help remove vagueness in the ordering relation. This explains the fact that we intuitively think that entities which are better in one conjunct and equally good in all the others are better in the conjunction. In particular, in adjectives which have multi-dimensional interpretations (like healthy, typical, similar, normal, etc.), we intuitively feel that the mean in the dimensions affects the ordering of entities. We saw in 7.5 that the categorization rule in such adjectives is the conjunction or disjunction of dimensions, not a requirement to reach a certain standard mean degree in the dimensions. But the ordering in conjunctions and disjunctions is not
completely given compositionally (mostly in conjunctions of relative-standard adjectives; their standard is highly context dependent and hence so is also the meaning and ordering of the conjunction). This allows us to assume that averaging over the conjuncts may be used in pairs that stand in different ordering relations in different dimensions. For example, if I compare a patient with cancer to a patient with the flu, I may weigh the cancer as more important, and judge the former patient as having a higher degree in sick. When context does not tell us how the conjuncts are to be weighed (for instance, if I compare a patient with cancer to a patient with serious heart problems), this strategy might fail. When it fails, we are left with a wide-scope interpretation like healthier wrt cancer and healthier wrt the heart, and we are likely to be reluctant to say about any of the patients that he is healthier.

Another strategy for removing the vagueness in the ordering relations of conjunctive or disjunctive adjectives $P$ is by using the mean in typicality dimensions of modified nouns. Any adjective can modify a trivial noun such as object, individual, one. Like other noun phrases, the noun phrase healthy entity is linked with a set of typicality dimensions such that averaging over them produces the intersective denotation, like calm, does not smoke, does not drink, does not eat fat, eats fruit and vegetables, is regularly involved in sport activities, etc. On the present proposal such dimensions may be directly linked to modified nouns such as healthy person, but not to the adjective healthy itself. They are only indirectly related to healthy, in that they emerge in virtue of the fact that the denotation of healthy is defined by respects (necessary dimensions) like blood pressure and pulse. The adjectival respects determine the denotation (by intersection), and then the mean on typicality dimensions such as eats fruit and vegetables and is regularly involved in sport activities may be discovered to correctly predict categorization under healthy people. The nominal and adjectival dimension-sets of healthy and healthy one represent two principled ways for building classification algorithms for the very same set (or a very similar set).

In sum, compositionality, in the case of the ordering in conjunctions and disjunctions, more often than not, fails. In many cases (for many pairs of entities) their ordering in a conjunction (or in a disjunction) cannot be predicted from their ordering in the constituents. Thus, conjunctions and disjunctions needs to be learnt directly, based on the order in which entities are classified in the conjunction (or disjunction). But what happens when this knowledge is not available? And what happens, for instance, when some conjuncts are relative adjectives whose standard is highly variable and therefore so are the classification facts in the conjunction? The ordering in conjunctive predicates cannot be based on the entities' weighted mean in the constituents (or in the union of their dimensions). However, crucially, ordering by mean in the constituents (or in the union of their dimensions) may be used in order to remove vagueness in the ordering relation in places where compositionality (as based on the learning principle) cannot help us. This explains the fact that in adjectives which have multi-dimensional interpretations (like healthy, typical, similar, normal, etc.), we intuitively feel that the mean in the dimensions affects the ordering of entities, despite the fact that their categorization rule is the conjunction or disjunction of dimensions, not a requirement to reach a certain standard mean degree in the dimensions.

8.8 Intermediate conclusions

Gradability is characterized as reflecting the order in which vagueness is removed (the order in which entities are learnt to be (super-) denotation members, either directly or by inference). This characterization is supported by developmental and experimental studies (the learning principle directly predicts learning-order effects, first impression effects, and maybe some
familiarity effects). In addition, the learning principle does not face the basic problems of previous vagueness models. It postulates intuitive connections between vagueness and gradability. Crucially, gradability can characterize non-vague predicates, providing that the denotations of these predicates may be learnt gradually, that is, providing that entities exemplify the properties in question to different extents and hence the membership of some of the entities in the denotation may already be inferred, when the membership of others may still be questioned. In virtue of this fact, our gradability theory captures the typicality effects (cf. 2.2), which are in essence gradability effects in (relatively) sharp predicates. Finally, the learning principle allows producing compositionality predictions for negated predicates, and it allows an account of the way multi-dimensional degree functions might be acquired.

8.9 Exemplars, kinds, typicality, and genericity: Suggestions for future research

8.9.1 Exemplar effects

Recall that the exemplar models (Medin and Schaffer 1978; Nosofsky 1992) assign a small (or no) role to the dimension set of the entire category (the summary representation), and a large role to the separate dimension sets of each one of the entities or sub-categories of the category (for instance, to the dimension sets of robin, duck, chicken, etc., for the category bird), or to the separate dimension sets of each one of the temporal stages of the instances of the category (for instance, to the dimension set of each occurrence of a bird). These items are called exemplars. As explain in 4.1, an exemplar is represented as a set of separable dimensions and selected values in these dimensions, thus an exemplar has the structure of a linearly separable (cf. 4.1.1.4 and 4.1.2), prototype-based category (this representation might change in time, because memory might degrade or strengthen). The exemplar approach argues that we categorize objects by comparing them to remembered exemplars. The more similar an object is to entities or sub-categories whose membership we have already encoded, the more likely it is to be a regarded as a member. The similarity of an entity d to a category P is given by the sum of d’s similarities to all the known exemplars k of P. A given exemplar k may be weighed by its importance to P (Weight(k,P)):

\[
\text{Exemplar-based similarity to a category P (Typicality):} \quad \text{Sim}(d,P) = \sum_{k \in [P]} (\text{Weight}(k,P) \times \text{Sim}(d,k))
\]

Since similarity to an exemplar is assumed to be multiplicative (cf. 4.1), it is best to be very similar (near to 1) to but few items than somewhat similar (dissimilar in but few respects and hence near to 0 similar) to many items. Thus, generally, if an item is highly similar to a known instance of P, it is highly similar to P. (However, if there are many highly similar exemplars, this raises the similarity score of newly encountered entities that are similar to them, and this raises their membership likelihood).

The tendency to employ a prototype or an exemplar strategy appears to vary across concepts, contexts, and speakers (Smith and Minda 1998; 2000; Malt 1989; Smith et al 1997).

First, the assumption that similarity is determined by one summary representation seems intuitive for some natural concepts (say robin or pet fish), while for others (say, games, or pets), the assumption that similarity is determined by a set of exemplars seems more viable.

Second, Smith and Minda (1998) have shown that if categorization judgments are assessed during learning, initially subjects rely on a prototype representation, and later on, when this
strategy fails they switch to an exemplar strategy. Thus, the very same people tend to employ different categorization strategies in different circumstances: A shift occurs from a prototype (or category) based strategy to an exemplar based strategy when categories are small and non-linearly separable (Smith and Minda 1998; 2000). This is typically the case in the experiments supporting the exemplar theory (Murphy 2002: 484).

Third, some people tend to employ prototype-based patterns of responding, while others employ exemplar patterns (Malt 1989; Smith et al 1997).

Most importantly, Smith and Minda (1998) argue that, in category learning, also sub-categories of the category are learnt. When membership in a sub-category is easy to determine based on similarity to the sub-category dimensions, membership in the big category is indirectly inferred. This description is based on the assumption that membership in the big category is necessary for membership (always hold) in the small category. In these ways, the exemplar effects can be incorporated into the prototype theory. I use this idea. I propose that we do not need to add to the model an additional mechanism to represent exemplars. Exemplar effects can be derived by explicitly representing the "sub-category" (or "sub-kind") relation.

In fact, from the semantic perspective, the prototype theory and the exemplar theory each seem to correctly describe a different interpretation of predicates. In the psychological literature the concepts robin and ostrich are normally referred to as exemplars of the category bird, though they are also categories in their own right, and hence also kind-terms. Each kind term k can become a predicate at several interpretation-levels. A predicate that denotes a set of sub-kinds subkinds of k, a set of entities instances of k, a set of entity stages stages of instances of k, etc. (cf. Carlson 1977; Dayal 2004). Crucially, each level needs to be associated with a separate prototype (dimension-set) in each context t and assignment g. For example, the dimension small characterizes (the prototype of) the predicate bird as a name for a set of bird sub-kinds (i.e. the predicate subkind of bird), rendering robins typical and ostriches atypical. But small does not characterize (the prototype of) the predicate bird as a name for a set of entities (i.e. the predicate instance of bird). Female birds are most often smaller, but not more typical, than (otherwise equally typical) males. Conversely, the dimension healthy (when interpreted as, for instance, not having a broken wing or a flu) may affect typicality in categories of bird-entities or entity stages, but not of bird-sub-kinds. Sub-kind categories may have dimensions like extinct, widespread, typically flying (λk. flying is typical of instances of k). Categories of ordinary entities cannot have such dimensions (but they can have properties like usually eats insects that categories of entity-stages cannot have). A statement like Tweety is a typical bird may be understood either as "Tweety is a typical bird-entity" (it averages well in Ft(bird,t,g)), or as "Tweety belongs to a typical bird-sub-kind" (Tweety's bird-sub-kind averages well in Ft(sub-kind-of-bird,t,g), etc. In both cases, Tweety may in principle be atypical of its sub-kind.

In sum, the question whether a category ought to be represented by a summary of its dimensions or by a set of exemplars or by both, needs to be answered in each speaker, context, category and category level, separately. E.g. a set of sub-kinds is clearly encoded in our memory under the category bird, but we can ask whether the sub-kinds of these sub-kinds are also encoded directly under bird or not, and for each bird stage (say a stage of a grey robin) we can ask whether it is encoded under bird, robin, grey robin, or under some or all of these categories. In particular, if in some t and g the dimension-set in the sub-kind level of a category like bird includes many dimensions (that order by typicality sub-kinds of birds, and constrain the membership in the set of sub-kinds of birds), but the dimension set in the entity-level interpretation is empty, the sub-kinds' prototypes (dimension sets) can be used in order to
indirectly constrain membership in the set of bird entities.

The interpretation of each predicate can be identified with the interpretation of the disjunction of its sub-kinds' names or of its instances' names. For example, if $[[\text{sub-kind of bird}]]_{t,g} = \{\text{robin}, \text{pigeon}, \text{chicken} \ldots, \text{ostrich}\}$, then the interpretation of (instance of) bird in $t$ and $g$ can be identified with the interpretation of the predicate (instance of) robin or pigeon or chicken ... or ostrich. The criterion for membership in any disjunction is membership in at least one of the disjuncts (cf. chapter 6). Thus, the criterion for membership in the sub-kind-disjunction robin or pigeon or ... , requires that the mean in the dimensions of at least one of the disjuncts would be high enough, that is, $[[\text{bird}]]_{t,g} = [[\text{robin}]]_{t,g} \cup [[\text{pigeon}]]_{t,g} \ldots = \cup\{[[Q]]_{t,g} \mid \exists Q \in [[\text{sub-kind of bird}]]_{t,g}\}$ (I suppress the use of "instance of" in the entity interpretations). Thus, nothing new needs to be stipulated in order to account for this (exemplar-based) categorization strategy, except for a restriction on the sub-kind relation:

$$\forall n \in \mathbb{N}, \forall P, Q_1, \ldots, Q_i \in \text{CONCEPT}^n, \forall t \in T, \forall g \in G$$

$$[[P]]_{t,g} = \cup\{[[Q]]_{t,g} \mid Q \in \text{CONCEPT}^n \text{ and } \exists Q \in [[\text{sub-kind of } P]]_{t,g}\}.$$  
(The set of $P$'s sub-kinds consists of $Q_1$ and ... and $Q_i$ iff the set of $P$ entities consists of the set of $Q_1$ or ... or $Q_i$ entities)

For example, in each context 'robin' is a kind of 'bird' iff every robin must be a bird. Presumably, the exemplar-based categorization strategy is matched well by the exemplar based similarity function (that maps each entity to the sum of its degrees of similarity to P's sub-kinds), only because in effect this function favors entities that are very similar to one exemplar over entities that are only fairly similar to many (due to the use of an exponential inverse similarity function, as explained in 4.1).

In principle, in any context $c$, any known – e.g. – bird, $b$, can be viewed as the unique instance of (and hence the basis for the formation of the prototype of) a category $B$ (or a bird sub-kind $B$). Thus, we may easily produce categories based on instances, and these categories may affect categorization in the categories they are sub-categories of.

If two categorization strategies yield contradicting results, they must correspond to two different total contexts, or be marked by two different values for some implicit semantic parameter. But note that the two predicates instance of a bird and instance of a sub-kind of a bird though their denotations in total contexts are identical, may be learnt in different orders. I may know that some item is a bird while not being sure which bird (sub-kind) it is an instance of. In such a case, in any extension $t$, the entity that this item turns out to be is part of the denotation of bird, but it is not the case that in any extension $t$ the entity it turns out to be is a member of robin, it is not the case that in any extension $t$ the entity it turns out to be is in pigeon, not ... in chicken, etc. (It is in the denotation of instance of a sub-kind of a bird, but still may be considered atypical of each sub-kind, when considered separately.) Thus, the item may be typical of a bird (or of the sub-kind-disjunction), but not typical of any bird sub-kind. On the other hand, I may learn that an item is a bird rather late (relative to other birds) while learning that it is an ostrich (and that ostriches are birds) early (relative to other ostriches). Future research should focus on such predicates and explicate the connections between them.

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8.9.2 Typicality and genericity

We have given the following analysis of typicality statements with dimension-readings.

(23) \( \forall t \in T, \forall P \in \text{CONCEPT}, \text{s.t. } P \text{ is nominal:} \)
    a. \( [[F \text{ is typical of } P]]_{t,g} = 1 \iff F \in F^+(P,t,g). \)
       A predicate F is typical of P iff F is in P's dimension set
    b. \( [[\text{Typically, Fs are Ps}}]_{t,g} = 1 \iff F \in F^+(P,t,g). \)
       Ps are typically Fs iff F is in P's dimension set

To the best of my knowledge, this is the first semantic analyses of the contribution of expressions like typical or typically to truth conditions. These expressions have not been well understood or much discussed in the semantic literature up until now (Cohen 1999, p. 11). For example, Krifka et al (1995) discuss the adverb typically only concerning a certain test for genericity (p. 9). It is claimed that if adding to a sentence like birds fly an adverb like usually or typically (to form typically birds fly) yields at most a slight change of meaning, then the original sentence is not episodic but generic (it is a generalization over instances of a kind). This test forms evidence for the idea that genericity and typicality are deeply connected. Because one can usually be sloppy enough to freely switch between the two types of statements (and still express a good enough generalization), the task of teasing apart the differences between the two statement types is rendered rather difficult.

Characterizing generic statements are often assumed to involve an implicit quazi-universal determiner, GEN, with a vague domain (Kratzer 1989; Kadmon and Landman 1993; Carlson and Pelletier 1995). For example, an interpretation of the example in (24a) as a characterizing generic involves the use of the implicit determiner GEN with its vague domain ranging over a set of contextually relevant situations with birds.

(24) a. Birds fly  
    \( \text{Gen}_{s,x}(x \text{ are birds in } s, x \text{ fly in } s) \)
    b. African teams at the World Cup level (do not) arrive with local coaches  
    \( \text{Gen}_{s,x}(x \text{ is African team in WCL in } s, x \text{ does not arrive with coaches in } s) \)

Typicality statements too are like quantified statements in that they involve two arguments, or a restriction and a nuclear scope. (Means such as intonation, extraction, or previous context determine what material is mapped into the restriction and what material is mapped into the nuclear scope.) However, an operator which is different from GEN is explicitly expressed by the adjective typical or adverb typically, and it is this operator that combines the meaning of the two arguments, not GEN. The result is a different type of generalization (as I propose in 7.4.5).

(25) a. Typically, birds fly  
    (Properties like) flying, small, perching, etc. are typical (dimensions) of birds
    b. Typically, African teams at the World Cup level (do not) arrive with local coaches  
    (The property) arriving with local coaches is (not) typical of African teams in WCL.

What are the differences?
    On the one hand, the statement in (24b) is intuitively stronger than (25b), in the sense that
(24b) implies that the property \((\text{not}) \text{ arrive with local coaches}\) actually applies to \text{African teams at the WCL} in some situations, while (25b) seems to hold on a more abstract theoretical level. \((\text{Not}) \text{ arriving with local coaches}\) is more like a tag that is linked to these teams. (25b) entails neither that \((\text{not}) \text{ arriving with local coaches}\) necessarily occurs, nor that it occurs often (or relatively frequently), but rather that whenever it occurs, it expresses something which is inherent to being an African team or to the identity of such a team. It is implied that one is more like an African team when one does (or does not) arrive with local coaches. In addition, evidence that typicality statements are weaker than statements with an implicit generic operator is presented in connection with Ferreira (2005)\'s analysis of bare plural habituals. Ferreira observes that (26a) may provide a defense for John against rumors that he arrive at work drunk, while (26b), in such a context, is likely to be understood as suggesting that sometimes John does arrive at work drunk. Thus, Ferreira analyses bare plural habituals such as the statement in (26a) as involving an implicit plural definite determiner, which is stronger than overt operators such as \text{typically}.

(26)  

\begin{enumerate}  
\item When John arrives at work, he is sober  
\item When John arrives at work, he is typically sober  
\end{enumerate}

On the other hand, it is well known that generic statements are inherently vague. Often they allow difference of opinion regarding their truth, because their domains are vague. Fixing their domain is a rather flexible pragmatic matter. In contrast, the typicality statements are not vague and pragmatic dependent in this sense. Their truth depends on whether a property (predicate) characterizes (is an element of the dimension set) of a category \(P\) or not. The conditions under which a property is considered typical of a predicate are a strict yes or no rule. Naturally, one might not know whether the answer is yes or know due to ignorance about the facts, but this is a different kind of vagueness, which applies to the verification of just any statement.

Finally, note that the adverb \text{typically} may also occur in episodic statements (or statements with completely episodic entailments). (In fact, the negated adverb \text{atypically} can occur only in episodic statements.) For example the statement \text{typically for African teams at the World Cup level, they arrived (at the game yesterday) with local coaches} states about a certain team and a certain game that the team arrived at that game with local coaches, and that "arriving with local coaches" is a typicality dimension of \text{African teams at the World Cup level} (and if we drop \text{typically} the result is not a generic statement).

Thus, typicality statements are \text{not} generic statements. An adequate analysis of typicality statements has to capture this, and explicate the characteristics that are unique to typicality statement, and that it what I aimed to achieve with the above analysis of \text{typical} and \text{typically}. At the same time, given the tight connections between typicality and genericity, our improved understanding of the typicality phenomenon may also shed light on the analysis of generics. This issue is undoubtedly beyond the scope of this dissertation. However, let me just sketch the general idea that I have in mind.

The incorporation of a mechanism that represents typicality allows for a systematic mechanism for domain narrowing or widening, both at the level of individual sets (contextually restricted denotations) and at the level of modal bases (contextually restricted sets of worlds or contexts). I propose that typicality judgments can affect the truth conditions of generic statements, by helping to fix the modal base and by shaping the generic domain.

A statement like \text{a duck lays whitish eggs} is usually considered true in a context \(t\) and assignment \(g\) iff, roughly, all the \text{contextually relevant} ducks which lay eggs in \text{any alternative of}
reality (total context) s which is part of the modal base of GEN in t and g (healthy ducks, instances of typical duck-types, not genetically mutated etc.), lay whitish eggs in s and g. The "contextually relevant" bit is due to the vague generic determiner. Evidence that generics have vague, contextually restricted, domains is formed by the fact that they allow exceptions. For example, I may truthfully assert the given statement while standing in front of a duck with non-whitish eggs, as long as this duck can be regarded sufficiently atypical, and, consequently, be ignored. Evidence that generics are modal (GEN quantifies over alternatives of reality s), is formed by their counterfactual entailments. For instance, the given example entails that if I were an egg laying duck, my eggs would be whitish.

I propose that the generic statement a duck lays whitish eggs states roughly that in any alternative of reality, entities that are sufficiently good in the typicality dimensions of duck in t and g are such that they lay whitish eggs. I.e., the statement is generic because it abstracts away from those entities that are or are not ducks in the context of evaluation. It applies to any total context s where the dimension set of duck (and of any duck dimension, and maybe other predicates) is equal to its dimension set in t and g. In addition, the relevant entity set in each alternative is the set of ducks that satisfy these dimensions (the truth of the generic is based on them, so entities that violate them are irrelevant). So the generic entails that in any alternative of reality in which I am a (typical) egg-laying duck I lays whitish eggs. We see that the dimension set can give us the means for restricting both the modal base and the entity-set. If the domain is restricted by the typicality features, entities that violate but one dimension become irrelevant. Thus, incorporating typicality dimension sets into the formal semantic interpretation of predicates, may allow for an elegant account for the mechanisms that shape the generic modal base and domain, and for the tight connections between genericity and typicality, without stipulating new mechanisms for the purpose of an account for the generic domain. I leave this topic to future research.

Greenberg (2007) has shown that bare plural generics may be used to express more accidental generalizations (e.g. madrigals are popular), which cannot be expressed using the bare singular form (the bare singular would not receive a generic interpretation; compare, for example, (27a) to (29b)). We can now explain why. Sometimes, all the category members may be represented as sub-kinds, and then each one has a separate prototype (possibly, itself). Then, more incidental properties (which are usually variable across kind instantiations) might form part of their degree sets. For example, in (27b), all the phonologists in a certain department U are claimed to have the digit 8 in their ID number. This feature is not likely to form part of the prototype of phonologist in U (intuitively, it does not raise one’s typicality in this predicate). Thus no plausible generic interpretation for (27b) is available. But it is likely to form part of the representation of an individual (the ID number is one possible way to distinguish between individuals). And if accidentally, all the plural-entities (kinds that possess but one instantiation) which are similar enough to the prototype phonologist in U have this feature in their prototype representation, then this forms good evidence for (27a) (note that when a predicate P applies to a plural individual P distributes to the atoms of the plural individual).

9 This proposal can be formulated roughly as follows:
[[a duck lays whitish eggs]]_{t,g} = 1 iff
\forall s \in T, s.t. F'(egg laying duck, s, g) = F'(egg laying duck, t, g):
[[egg laying duck]]_{s,g} \cap [[\land F'(egg laying duck)]]_{s,g} \subseteq [[lays whitish eggs]]_{s,g}
(27) Singular and plural generics:

a. \([[Phonologists \text{ in this department have the digit 8 in their ID number}]]^+_{t,g} = 1\) 
   iff \(\exists k \subseteq (\cap Q | \cap Q \text{ is a sub-kind of phonologist in } U \text{ in } t,g),\) 
   \(\forall x \text{ s.t. } x \text{ is part of the sum } k:\) 
   \([[\text{Gen}_{y,s}(y \text{ is a realization of } x \text{ in } s, y \text{ has 8 in } y's \text{ ID in } s)]]^+_{t,g} = 1\) 
   iff \(\exists X \subseteq [[\text{phonologist in } U]]^+_{t,g}, \forall x \in X: \forall s \in MB(\text{Gen},t,g),\) 
   \(x \in [[\text{has 8 in its ID}]]_{t,g} \) 
   iff \(\exists X \subseteq [[\text{phonologist in } U]]^+_{t,g}, \forall x \in X: x \in [[\text{has 8 in its ID}]]^+_{t,g} \) and \"has 8 in its ID\" \(\in F'(\lambda y.y = x,t,g)\)

b. \([[A \text{ Phonologist in this department has the digit 8 in his/her ID number}]]^+_{t,g} = 1\) 
   iff \(\exists k \in (\cap Q | Q \text{ is a sub-kind of phonologist in } U \text{ in } t,g):\) 
   \([[\text{Gen}_{y,s}(y \text{ is a realization of } k \text{ in } s, y \text{ has 8 in } y's \text{ ID in } s)]]^+_{t,g} = 1\) 
   iff \(\exists x \in [[\text{phonologist in } U]]^+_{t,g}, \forall s \in MB(\text{Gen},t), x \in [[\text{has 8 in its ID}]],\) 
   \(x \in [[\text{has 8 in its ID}]]^+_{t,g} \) and \"has 8 in its ID\" \(\in F'(\lambda y.y = x,t,g)\)

Greenberg (2007) observes that the exceptions allowed by singular indefinite ("in virtue of") generics can easily be characterized compared to the exceptions allowed by bare plural generics. This is but natural, if indeed the cluster features of the kind denoted by the predicate in the restriction are used in singular indefinite kind interpretations, while the different clusters of the different sub-kinds are used in the bare plural. In the singular case, the exceptions are simply instances that violate features of the kind denoted by the predicate in the restriction. In the plural case, the exceptions depend on the sub-kind, and cannot be characterized as a single group.

Finally, this proposal explains the fact that if a kind denoting bare plural or mass noun must occur with a definite marker in a given language, then the kind denoting singular noun must occur with a definite marker too, but not vice versa (Dayal 2004). The set of sub-kinds of P may consist of the category members themselves. In the use of a bare plural, we will still consider the statement to be generic (it will indeed be a generalization over the category members). In the use of a bare singular we will not (it will only be a statement about one category member). Thus, in indefinite singualrs like a phonologist in U, generic interpretations (kind readings) much more often refer to the kind \(\cap \text{phonologist in } U\), rather than to sets of singletons (sub-kinds), because when they refer to sets of sub-kinds, we simply do not feel them to be kind readings. Thus, the singular forms are much more likely to be used as definite (and consequently more likely to be required to be used as definite), than the plural forms.
9 GRADABILITY IN POSITIVE AND NEGATIVE PREDICATES: MY PROPOSAL

9.1 Motivations for my proposals

9.1.1 Polarity

We have seen in 3.3-3.4 that there are polarity effects. First, some predicates are regarded as positive (*tall*) while others as negative (*short*). Second, the positive form of negative predicates does not combine with numerical degree phrases, even when the positive form of their positive counterparts does (cf. *two meters short* versus *two meters tall*), for reasons which are not completely clear. What is more, numerical degree phrases are always fine in the comparative (as in *two meters shorter*), and they also occur in argument position (as in *Dan is shorter than two meters*). These facts still await an explanation. Third, often the positive form of negative predicates does not combine with ratio phrases (cf. *twice as short* versus *twice as tall*). Fourth, negative and positive antonyms are not good in between predicate comparisons (*Dan is taller than Sam is short*), unless the negative predicate comes first, and the positive predicate lacks emphatic stress (as in *The ladder is shorter than the house is high*; Buring 2007), etc. In this chapter, I show that my proposals regarding the transformation value and the degree functions of positive and negative predicates explain all the above facts. Finally, clausal-, but not phrasal-, comparatives license negative polarity items (as in *today was hotter than it ever was in this area*, versus *today was hotter than any day*). In order to account for this, Landman (2005) has enriched the interpretation of predicates with supremum and difference operations. I show that given my proposal, Landman’s (2005) supremum theory can be incorporated without these stipulations.

We have seen that semantic theories regard degrees as representing quantities of something (the denotation of the nominalization of the predicate or its antonym) and, in accordance, they postulate additivity as a general feature of predicates’ degree functions. This is good for predicates like *long*:

(1) Quantity functions are additive (Klein 1991):

\[ \forall g \in G, \forall t \in T, \ f^*(long,t,g) \text{ is additive } \iff \text{it satisfies the following two conditions:} \]

a. \( \forall d_1, d_2 \in D: \ f^*(d_1 \oplus d_2, long, t, g) = f^*(d_1, long, t, g) + f^*(d_2, long, t, g) \)

The degree of length of the concatenation \( d_1 \oplus d_2 \) equals the degree of length of \( d_1 \) plus the degree of length of \( d_2 \) in \( t \) and \( g \).

b. \( f^*(d_1, long, t, g) = f^*(d_2, long, t, g) \iff f^*(d_1 \oplus d_2, long, t, g) = 2 \times f^*(d_1, long, t, g) \)

If \( d_1 \) and \( d_2 \) are equally long the degree of length of \( d_1 \oplus d_2 \) equals two times the degree of length of \( d_1 \).

But we have seen that the quantity metaphor is misleading in that many natural language predicates are not additive wrt their (presumed) dimension (or nominalization). In particular, in negative predicates like *short* the mapping of entities to degrees is not additive wrt quantities of height in every \( t \) and \( g \). We know (or we have very strong intuitions) about the entity ordering of *short* that it is reversed to that of *tall* (*Dan is taller than Sam iff Sam is shorter than Dan*). Thus, the degrees are reversed (if Dan is mapped to a higher degree in *tall* Sam is mapped to a higher degree in *short*). But we do not know how they are reversed (there are many possible functions that are reversed wrt to the degree function of *tall*). To account for these intuitions, I have
I. I have proposed that in each total context t and assignment g, the additive degree function P is linked with, \( f^+(P,t,g) \), the thing that in our model represents quantities of something in the world (for instance, \( P\text{-hood} \), the thing denoted by the nominalization of P or its antonym). I have taken this to be an axiom. That's what \( f^+(P,t,g) \) does by its definition. In addition to \( f^+(P,t,g) \), in each t and g, P is associated with a transformation value (a real number), \( \text{Tran}^+(P,g,t) \), which together with \( f^+(P,t,g) \) helps to fix the values that degree-terms (expressions of the form \( \deg(\alpha,P) \)) denote. I have proposed that we know about the degrees of negative predicates that they are produced by reversed functions, but we do not know which reversed function. There are many candidates. For any g and t, for any constant \( \text{Tran}\in R \), a function \( f_{\text{Tran}} \) that assigns any d the degree (\( \text{Tran} – f^+(d,tall,t,g) \)) can do the job of reversing the degrees. When the constant is not zero, these functions are not additive wrt height. Consider, for example, the function \( f_{1–f}(long,t,g) \) that maps each d in d to the value \( (1 – f^+(d,long,t,g)) \). It does not represent adequately quantities of length (that thing quantities of which \( f^+(long,t,g) \) adequately represents) – it is not additive wrt length. If two entities \( d_1 \) and \( d_2 \) have the same length, say, \( f^+(d_1,long,t,g) = f^+(d_2,long,t,g) = 5 \), then by additivity, \( f^+(d_1\oplus d_2,long,t,g) = 10 \). But by the definition of \( f_{1–f} \), \( f_{1–f}(d_1) = f_{1–f}(d_2) = -4 \), and \( f_{1–f}(d_1\oplus d_2) = 1 – f(d_1\oplus d_2) = -9 \neq (2 \times -4) \). We see that a function that maps entities to their length quantity transformed by a constant (and, in the given example, reversed) is not additive wrt length. The ratios between the degrees that this function assigns to entities (like \( d_1 \) and \( d_1\oplus d_2 \)) do not adequately represent the ratios between the quantities of length in them (e.g. the ratio between the degrees of \( d_1\oplus d_2 \) and \( d_1 \) is 9/4 and the ratio between their quantities of length is 8/4).

Do we have intuitions that tell us that \( \text{Tran}^+(short,g,t) \) is zero (in any total extension t of an actual context c and any g)? Well, I do not think so. This can be tested by checking our intuitions concerning the value of individuals with zero height. If for any extension t of an actual context c and any g, \( \text{Tran}^+(short,g,t) = 0 \), and \( f^+(tall,t,g) \) is additive (it maps entities with no height to 0), then the degree of entities with no height in short should be zero. But is it? I do not know. Maybe they are mapped to a number that approximates infinity? This view is taken by some well-known semantic theories (von Stechow 1984; Kennedy 1999). But if so, then the mapping function of short is not additive, it is a function that transforms height quantities by a constant, \( \text{Tran}^+(short,g,t) \). To be honest, we do not know anything about the constant; it may be any real number from zero to infinity. It should be represented as a value that varies between total contexts, \( \text{Tran}^+(short,t,g) \), and that may be unknown in a partial context (as we have represented it in previous chapters).

We see then that there is a sharp semantic distinction between positive and negative predicates. Negative predicates denote reversed degree functions, where the precise reversed function is unknown. In other words, their transformation value is unknown. In what follows I show that by representing this distinction, we can explain the polarity effects.

**Proposed** that the interpretation of degree terms is given by the following rules, where negative predicates denote functions that are reversed wrt their positive antonyms.

\[
(2) \quad \text{Positive and negative (reversed) degree functions:}
\]

\[
\forall g \in G, \forall t \in T, \forall n \in N, \forall P \in \text{CONCEPT}^n, \forall \alpha \in \text{TERM}: \\
\text{a. If } P \text{ is a positive predicate:} \\
\quad \left[ \deg(\alpha,P) \right]_{tg}^* = f^+(\left[ \alpha \right]_{tg}^*, P,t,g) - \text{Tran}^+(P,g,t) \\
\text{b. If } P \text{ is a negative predicate:} \\
\quad \left[ \deg(\alpha,P) \right]_{tg}^* = \text{Tran}^+(P,g,t) - f^+(\left[ \alpha \right]_{tg}^*, P,t,g)
\]

I.e., I have proposed that in each total context t and assignment g, the additive degree function P is linked with, \( f^+(P,t,g) \), is the thing that in our model represents quantities of something in the world (for instance, \( P\text{-hood} \), the thing denoted by the nominalization of P or its antonym). I have taken this to be an axiom. That's what \( f^+(P,t,g) \) does by its definition. In addition to \( f^+(P,t,g) \), in each t and g, P is associated with a transformation value (a real number), \( \text{Tran}^+(P,g,t) \), which together with \( f^+(P,t,g) \) helps to fix the values that degree-terms (expressions of the form \( \deg(\alpha,P) \)) denote. I have proposed that we know about the degrees of negative predicates that they are produced by reversed functions, but we do not know which reversed function. There are many candidates. For any g and t, for any constant \( \text{Tran} \in R \), a function \( f_{\text{Tran}} \) that assigns any d the degree (\( \text{Tran} – f^+(d,tall,t,g) \)) can do the job of reversing the degrees. When the constant is not zero, these functions are not additive wrt height. Consider, for example, the function \( f_{1–f}(long,t,g) \) that maps each d in d to the value \( (1 – f^+(d,long,t,g)) \). It does not represent adequately quantities of length (that thing quantities of which \( f^+(long,t,g) \) adequately represents) – it is not additive wrt length. If two entities \( d_1 \) and \( d_2 \) have the same length, say, \( f^+(d_1,long,t,g) = f^+(d_2,long,t,g) = 5 \), then by additivity, \( f^+(d_1\oplus d_2,long,t,g) = 10 \). But by the definition of \( f_{1–f} \), \( f_{1–f}(d_1) = f_{1–f}(d_2) = -4 \), and \( f_{1–f}(d_1\oplus d_2) = 1 – f(d_1\oplus d_2) = -9 \neq (2 \times -4) \).
9.1.2 Difference operators: "One-dimensionality" versus "Multi-dimensionality"

We have seen that semantic theories regard degrees as representing quantities of something (the denotation of the nominalization of the predicate or its antonym) and, in accordance with that, they postulate additivity. We saw that the quantity metaphor is misleading in that some predicates are not associated with additive final degree functions in some total contexts. The second respect in which the quantity metaphor is misleading is that some predicates may well have additive final degree functions in any total context, but the thing wrt which their function is additive (quantities of which it represents) may vary considerably between total contexts (not only in funny counterfactual contexts, but even in total extensions of contexts that represent the actual knowledge of a competent speaker). This is typically the case with multi-dimensional predicates.

Consider, for instance, the predicate *bird*. The set of bird dimensions and their weights is partial. They vary considerably between contexts. The things quantities of which the additive degree function of *bird* measures and their relative importance varies between different total contexts t and assignments g. Hence, there is no one thing quantities of which the predicate *bird* can be said to represent. Our knowledge about this predicate is like a recipe for a cake that does not specify the quantities of the ingredients. In one total context, the cake may include two cups of sugar, in another three cups, and in yet another no sugar at all. In one context it may include chocolate, in another cheese. In a partial context, this recipe adequately represents no cake.

Intuitively, predicates are multi-dimensional iff they may have in their dimension set several different dimensions (i.e. predicates whose degree functions are not identical). Thus, nominal concepts like *bird* are inherently multi-dimensional. They are known to be associated with many dimensions. But can't there exist a context structure such that in each total context we represent the dimension set of *bird* as consisting only of predicates whose meaning is identical to that of *bird* (i.e. where the dimension set is one-dimensional like the dimension set of *long*)? I propose that we do not ever represent *bird* as one-dimensional, because there is a principled difference between *bird* and *long*.

Let us call the contexts that represent the world knowledge of competent speakers actual (these contexts contain all and only the knowledge that speakers possess "out of the blue" in the lack of additional context). Let us call their total extensions actual total contexts. Other contexts are counterfactual, in the sense that some piece of knowledge in them is already known to speakers to be wrong.

I have proposed that in each total context t and assignment g, the additive degree function that P is linked with, $f^+(P,t,g)$, is the thing that in our model represents quantities of something in the world (maybe the thing denoted by the nominalization of P or its antonym). That's what $f^+(P,t,g)$ does by its definition (cf. 6.1). Based on this, I propose that a predicate P is felt to be one-dimensional iff there is something in the world quantities of which it represents in every total extension (of any actual partial context). It is important to realize that this does not mean that an adjective P is one-dimensional iff $f^+(P,t,g)$ is identical in every t and g. Let me explain.

I would claim that all the predicates are associated with numerical degree functions per context t. How do I know that, though? In previous vagueness based gradability theories (cf. Kamp 1975), a predicate is assigned a degree function once per model (the values of its degree function do not vary between total contexts). This represents the intuition that the ordering of entities along predicates like *tall* is not context dependent. I do agree with this basic intuition. Furthermore, intuitively, the ratios between degrees of height of pairs of entities do not vary with
context (because these ratios represent the ratios between quantities of height in entities in D, and all the total contexts represent information about this one and the same universe D). For the same reason, the ratios between differences between degrees of height do not vary, etc.

But what about the actual numbers that represent height? We can tell that in any two contexts \( t_1 \) and \( t_2 \), objects with no height are mapped to 0 (because \( f \) represents height quantities adequately). Yet, the numbers to which \( \text{tall} \) maps entities with non-zero quantities of height are unknown. \( \text{Height, length, heat, happiness} \), etc. are all mass nouns. They denote 'stuff' (so to speak), quantities of which you cannot count directly. Thus, if we are to associate predicates with numerical degree functions, these must be context dependent. \( \text{Tall} \) can map any non-zero "amount of length" to any positive real number. We can positively say about no non-zero "amount of P-hood" that it has a known value, say, 1, or 2 or 3. Consider, for example, the meter. On some rulers its degree is 1, on others 100, on others 100000, on rulers with inches it has yet other degrees, etc. If we postulate degree functions once per model we fail to represent this fact. Sure, the object(s) that form the basis for defining the unit meter (the original meter, and any ruler that is based on it) do not change their length from one context to another (the same set of things are \( \text{meters} \) or \( \text{equally long as the meter} \) in any actual total context, and perhaps even in any counterfactual context). But the numbers they are mapped to (co-)vary: In any two contexts \( t_1 \) and \( t_2 \), objects with some given non-zero quantity of height in them (say, entities \( d \) that are one meter tall) are mapped to some numeral in \( t_1 \), \( f^+(d, \text{tall}, t_1, g) = r_1 \) and to some numeral in \( t_2 \), \( f^+(d, \text{tall}, t_2, g) = r_2 \) (and \( r_1 \) and \( r_2 \) are not necessarily equal), and objects with \( n \) times this quantity of P-hood are mapped to the numeral \( n \times r_1 \) in \( t_1 \) and to the numeral \( n \times r_2 \) in \( t_2 \) (so the ratios between quantities of height are adequately represented in each total context).

Given this background, here is my way of making the notion of one-dimensionality precise. When I say that a predicate \( P \) is \( \text{one-dimensional} \) in a context \( c \), i.e. that "there is something in the world quantities of which it represents in every total extension of \( c \)", I mean that the ratios between the values that \( f^+(P, t, g) \) assigns to entities are kept equal in all the total extensions of \( c \) (for any \( g \)). And if these ratios are kept equal in all these total contexts, that means that "there is something in the world quantities of which it represents in every total extension of \( c \)" (it is one-dimensional in \( c \)).

(3) \text{An n-place predicate P is one-dimensional in c iff:}

There is one known thing quantities of which \( f^+(P, t, g) \) represents in every \( t \in T_c \) and \( g \), i.e., the ratios between values that \( f^+(P, t, g) \) assigns to entities are kept equal in all the extensions of \( c \) (they are known numbers in \( c \)):

\[
\forall d_1, d_2 \in D^n, \exists r \in R: \forall t \in T_c, \forall g \in G: f^+(d_1, P, t, g) = r \times f^+(d_2, P, t, g)
\]

In other words:

\[
\forall d_1, d_2 \in D^n, \forall t_1, t_2 \in T_c, \forall g \in G: f^+(d_1, P, t_1, g) / f^+(d_2, P, t_1, g) = f^+(d_1, P, t_2, g) / f^+(d_2, P, t_2, g)
\]

Otherwise, \( P \) is \( \text{multi-dimensional} \) in \( c \).

If we consider actual contexts, then examples of predicates that are one-dimensional in them include predicates like \( \text{tall, long, short} \), etc. Examples of multi-dimensional predicates include all the nominal concepts, conjunctions and disjunction, and multi-dimensional concepts whose respect-argument is a multi-dimensional dimension (like \( \text{healthy wrt blood pressure and pulse and} \) ...). Thus, even if a predicate has no two dimensions with different degree functions in its dimension set, it may not count as one-dimensional, unless the condition in (3) is satisfied.
For example, if the degree function of the predicate fat in a context c measures mere 'weight', then fat is one-dimensional in c. Furthermore, if the degree function of the predicate fat measures the ratio between one's weight and one's height, such that there is a formula that says exactly how these two dimensions combine (say, for any d, extension t of c and g, $F^*(fat,t,g)$ maps d to d's weight divided by half its height), then fat is still one-dimensional (and it has in its dimension set one dimension, say – weight divided by height, multiplied by 2). Fat may even have several different senses (perhaps lexical entries), and the dimension-set of each sense may consist of a different, but a unique, dimension in c. The predicate long has two senses (in one sense it measures spatial length and in another it measures temporal duration), but in each sense, it is one-dimensional in any actual context c. However, if in an actual context for (a given sense of) fat, there is no formula that says how two dimensions (like weight and height or weight and volume) combine, then this entry of fat is multi-dimensional in that context. Nominal concepts are almost all multi-dimensional in partial actual contexts (cf. chapter 2 and 4). Consider for example, the interpretation of the nominal concept fat man. If the two dimensions weight and volume form part of its dimension-set together, then in different contexts, the degree function of fat man corresponds to different recipes, so to speak. Different weights are assigned to the two dimensions and maybe to many others, too, (say, waist line, general look, physical condition, etc.), and in each total extension t and g, entities are mapped to their average in these dimensions. The degree function in multi-dimensional adjectives too (and hence the ordering between entities) may vary between different actual total contexts (so the ratios between degrees are not preserved). For example, if one individual has more volume and another one is heavier it is not clear who is fatter (wrt weight and volume).

Distance-predicates (predicates whose additive function maps entities to degrees that represent their distance from a selected value, Value*(P,t,g)), are one-dimensional in a context c iff Value*(P,t,g) is known by convention. It does not vary through total extensions of c. In the lack of knowledge about the ideal blood pressure, I cannot tell who is healthier wrt bp than whom, so even the ordering between entities is unknown. Most of the discussion below focuses on one-dimensional predicates. I specifically discuss distance-predicates in 9.5.2, and nominal concepts in 9.5.4.

In what follows, I show that by representing the distinction between predicates for which there is something they measure in any actual t and g, and predicates for which there is no such thing, we can explain why some predicates combine with comparative morphemes and others do not in within predicate comparisons.

9.2 My proposal

In 9.2.1-9.2.2 I define three types of predicates that differ wrt features of their transformation value and additive degree function. I describe a set of facts concerning each predicate type (these facts are derived from the theoretical proposals that I have described in 9.1). These facts are then used in 9.3-9.5 in explaining a long list of polarity effects and distributional facts.

9.2.1 Ratio versus interval predicates

Let us call predicates ratio predicates iff in all the actual total contexts their final degree function ([(deg(P))]t,g) does not transform values relative to their additive degree function (i.e., predicates P s.t. Tran*(P,t,g) is known to be zero). Otherwise let us call P an interval predicate.
A predicate $P$ is a *ratio predicate in $c$* iff

In no total context $t$ in $T_c$ and assignment $g$, does $[\deg(P)]^+_{t,g}$ assigns transformed degrees:

$$\forall t \in T_c, \forall g \in G: \forall d \in D: \deg^+(d,P,t,g) = f^+(d,P,t,g) \quad \text{or} \quad \deg^+(d,P,t,g) = -f^+(d,P,t,g)$$

iff $\forall t \in T_c, \forall g \in G$: $\text{Tran}^+(P,t,g) = 0$

Otherwise, $P$ is an *interval predicate in $c$*.

Examples of ratio predicates (predicates which are ratio in any actual context $c$) include *tall, long* and *wide*, but not predicates like *short*.

How can we tell that a given positive predicate is mentally represented as *ratio* (that $[\deg(P,t,g)]^+_{t,g}$ is additive, rather than transformed, in any $t$ and $g$ above the contexts $c$ that represent our "out of the blue" knowledge)?

In positive predicates like *long* we have measuring tools that do the job in an additive way (you can measure the concatenation of two rods, you can measure the two separately, and you see that the former value equals the sum of the latter two values. This would not have been the case had there been a hidden non-zero transformation value). The situation is more obscure for predicates like *happy*. How can you tell that *happy* links entities with numerical values that are not transformed? We saw that some basic intuitions correspond to the ratio / interval distinction.

First, we have the zero-test. If in a context $c$, I map to zero only stones, non-living objects, or living objects that clearly have no happiness in them this is a sign of a ratio scale. *Happy* passes this test. But consider *warm*. Zero heat should in principle correspond to the minimal degree on the scale of *warm*, but intuitions are blurred over here. What is zero heat? Is there such a thing at all? These question-marks show that *warm* fails the test. Entities are not necessarily mapped to non-transformed degrees. Scientists can tell us the facts. They can tell us that the degree zero on the Kelvin scale represents zero heat. But many speakers may not be aware of this scientific fact, or may not be willing to change their mental representations by mapping to zero the entities about which scientists talk (in addition, the scientists themselves use transformed units: the 0 on the Celsius scale does not correspond to zero heat; the existence of this unit may form further evidence for the hypothesis that intuitively temperature-predicates like *warm* are interval, not ratio, predicates). The same argument applies to *cold*, which is also a reversed function.

Note that these predicates have two (related) senses. The sense I have spoken about corresponds to their interpretation when they are associated with units, as in *today was 30 degrees Celsius warmer than yesterday*. In this sense, these predicates measure the temperature out there in the world, and their intuitive representation seems to be as interval-predicates. In another sense, they refer to mental states, extents of personal experiences with things with temperature (as in: *I think today was (twice) as cold/ warm as yesterday / not cold/ not warm at all*). In this sense, these predicates measure the extent to which one feels that something is warm or cold. Such mental quantities cannot be measured by units like Celsius or Kelvin. For instance, the extent of my experience when I touch things that are 40 degrees Kelvin may not be exactly twice the extent of my experience when I touch things that are 20 degrees Kelvin (in addition, I may not know how a unit Kelvin feels at all). If we apply the zero test to this sense, then intuitively, *cold* maps to zero

---

1 The interpretation of the predicate *warm* is more similar to that of *not cold* than to that of *hot*, in that you do not require that warm entities will have much heat (will reach the standard of *hot*), you only require that they will not have too little heat (i.e. that they will not be *cold*). Thus, perhaps *hot* is a ratio predicate, *cold* is reversed once (so it is a negative interval predicate), and *warm* is reversed twice (so it is positive, but also an interval predicate).
all and only things that do not feel cold at all, and \textit{warm} maps to zero all and only things that do not feel warm at all. So in this sense, these predicates are intuitively ratio predicates.

Second, the intuitions that are based on the zero-test are further supported by the fact that ratio modifiers like \textit{twice} or \textit{four times} combine more easily with ratio-predicates (like \textit{happy}) than with interval-predicates (like \textit{short}, and maybe even \textit{warm}). You can say that someone is \textit{twice as happy}, but saying that something is \textit{twice as short} is less natural, and the combinations *\textit{twice as unhappy}, \textit{unsafe} or \textit{not happy} are even worse. Predicates like \textit{unhappy} are explicitly negated, so they cannot be interpreted as positive (while perhaps \textit{cold} can, when it refers to mental states). They are the most likely to be linked with reversed functions with no conventions concerning their transformation value. As we saw in 2.1, these intuitions are supported by the fact that negative predicates usually occur less often with ratio predicates (this generalization is based on the number of entries of the form “\textit{twice as P as}” and “\textit{half as P as}” that I have found in a Google-search and an MSN-search for 16 pairs of antonym predicates). There are other factors that affect the distribution of ratio-modifiers, but the ratio–interval distinction is certainly an important one.

Third, numerical degree predicates are good with ratio predicates (when these have established units). For example, you can say that something is \textit{n meters tall}, but not that something is \textit{n meters short}, and as Kennedy (2001) observes, the combinations #\textit{two degrees warm} and #\textit{two degrees cold} are not so natural. In this chapter I give an analysis of unit names and of the semantics of ratio-modifiers, and I show how this explains the facts.

The following facts hold true of ratio predicates:

(5) Facts: For an actual context c, all and only the \textbf{ratio predicates P in c} are such that:

\begin{enumerate}
\item a. They are positive predicates iff in any \( t \in T_c \) and g: \([\deg(P)]^+_{t,g} = f^+(P,t,g)\)
\end{enumerate}

(because in positive predicates \([\deg(\alpha,P)]^+_{t,g} = f^+([\alpha])^+_{t,g,P,t,g} – \text{Tran}^+(P,g,t))\)

\begin{enumerate}
\item b. They are negative predicates iff in any \( t \in T_c \) and g: \([\deg(P)]^-_{t,g} = \lambda d.– f^-(d,P,t,g)\)
\end{enumerate}

(because in negative predicates \([\deg(\alpha,P)]^-_{t,g} = \text{Tran}^-(P,g,t) – f^-([\alpha])^-_{t,g,P,t,g})\).

\begin{enumerate}
\item c. They are one-dimensional in c iff the ratios between values that \([\deg(P)]^-_{t,g}\)
\end{enumerate}

assigns to entities are the same in all the contexts in \( T_c \) (they are known numbers):

\( \forall d_1,d_2 \in D, \exists R: \forall t \in T_c, \forall g \in G: deg^+(d_2,P,t,g) = r \times deg^+(d_1,P,t,g)\) \hspace{1cm} (Because in one-dimensional predicates ratios between values assigned by \( f^+(P,t,g)\) are always the same in all the contexts in \( T_c \):

\( \forall d_1,d_2 \in D, \exists R: \forall t \in T_c, \forall g \in G: \lambda t,g: \deg^+(d_1,P,t_1,g) / \deg^+(d_2,P,t_1,g) = \deg^+(d_1,P,t_2,g) / \deg^+(d_2,P,t_2,g)\)

These facts do not hold true of interval predicates. For example, even if P’s transformation value is fixed to be one and the same number in every t and g, say Tran\(^+(P,t,g) = 1\), since P’s additive degree function may map two entities \( d_1 \) and \( d_2 \) to 2 and 4 in \( t_1 \) and \( g \) and to 4 and 8 in \( t_2 \) and \( g \), it follows that \( \deg^+(d_1,P,t_1,g) / \deg^+(d_2,P,t_1,g) = (\text{Tran}^+(P,t_1,g) / f^+(d_1,P,t_1,g) – \text{Tran}^+(P,t_1,g) / f^+(d_2,P,t_1,g)) = 4 – 1 / 2 – 1 = 3/1 \neq \deg^+(d_1,P,t_2,g) / \deg^+(d_2,P,t_2,g)\) = \{\text{Tran}^+(P,t_2,g) / f^+(d_1,P,t_2,g) – \text{Tran}^+(P,t_2,g) / f^+(d_2,P,t_2,g)\} / \{f^+(d_1,P,t_2,g) – \text{Tran}^+(P,t_2,g)\} = 8 – 1 / 4 – 1 = 7/3. Thus, we do not know the ratios between the values that P’s final degree function gives to entities in interval predicates P (these ratios are not kept equal in all total contexts).

In 9.2.3, I propose that unit names like \textit{meter} denote (characteristic functions of sets of) agreed upon objects, and since \textit{long} is a ratio predicate, the ratios between any object’s degree and the meter unit objects’ degree is always known in actual contexts. This fact allows us to map entities
to known numbers (i.e. when we say that Dan is 1.87 meter tall, this does not mean that Dan’s degree of height is 1.87. Rather, it means that whatever Dan’s degree and the meter unit-objects’ degree of height are, the ratio between them is 1.87). In 9.3 I show that this observation explains the infelicity of numerical degree predicates with negative (and some positive) interval predicates (as in *two meters short). It also explains their felicity with ratio predicates.

9.2.2 Interval versus ordinal predicates

When we calculate the difference between the degrees of two entities in an interval predicate, the two transformation values cancel one another. For example, for any positive predicate P, two terms α and β, and any t and g:

\[
[deg^+(\alpha, P)]^t_{tg} - [deg^+(\beta, P)]^t_{tg} = (f^+([\alpha]^{t,g}, P, t, g) - Trans^+(P, g, t)) - (f^+([\beta]^{t,g}, P, t, g) - Trans^+(P, g, t))
\]

Similarly, if P is negative,

\[
[deg^-(\alpha, P)]^t_{tg} - [deg^-(\beta, P)]^t_{tg} = (Trans^+(P, g, t) - f^+([\alpha]^{t,g}, P, t, g)) - (Trans^+(P, g, t) - f^+([\beta]^{t,g}, P, t, g))
\]

Thus, we can state that the following fact holds true of any one-dimensional predicate in an actual context c, including interval predicates whose transformation value is unknown:

(6) Fact: For an actual context c, all and only the one-dimensional predicates in c are s.t.: a. The ratios between differences between values that \([deg(P)]^t_{tg}\) assigns to entities are the same in all the total extensions t of c (they are known numbers):

\[
\forall d_1...d_4 \in D, \exists \in R: \forall t \in T_c, \forall g \in G: (deg^+(d_1, P, t, g) - deg^+(d_2, P, t, g)) = r \times (deg^+(d_3, P, t, g) - deg^+(d_4, P, t, g))
\]

In other words:

\[
(\text{deg}^+(d_1, P, t_1, g) - \text{deg}^+(d_2, P, t_1, g)) / (\text{deg}^+(d_3, P, t_1, g) - \text{deg}^+(d_4, P, t_1, g)) = (\text{deg}^+(d_1, P, t_2, g) - \text{deg}^+(d_2, P, t_2, g)) / (\text{deg}^+(d_3, P, t_2, g) - \text{deg}^+(d_4, P, t_2, g))
\]

b. The entities’ orderings are the same in all total extensions t of c (they are known):

\[
\forall d_1, d_2 \in D, \forall t_1, t_2 \in T_c, \forall g \in G: \text{deg}^+(d_1, P, t_1, g) > \text{deg}^+(d_2, P, t_1, g) \quad \text{or} \quad \forall t \in T_c, \forall g \in G: \text{deg}^+(d_1, P, t, g) < \text{deg}^+(d_2, P, t, g) \quad \text{or}
\]

\[
\forall t \in T_c, \forall g \in G: \text{deg}^+(d_1, P, t, g) = \text{deg}^+(d_2, P, t, g)
\]

For example, even if P's transformation value varies between total contexts, say \(Trans^+(P, t_1, g) = 1\) and \(Trans^+(P, t_2, g) = 8\), for four entities \(d_1 \ldots d_4\) (say, such that P's additive degree function may map \(d_1 \ldots d_4\) to 2, 3, 4, and 5 in \(t_1\) and \(g\) and to 4, 6, 8, and 10 in \(t_2\) and \(g\), the ratios between (i) the difference between \(d_1\) and \(d_2\) and (ii) the difference between \(d_3\) and \(d_4\) does not depend on the transformation value, and hence it is kept equal:

\[
\text{deg}^+(d_1, P, t_1, g) - \text{deg}^+(d_2, P, t_1, g)) / \text{deg}^+(d_3, P, t_1, g) - \text{deg}^+(d_4, P, t_1, g)) = (2 - 3) / (4 - 5) = 1/1 = 1 = \text{deg}^+(d_1, P, t_2, g) - \text{deg}^+(d_2, P, t_2, g)) / \text{deg}^+(d_3, P, t_2, g) - \text{deg}^+(d_4, P, t_2, g)) = (4 - 6) / (8 - 10) = 2/2 = 1
\]
Thus, even in interval predicates whose transformation value is unknown (i.e., many negative predicates), where the ratios between degrees of entities do vary between total extensions of actual contexts as explained above, if they are one-dimensional in an actual context \( c \), the entities’ ordering does not vary between total extensions of \( c \) (the transformation constant cannot change the ordering between the degrees of different entities), and the ratios between differences between degrees (values that \( P \)'s final degree function assigns to entities) are kept equal in all these contexts (they are known numbers).

These facts do not hold true of multi-dimensional predicates in \( c \). In predicates like \( \text{bird} \), the ordering between some of the entities varies between total extensions of \( c \) (depending on the entities’ values in the predicates that are \( \text{bird} \) dimensions in each extension of \( c \) and their weights). Thus, even if their final degree functions are additive in every \( t \) in \( T_c \), these functions represent quantities of different things in different total contexts, and for that reason, even when the transformation values are canceled out (when differences between entities’ degrees are considered), and even when the ordering between entities is known (it is the same in all the total contexts), the ratios between degrees are not constant across different total contexts, and the ratios between differences between degrees are not constant across different total contexts. Let us call these predicates ordinal in \( c \).

For example, even if \( P \)’s transformation value is zero in any \( t \), for four entities \( d_1 \ldots d_4 \) \( P \)'s additive degree function may map \( d_1 \ldots d_4 \) to 2, 3, 4, and 5 in \( t_1 \) and 4, 5, 7, and 10 in \( t_2 \) and \( g \), so the ratios between (i) the difference between \( d_1 \) and \( d_2 \) and (ii) the difference between \( d_3 \) and \( d_4 \) are not the same in \( t_1 \) and \( t_2 \):

\[
\frac{\deg^+(d_1,P,t_1,g) - \deg^+(d_2,P,t_1,g))}{\deg^+(d_1,P,t_1,g) - \deg^+(d_4,P,t_1,g))} = \frac{2 - 3}{4 - 5} = \frac{1}{1} = 1 \neq \frac{\deg^+(d_1,P,t_2,g) - \deg^+(d_2,P,t_2,g))}{\deg^+(d_3,P,t_2,g) - \deg^+(d_4,P,t_2,g))} = \frac{4 - 5}{7 - 10} = \frac{1}{3}
\]

We see then that there is a sharp semantic distinction between one-dimensional and multi-dimensional predicates. In 9.5 I show that this distinction explains the infelicity of difference modifiers (comparative morphemes) with multi-dimensional concepts (namely, nominal concepts, or adjectives when they are interpreted relative to several dimensions simultaneously).

To summarize, I propose that the different types of degree functions that we have introduced in chapter 7 produce three important predicate types, ratio, interval and ordinal predicates (we do not need to stipulate anything new in order to get these distinctions). Table 6 gives examples of these three predicate types.

<table>
<thead>
<tr>
<th>Table 6: Predicate types</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One dimensional</strong></td>
</tr>
<tr>
<td><strong>Ratio</strong></td>
</tr>
<tr>
<td>- Many measures of physical quantities, such as mass, length, and energy</td>
</tr>
<tr>
<td>- Measures of age</td>
</tr>
<tr>
<td>- Length of residence in a place.</td>
</tr>
<tr>
<td>Tall, long, wide, big, hot, weighs, happy, coldmental</td>
</tr>
</tbody>
</table>
I have borrowed the names for these predicates types from the well-known taxonomy of scale types, which goes back to Stevens (1946; 1975), and is widely used in statistics, as a cue as to which operations can apply to which property-scales (Babbie 2004). The taxonomy consists of *ratio*, *interval*, *ordinal* and *nominal* scales. I define these notions per *partial context*, but originally they are not defined per partial contexts. Originally, *interval-scales* (or measuring functions), are defined to be scales in which the zero point is arbitrary and negative values can be used (these functions are not additive), so ratios between numbers on the scale are not meaningful. For that reason, *operations such as multiplication and division cannot be carried out directly* (only ratios between differences between degrees on the scale are meaningful). Operations of division and multiplication can only be used in *ratio-scales* where the zero is not arbitrary and the ratios between numbers on the scale are meaningful (and for that reason only ratio scales can have real units; see discussion of this point in ongoing sections). Finally, in *ordinal-scales*, the differences between any two adjacent degrees are arbitrary (only their ordering is meaningful), so differences between numbers on the scale are not meaningful. For that reason, *operations such as difference and plus cannot be carried out*.

According to my proposal, all the final degree functions that are linked with predicates in total contexts are ratio-scale functions (they consist of the real numbers, for which, as we all know, we have operations such as division and multiplication). However, *the knowledge that speakers have about these degree functions is never total*. Crucially, the knowledge that speakers actually have about entities' degrees in predicates which I call *ratio* includes knowledge about the ratios between entities' degrees. The knowledge that speakers actually have about entities' degrees in predicates which I call *interval* does not include knowledge about the ratios between entities' degrees, only about the ratios between degree-differences in entity-pairs. Finally, the knowledge that speakers actually have about entities' degrees in predicates which I call *ordinal* includes knowledge about neither the ratios between entities' degrees, nor the ratios between degree-differences in entity-pairs. The distinction between ratio, interval and ordinal scales plays a very important role in science (in statistics, research methods, etc.). For that reason, I propose that we need to check whether these distinctions can also provide constraints on the distribution of difference- and ratio-modifiers in natural languages. The hypothesis that they do is the minimal, most obvious, explanation for the distribution of such modifiers, and it should be ruled out before more complex accounts are considered. In 9.3-9.5, I take up this task, and I show that this minimal explanation is adequate. In 9.5, I propose compositional semantics for difference- and ratio-statements and I propose felicity conditions for these statements, which correctly predict the facts based on the distinctions between ratio, interval, and ordinal predicates. In 9.2.3, I propose a general rule for the interpretation of unit names, which, crucially, involves in it a ratio operation. In 9.3 I show that the distinctions between ratio and interval predicates give us precisely what we need to account for the use of unit names and numerical degree predicates (which are based on unit names) in positive and negative predicates. Finally, in 9.4, I show that the hypothesis that negative predicates have reversed and (usually) transformed final degree functions allows for a natural explanation of cross polar (a)nomalies and of the difference between phrasal and clausal comparatives (with no stipulation of supremum, difference or any other operation or relation, as part of the interpretation of predicates).
9.2.3 Unit names and numerical degree predicates

In what follows, I am going to treat a number of natural language expressions as denoting functions, including functions that are not characteristic functions (that do not correspond to sets). Naturally, I will want to use \( \lambda \)-expressions for that purpose. But in the logical language I have been using so far, the basic predicates as well as the only \( \lambda \)-expressions that we had were all treated as denoting sets. So, for convenience, I will now switch to a typed language where predicates are analyzed as denoting functions (the characteristic functions of those sets), and we have the full range of \( \lambda \)-expressions.

We assume that every syntactic category in the language is of one of the following types.

(7) A typed language

a. Let TYPE be the set of types. \( e \in \text{TYPE}; t \in \text{TYPE} \); for any \( a,b \in \text{TYPE} \), \( <a,b> \in \text{TYPE} \); nothing else is in TYPE.

b. \( \forall g \in G, \forall c \in C: \forall \alpha \text{ of type } e: \quad \llbracket \alpha \rrbracket^+_{t,g} \in D_e = D \)
\( \forall \alpha \text{ of type } t: \quad \llbracket \alpha \rrbracket^+_{t,g} \in D_t = \{1, 0, \text{unknown}\} \)
\( \forall \alpha \text{ of type } <a,b>: \quad \llbracket \alpha \rrbracket^+_{t,g} \in D_{Da} \)

So in any \( t \) and \( g \), terms \( k_e \in \text{TERM} \) denote individuals; one-place concepts \( P_{<e,t>} \in \text{CONCEPT}^1 \) denote functions from D to truth values (characteristic functions of sets of entities); two-place concepts \( P_{<e,<e,t>} \in \text{CONCEPT}^2 \) denote functions from D to functions from D to truth values (characteristic functions of sets of pairs of entities), etc. (The reader can imagine what the type and set of possible denotations of all the other categories are, under the functional perspective.)

In accordance with that, instead of the predication rule formulated for sets, we now have it that for any \( n \) terms \( \alpha_1 \ldots \alpha_n \) and an \( n \) place concept \( P \in \text{CONCEPT}^n, \llbracket P(\alpha_n)(\ldots(\alpha_1) \ldots) \rrbracket^+_{t,g} = 1 \text{ iff } \llbracket P \rrbracket^+_{t,g}(\llbracket \alpha_n \rrbracket^+_{t,g}) \ldots (\llbracket \alpha_1 \rrbracket^+_{t,g}) = 1 \text{ (otherwise } \llbracket P(\alpha_n)(\ldots(\alpha) \ldots) \rrbracket^+_{t,g} = 0) \). Instead of the old rule that interpreted complex predicates as sets, we now have a general rule for \( \lambda \)-expressions. For that purpose, the following expressions are added to the vocabulary:

c. For any type \( a \in \text{TYPE} \) the language contains infinitely many variables \( \alpha \) of this type.

The following syntactic rules are added,

d. For any two expressions \( \alpha \) and \( \beta \), s.t. \( \alpha \) is of type \( <a,b> \) and \( \beta \) of type \( a \), or vice versa (\( \alpha \) of type \( a \) and \( \beta \) of type \( <a,b> \)), \( [\alpha \beta] \) is an expression of the language, of type \( b \).

e. For any variable \( \alpha \) of type \( a \) and expression \( \beta \) of type \( b \), \( \lambda \alpha \beta \) is an expression of the language, of type \( <a,b> \).

together with their corresponding semantic rules:

Functional application:

f. \( \forall g \in G, \forall t \in T, \text{ for any two expressions } \alpha \text{ and } \beta: \)
\( \llbracket (\alpha \beta) \rrbracket^+_{t,g} = \llbracket \alpha \rrbracket^+_{t,g}(\llbracket \beta \rrbracket^+_{t,g}) \text{ iff } \exists a,b \in \text{TYPE}: \beta \text{ is type } a \text{ and } \alpha \text{ is } <a,b> \)
\( \llbracket (\alpha \beta) \rrbracket^+_{t,g} = \llbracket \beta \rrbracket^+_{t,g}(\llbracket \alpha \rrbracket^+_{t,g}) \text{ iff } \exists a,b \in \text{TYPE}: \alpha \text{ is type } a \text{ and } \beta \text{ is } <a,b> \)
Interpretation of $\lambda$ expressions:

For any variable $\alpha$ of type $a$ and expression $\beta$ of type $b$,

$$[\lambda \alpha. \beta]_{\tau_g}$$ is that function $f$ from $D_a$ to $D_b$ s.t. for all $d \in D_a$, $f(d) = [[\beta]]_{\tau_g(\alpha/d)}^+$.

In case of type mismatch, expressions can type-shift before functional application (throughout the chapter I present some conventional types shifted interpretations where needed).

We can now turn to discussing unit names.

First note that the word *meter* in English is ambiguous. It is read as *meter*$_{long}$, or *meter*$_{wide}$, or *meter*$_{tall}$, etc. The predicate name can often be omitted when a unit name for this predicate is mentioned. For example, saying that *the box is one meter-long longer than the shelf* is superfluous. However, saying that *the box is one meter long* is less superfluous, as the box may be one meter$_{wide}$, rather than one meter$_{long}$.

Furthermore, I propose that natural language expressions like *meter* (understood as *meter*$_{tall}$) have two different translations into the logical language: $meter^1_{tall}$ and $meter^2_{tall}$. The first one, $meter^1_{tall}$, is a one-place predicate, whose semantic value (called the ‘unit interpretation’ of $meter_{tall}$) is (a characteristic function of) a set of equally tall entities; it intuitively represents a set of selected entities, viz., those entities whose height we call ‘one meter’. I will call these entities *meter unit-objects*. The second translation, $meter^2_{tall}$, is a two place predicate, called a *numerical degree modifier*. It is equivalent to $\lambda n.\lambda x.meter^2_{tall}(n,x)$ (a relation between a number $n$ and an object $x$, such that $x$ is $n$ times as tall as the meter unit-objects).

(8) Unit names and numerical degree modifiers,

a. Let us add to the language the category UNIT$^1$ that consists of words like $meter^1_{long}$, $meter^1_{tall}$, $meter^1_{wide}$, Kelvin$^1_{hot}$, gram$^1_{weigh}$, etc.

b. Let us add to the language the category UNIT$^2$ that consists of words like $meter^2_{long}$, $meter^2_{tall}$, $meter^2_{wide}$, Kelvin$^2_{hot}$, gram$^2_{weigh}$, etc.

c. UNIT$^1$ $\subseteq$ CONCEPT$^1$ and UNIT$^2$ $\subseteq$ CONCEPT$^2$

d. $meter^1_{tall}$ $\iff$ $\lambda x.meter^1_{tall}(x)$

e. $meter^2_{tall}$ $\iff$ $\lambda n.\lambda x.meter^2_{tall}(n,x)$

$f. \quad meter^2_{tall}(n) \iff \lambda x. \exists y, meter^1_{tall}(y) : \text{deg}(x,\text{tall}) = n \times \text{deg}(y,\text{tall}))$

The set of meter unit-objects (the set of entities for which the characteristic function that forms semantic value of $meter^1_{tall}$ assigns truth value 1) does not vary through total extensions of contexts $c$ that represent the knowledge states of competent speakers. For any total context in $T_c$ and time point $h$, the denotation of *Celsius* contains all the (possible) individuals for which a thermometer will assign degree 1 in $h$, and the denotation of $meter^1_{tall}$ contains all those for which a conventional meter-ruler will assigns degree 1. If I see something and I cannot tell whether it is one meter or not, that only means that some of its property-values are not accessible to me, so it may be any one of a set of possible individuals with some identical property values, but with different lengths (I can tell that there exists an individual but I cannot tell precisely which individual).

---

2 Note that I assign $meter^2_{tall}$ an ‘exactly’ reading: $\lambda n.\lambda x. \forall y, meter^1_{tall}(y) : \text{deg}(x,\text{tall}) = n \times \text{deg}(y,\text{tall}))$. I think this should be the case, certainly for predicate positions. The ‘at least’ interpretation is the result of discourse existential closure (cf. Kadmon 1987), or of a type-shift in argument positions (cf. Landman 2004).
Yet, in each $t$ in $T_c$ and $g$, $tall$ may be linked with a different final degree function. So we do not know the meter unit-objects’ degree of height in a partial context (in principle, for each real number we may have a total context in which they are mapped to that number). But all the final degree functions that are linked with $tall$ in some total extension of $c$ yield the same ordering between entities and the same ratios between their degrees (these orderings and ratios are known). We have rulers that for each actual entity can tell us the ratio between its degree in $tall$ and the meter unit-objects’ degree in $tall$ in $c$. I therefore propose that English expressions of the form $n$ meters tall denote predicates which map entities to numerical degrees that represent the ratio between their degree and the degree of certain selected objects (the unit-objects).

The expression $meter^2_{tall}$ denotes a relation between a number $n$ and an entity $x$ such that, $x$’s height is $n$ times the meter unit-objects' height. Expressions in English like one meter tall, two meters tall, three meters tall, $n$ meters tall are translated to the logical language expressions $meter^2_{tall}(1)$, $meter^2_{tall}(2)$, $meter^2_{tall}(n)$, etc. When the relation $meter^2_{tall}$ is fed with a numeral $n$ the result, $meter^2_{tall}(n)$, is a predicate that denotes (the characteristic function of) the set of entities such that the ratio between their final degree in $tall$ and the meter unit-objects' final degree in $tall$ is $n$. That is, $meter^2_{tall}(n)$ denotes (the characteristic function of) the following set of entities:

$$\{d \in D: [(\exists y, meter^1_{tall}(y): \deg(x, tall) = n \times \deg(y, tall))]^{t,g(x/d)} = 1\}$$

Since the transformation value of $tall$ is zero, and the final degree function of $tall$ is identical to its additive degree function, the set just specified equals the following:\footnote{Recall that $\sigma$ is an operation from a singleton set to its unique member: $\forall A \subseteq D, \sigma(A) = d$ iff $A = \{d\}$; else undefined.}

$$\{d \in D: f^*(d, tall, t, g) = n \times \sigma(\{f^*(d_m, tall, t, g): d_m \in [[meter^1_{tall}]]^{t,g}\}) \}$$

Similarly, the predicate $Kelvin^2(n)$ denotes (the characteristic function of) the following set of individuals $d$ in $D$:

$$\{d \in D: [(\exists y, Kelvin^1(y): \deg(x, hot) = n \times \deg(y, hot))]^{t,g(x/d)} = 1\} = \{d \in D: f^*(d, hot, t, g) = n \times \sigma(\{f^*(d_m, hot, t, g): d_m \in [[Kelvin^1]]^{t,g}\}) \}$$

In addition, $meter^2_{tall}$ may denote a relation, $[meter^2_{tall}]_{num}$, between a number $n$ and a number $m$ such that $m$ is $n$ times the meter unit-objects’ height:\footnote{This number interpretation is probably the basic one, while the entity interpretation is a result of type-shift: $TS_{meter^2_{tall}} = \lambda x. [Am, \exists y, meter^1_{tall}(y): m = n \times \deg(y, P)](\deg(x, P) = \lambda x. \exists y, meter^1_{tall}(y): \deg(x, P) = n \times \deg(y, P)$.
If the predicate variable is filled with $tall$ we get: $\lambda x. \exists y, meter^1(tall): \deg(x, tall) = n \times \deg(y, tall)$.
But below I usually suppress the type shift marking in $TS_{meter^2_{tall}}$}

$$g. \ [meter^2_{tall}(n)]_{num} \iff \lambda m. \exists y, meter^1_{tall}(y): m = n \times \deg(y, tall)$$

We can give a general interpretation rule for unit names and numerical degree predicates:
Semantic values of unit names:

\[ \forall g \in G, \forall t \in T, \forall n \in N, \forall P \in \text{CONCEPT}^n, \forall u^1_p \in \text{UNIT}^1 \text{ (up is a unit name for P):} \]

a. \[ \exists r_{up} \in \mathbb{R}, r_{up} \neq 0: \] 

\[ \llbracket u^1_p \rrbracket^{+}_{t,g} \text{ is that function from } D^n \text{ the value } 1 \text{ iff } f^+(d^1, P, t, g) = r_{up}. \]

In any g and t, the denotation of a conventional unit names \( u^1_p \) (like meter tall, Kilometer long, inch wide etc.), \( \llbracket u^1_p \rrbracket^{+}_{t,g} \), is the characteristic function of a set of objects, such that for some real number \( r_{up} \), any object \( d \) is an element of this set iff \( f^+(P, t, g) \) maps \( d \) to \( r_{up} \).

b. \[ \llbracket [u^2_p]^{num} \rrbracket^{+}_{t,g} = \llbracket \lambda n. \lambda x. \exists y, u^1_p(y): \deg(x, P) = n \times \deg(y, P) \rrbracket^{+}_{t,g} \]

In any g and t, the denotation of a numerical degree predicate that is based on \( u^1_p \) (like \( n \) meter tall), is a relation between numbers \( n \) and objects \( d \), such that \( d \)'s degree in \( P \) equals \( n \) times the degree of any unit-object \( d_u \) such that \( \llbracket u^1_p \rrbracket^{+}_{t,g}(d_u) = 1. \)

c. \[ \llbracket [u^2_p]^{num} \rrbracket^{+}_{t,g} = \llbracket \lambda n. \lambda m. \exists y, u^1_p(y): m = n \times \deg(y, P) \rrbracket^{+}_{t,g} \]

For any quantity of \( P \)-hood, the set of objects possessing it can form a basis for a unit name. Therefore, any conventional mapping of entities to unit-based numbers has to be associated with a name (meter, centimeter, Celsius, kilometer etc.)

The set of objects one meter long and the set of objects one meter wide are different (in any \( t \) and \( g \), \( \llbracket \text{meter}_{long} \rrbracket^{+}_{t,g} \) need not be identical to \( \llbracket \text{meter}_{wide} \rrbracket^{+}_{t,g} \)). Why? Because long and wide differ wrt the relevant pair of points the distance between which their additive degree function measures.

Some unit names (like Celsius) are interpreted by other interpretation rules. Yet we will see in ongoing sections that speakers often wrongly presuppose that the unit name Celsius is interpreted by the rule in (8). In this sense, the rule in (8) is productively used by speakers, while other rules that scientists invent are not.

9.3 Direct consequences:

Unit names and numerical degree relations in positive and negative predicates

We have seen that different types of predicates can occur with numerical degree predicates (like two meters) in different types of linguistic contexts (cf. 2.1 and 3.2.4): Predicates like happy combine with numerical degree predicates in no context at all, predicates like short do so only in comparison statements (as in two meter shorter), and predicates like tall do so in any context (including their positive forms, as in two meter tall). My analysis has an advantage in that it allows a straightforward account of this distribution of numerical degree predicates.

Naturally, a pre-condition for the creation of numerical degree predicates is the existence of unit names. Our analysis also allows for an account also of the distribution of all the unit-names that are interpreted by the productive rule in (8).

9.3.1 The distribution of unit names

We have seen that the ratios between the degrees of any two entities in a given predicae \( P \) are known (are constant across total extensions of actual contexts \( c \)) iff \( P \) is a one-dimensional ratio predicate. That is the case because the transformation value of ratio predicates is taken to be zero (it is zero in any \( t \) in \( T_c \) and \( g \), so their final degree function \( [[\deg(P)]^{+}_{t,g}} \) equals either \( f^+(P, t, g) \)
(in positive predicates) or \( \lambda d. - f^*(d,P,t,g) \) (in negative predicates). As both these functions are additive (by definition) in any \( t \) and \( g \), it follows that \([\deg(P)]^t,g\) too is additive in any \( t \) and \( g \). In one-dimensional predicates, ratios between values that \( f^*(P,t,g) \) assigns to entity pairs are constant across all \( t \) in \( T_c \) and \( g \) (they are known numbers), and therefore that is also the case for \( \deg^*(P,t,g) \):

\[(10) \quad \text{Facts: All and only one-dimensional ratio predicates in } c \text{ are such that the ratios between values that their final degree function assigns to entities are constant across all the total extensions of } c \text{ (they are known numbers):}
\]

\[\forall d_1,d_2 \in D, \exists r \in R: \forall t \in T_c, \forall g \in G: \deg^*(d_2,P,t,g) = r \times \deg^*(d_1,P,t,g)\]

In other words:

\[\forall d_1,d_2 \in D, \forall t_1,t_2 \in T_c, \forall g \in G: \frac{\deg^*(d_2,P,t_1,g)}{\deg^*(d_1,P,t_1,g)} = \frac{\deg^*(d_2,P,t_2,g)}{\deg^*(d_1,P,t_2,g)}\]

On the present proposal, unit names like \( \text{meter}^1_{\text{tall}} \) denote (characteristic functions of) sets of agreed upon objects, and since the ratios between any object's degree and the meter unit-objects' degree is always known in ratio predicates, we can map entities to known numbers. When we say that Dan is 1.87 meter tall, this does not mean that Dan's degree of height is 1.87. Rather, it means that whatever Dan's degree and the meter unit-objects' degree of height are in any \( t \) and \( g \) above \( c \), the ratio between them is 1.87.

We cannot create interpretable unit names for interval predicates, because this ratio is always unknown in interval predicates (predicates whose transformation value is not set to zero in at least some total context above \( c \)). For example, even if \( P \)'s transformation value is fixed to be one and the same number in every \( t \) in \( T_c \) and \( g \), say \( \text{Tran}^*(P,t,g) = 1 \). \( P \)'s additive function may map two entities \( d_1 \) and \( d_2 \) to 2 and 4 in \( t_1 \) and \( g \) and to 4 and 8 in \( t_2 \) and \( g \). In this case:

\[
\frac{\deg^*(d_1,P,t_1,g)}{\deg^*(d_2,P,t_1,g)} = \frac{(f^*(d_1,P,t_1,g) - \text{Tran}^*(P,t_1,g))}{(f^*(d_2,P,t_1,g) - \text{Tran}^*(P,t_1,g))} = \frac{4 - 1}{2 - 1} = \frac{3}{1} \neq \frac{8 - 1}{4 - 1} = \frac{7}{3}.
\]

Knowing the ratios between the values that \( P \)'s final degree function gives to entities in \( P \) is the pre-condition for the creation of unit-names for \( P \) that are interpreted by the rule in (8). But in the case of interval predicates, this pre-condition is not met. So they cannot have unit names governed by the rule in (8).

### 9.3.2 The licensing of numerical degree relations with predicates in the positive form

On the present analysis, a statement like \textit{Sam is two meters tall} is true in a context \( c \) and an assignment \( g \) iff in any way of extending \( c \) to totality (whatever \( f^*(\text{tall},t,g) \) turns out to be), the ratio between the final degree of the meter unit-objects and the final degree of Sam is 2.

\[(11) \quad \text{Sam is 2 meters tall}
\]

\[\Leftrightarrow [\lambda n.\lambda x. \exists y, \text{meter}^1_{\text{tall}}(y): \deg(x,\text{tall}) = n \times \deg(y,\text{tall})(2)](Sam)
\]

\[\Leftrightarrow \exists y, \text{meter}^1_{\text{tall}}(y): \deg(Sam,\text{tall}) = 2 \times \deg(y,\text{tall})\]
Thus, the present analysis of unit-names has the advantage that it allows for a straightforward account of the set of predicates that can occur with numerical degree predicates in the positive form (e.g. *tall*). Again, these are the ratio predicates P, in which the ratios between values that P's final degree function assigns to entities are known numbers. Thus it is possible to say that the ratio between, e.g., Sam's degree in *short* and the meter unit-objects' degree in *tall* is 2 (because it is 2 in any t in Tc and g, for any actual context c).

But in interval predicates (predicates whose transformation value is not always set to zero) like *short*, we have no unit names which are directly based on degrees assigned by [(deg(*short*))]+tg, as explained in 9.3.1. Even if we regard some set of entities (say, the entities that are 1 meter tall, and hence always end up equally short) as unit-objects for *short*, we still cannot make a rule that will tell us for each entity what the ratio between its degree and the meter unit-objects' degree in *short* is in a partial context c, because this ratio varies across the total extension of c, in accordance with the transformation value. Thus, we cannot associate with entities numbers which are based on the ratio between their degree in *short* and the meter unit-objects' degree in *short*. The meaning of numerical degree relations like *n meter2 short*, which is based on our ability to say what these ratios are, remains completely vague.

\[
(12) \quad [[\text{Sam is 2 meters tall }]]_{tg}^+ = 1 \text{ iff } \\
\exists d_m \in [[\text{meter}^1_{\text{tall}}]]_{tg}^+ \quad \text{deg}^+([[\text{Sam}]]_{tg}^+, \text{tall}, t, g) = 2 \times \text{deg}^+(d_m, \text{tall}, t, g) \quad \text{iff} \\
\exists d_m \in [[\text{meter}^1_{\text{tall}}]]_{tg}^+ \quad \text{f}^+([[\text{Sam}]]_{tg}^+, \text{tall}, t, g) = 2 \times \text{f}^+(d_m, \text{tall}, t, g) \quad \text{iff} \\
\text{f}^+([[[\text{Sam}]]]_{tg}^+, \text{tall}, t, g) = 2 \times r_{\text{meter},tg}
\]

\[
(13) \quad [[\text{Sam is 2 meters tall }]]_{cg}^+ = 1 \text{ iff } \\
\forall t \in T, t \geq c: [[\text{Sam is 2 meters tall }]]_{tg}^+ = 1 \text{ iff } \\
\forall t \in T, t \geq c: \text{f}^+([[[\text{Sam}]]]_{tg}^+, \text{tall}, t) = 2 \times r_{\text{meter},tg}
\]

Since we are always found in a partial context c, given the indeterminacy in the value of Tran^+(short,c), the statements can never be true in every total extension of c. Generally, we can say that in the case of interval predicates, for any d_u and d which are not equally P we can find two total extensions of c in which the ratios between their degrees are not identical.
For interval predicates P, the ratios between values that the final degree function assigns to any two entities d_u,d which are not equally P (\(\deg^*(d_u,P,t,g) \neq \deg^*(d,P,t,g)\)) in some extension t of c and g change with the context:

\[\neg\exists \in R, \forall t \in T_c, \forall g \in G: \deg^*(d,P,t,g) = r \times \deg^*(d_u,P,t,g)\]

For example, if in t_1 the transformation value of short is set to 1, in t_2 it is set to 2, and in both, \(f^*(tall,t,g) = f^*(short,t,g)\) assigns d_u the number 4 and d the number 8, then \(\deg^*(d_u,short,t_1,g) = 1 – 4 = -3, \deg^*(d,short,t_1,g) = 1 – 8 = -7\) and \(\deg^*(d,short,t_2,g) = 2 – 8 = -6\). Crucially, the ratio between the degrees of d_u and d in t_1 is 3/7 and in t_2 is 2/6 \(\neq 3/7\). It follows that for no actual partial context c (one that reflects the knowledge of competent speakers), \(\text{[Sam is 2 meters short]}^*_{c,g} = 1\) (it is never true in any total extension of c).

For this reason, the use of numerical degree relations in statements like Dan is 2 meters short is infelicitous. Of course, if we set the transformation value of short to zero, we can make sense of statements with such relations. But we have no basis for doing that. And if the transformation values vary between total contexts, the ratios between the degrees of entities and the degree of the meter unit-objects vary with it between different total contexts. The only numerical degree predicate that can produce truth values 1 is one meter\(^2\) short, because it trivially holds of all the entities that are one meter tall (the unit-objects themselves). But this is not sufficient to justify the use of the meter\(^2\) short relation. This is simply the meaning of meter\(^1\) tall (if you want to say that something is one meter tall, you can say so without forcing your listener to use the transformation value of short and then eliminate it from the calculation). Thus, it is clear why negative predicates cannot occur with numerical degree modifiers.

To summarize the points we have made so far: The use of unit-based relations presupposes that the transformation value is known to be 0. Obviously, if the transformation value is not known to be 0, the unit name has no use. But given a zero transformation value and a specific conventional (characteristic function of a) unit-set \([u_1^P])^*_{c,g}\), numbers can be associated with all the entities (by using the \(u_2^P\) relation). For each individual in D, we can talk about its degree in \(u_1^P\) unit objects in P, for any assignment g and context t in T_c. Thus, we feel that P's (final) degree function is known, although in fact, we only know the ratios between the degrees of entities and the degrees of the unit-objects. Finally, predicates whose degree functions transform additive values by a constant that varies with context cannot have conventional unit names. The ratios between the degrees of entities and the degrees of any given object (any possible unit-object) vary between total contexts in T_c (with the transformation value), and so these predicates cannot combine with numerical-degree modifiers in their positive form. For that reason we feel that the degree function (the numbers that objects map into) is not known to us.

9.3.3 The licensing of numerical degree predicates with comparative predicates

What happens in the comparative form? How can negative predicates combine with numerical degree predicates in the comparative?

The interpretation of comparative statements like 20 meters is 5 meters longer than 15 meters, seems to be, roughly: "the difference between (anything that is) 20 meters long and (anything that is) 15 meters long is 5 meters long". Given this intuition, and following von Stechow (1984) and many others since (for further discussion see 3.2), I assume that the comparative morpheme more or er has the difference operation \(\lambda n_2, \lambda n_1. n_1 – n_2\) as part of its interpretation, and less has the reversed difference operation, \(\lambda n_2, \lambda n_1. n_2 – n_1\), as part of its interpretation (Landman 2005).
Inspired by Schwarzschild and Wilkinson (2002) and Landman (2005), I propose that the comparative morpheme denotes a three place relation, which takes a degree predicate \( M \) (say, \( n \) meters) and two degrees \( n_1 \) and \( n_2 \) as its three arguments, and returns truth value 1 iff \((n_1 - n_2) = M\) (e.g., iff the difference between \( n_1 \) and \( n_2 \) is \( M \), e.g., if the difference between \( n_1 \) and \( n_2 \) is \( n \) meters, i.e. \( n \) times the degree of the meter unit-objects in \( long \)).

(18) Difference modifiers:

a. Let us add to the vocabulary the expressions \([More]^{num}, \{less\}^{num}, more, less, \{Than\}^{num}, than, \{as\}^{num}, as, \{as\_2\}^{num}, as_2, than\_clausal\)

b. \([More]^{num} \iff \lambda M. \lambda n_2. \lambda n_1. M(n_1 - n_2)\)

When combined with a predicate like \( tall \), comparative morphemes type shift into operations that take entities instead of degrees, and return 1 iff the difference between their degrees is \( M \).

d. \( More \iff \lambda P. \lambda M. \lambda x_2. \lambda x_1. (\lambda M. \lambda n_2. \lambda n_1. n_1 - n_2)(M)(\lambda (x_2,P))((\lambda (x_1,P)) = \lambda P. \lambda M. \lambda x_2. \lambda x_1. \lambda (deg^+(x_2,P)) - \lambda (deg^+(x_1,P))\)

e. \( less \iff \lambda P. \lambda M. \lambda x_2. \lambda x_1. \lambda (deg^+(x_2,P)) - \lambda (deg^+(x_1,P))\)

f. \( more \_short \iff \lambda P. \lambda M. \lambda x_2. \lambda x_1. \lambda (deg^+(x_1,P)) - \lambda (deg^+(x_1,P))\)

g. \( Less \_short \iff \lambda M. \lambda x_2. \lambda x_1. M((\lambda (x_2,P)) - deg^+(x_1,P))\)

Given this analysis, the factor affecting the distribution of numerical degree predicates in comparative statements should be whether the ratios between the degree-differences for any two entity-pairs are constant across total contexts. Crucially, in one-dimensional predicates, they are constant, even when the transformation value varies, as we saw in 9.1.2.

(19) In both ratio and interval one-dimensional predicates \( P \), the ratios between the differences between values that the final degree function assigns to entities do not change across total extensions of actual contexts \( c \): \( \forall d_1, d_2, D, T_e, G: \forall d_1, d_2, D, T_e, G: \exists e \in R: \forall t, e \in T_e, g \in G: \deg^+(d_1, P, t, g) - \deg^+(d_2, P, t, g) = r \times (\deg^+(d_1, P, t, g) - \deg^+(d_2, P, t, g))\)

This is the case because by applying the difference operation on two transformed degrees, we get rid of the Transformation value. The difference between two entities’ degrees in \( short \) can be expressed using \( f^*(tall, t, g) \) (it can also be seen as the degree of some objects in \( tall \)):

\[
\text{If P is positive: } \deg^+(d_1, P, t, g) - \deg^+(d_2, P, t, g) = (f^*(d_1, P, t, g) - \text{Tran}^*(P, t, g)) - (f^*(d_2, P, t, g) - \text{Tran}^*(P, t, g)) = (f^*(d_1, P, t, g) - f^*(d_2, P, t, g))
\]

\[
\text{If P is negative: } \deg^+(d_1, P, t, g) - \deg^+(d_2, P, t, g) = (\text{Tran}^*(P, t, g) - f^*(d_1, P, t, g)) - (\text{Tran}^*(P, t, g) - f^*(d_2, P, t, g)) = (f^*(d_2, P, t, g) - f^*(d_1, P, t, g))
\]

Therefore, numerical degree predicates \( u_2^{as(n)} \) which are based on ratio-predicates \( P \) (that do not bring in a transformation variable) like \( \text{meter}^2_{tall}(n) \) can be used as means of associating a number with degree-differences, like \( f^*(d_1, P, t, g) - f^*(d_2, P, t, g) \), or \( f^*(d_2, P, t, g) - f^*(d_1, P, t, g) \).
On the present analysis, the predicate in *Sam is two meters short* translates into \( \text{meter}^2_{\text{short}} \) (and not into \( \text{meter}^2_{\text{tall}} \)). Conversely, in *Sam is 2 meters shorter than Dan*, the predicate *short* serves as the argument of the comparative morpheme *more* (the sentence expresses a comparison between Sam’s and Dan’s degrees in *short*), and not as part of the degree predicate 2 *meters*. Hence, 2 *meters* can translate into the relation \( \text{meter}^2_{\text{tall}} \). Given that using the relation \( \text{meter}^2_{\text{short}} \) will lead us nowhere, this is the only possibility we are left with, always, and for that reason saying explicitly that *Sam is 2 meter-tall shorter than Dan* is superfluous. (*Meter* is further ambiguous between \( \text{meter}_{\text{long}}, \text{meter}_{\text{wide}} \), etc., but we will see in the next section that the degrees of entities that are assigned truth value 1 by \([\text{meter}^1_{\text{long}}]\), \([\text{meter}^1_{\text{wide}}]\), \([\text{meter}^1_{\text{tall}}]\), etc., co-vary, so it makes no difference whether we use \( \text{meter}^2_{\text{long}}, \) or \( \text{meter}^2_{\text{wide}} \)). Let us see step by step the derivation of *Dan is 2 meter-tall shorter than Sam*.

Recall that I have assigned the predicate \( n \text{ meter}^2_{\text{tall}} \) an 'exactly' interpretation "\( \lambda k. \exists x, \text{meter}^1_{\text{tall}}(x): k = n \times \text{deg}(x,\text{tall}) \)". However, in the following I replace this interpretation with the "at least" interpretation, namely the one equivalent to "\( \lambda k. \exists x, \text{meter}^1_{\text{tall}}(x): k \geq n \times \text{deg}(x,\text{tall}) \)". This is not because I think it is more correct, but because in this way the discussion will be more informative in that the derivations will demonstrate the effect of the use of positive versus negative predicates with 'at least' relations, while at the same time, if the reader replaces all the occurrences of \( \geq \) and '≤' with '=' in the derivations, the result will readily illustrate the truth conditions with 'exactly' interpretations for numerical degree relations (those readers that are convinced in an 'at least' semantics for numerical predicates will be able to see that the theory works just as well with this assumption).

When numerical degree predicates like \( n \text{ meter}_{\text{tall}} \) combine with difference modifiers, the result is a degree-relation like \( n \text{ meters more} \):

\[
(20) \quad \text{Degree relations with } \text{more}: \\
\text{a. n meters more:} \quad \iff \quad [\lambda M, \lambda k_2, \lambda k_1. M(k_1 - k_2)](\lambda k. \exists x, \text{meter}^1_{\text{tall}}(x): k \geq n \times \text{deg}(x,\text{tall})) = \\
\quad \lambda k_2, \lambda k_1. (\lambda k. \exists x, \text{meter}^1_{\text{tall}}(x): k \geq n \times \text{deg}(x,\text{tall}))(k_1 - k_2) = \\
\quad \lambda k_2, \lambda k_1. \exists x, \text{meter}^1_{\text{tall}}(x): (k_1 - k_2) \geq n \times \text{deg}(x,\text{tall}) = \\
\quad \lambda k_2, \lambda k_1. \exists x, \text{meter}^1_{\text{tall}}(x): k_1 \geq k_2 + (n \times \text{deg}(x,\text{tall})) \\
\text{b. n meters more P:} \quad \iff \quad [\lambda M, \lambda x_2, \lambda x_1. M(\text{deg}(x_1, P) - \text{deg}(x_2, P))] \\
\quad (\lambda k. \exists y, \text{meter}^1_{\text{tall}}(y): k \geq n \times \text{deg}(y, \text{tall})) = \\
\quad \lambda x_2, \lambda x_1. (\lambda k. \exists y, \text{meter}^1_{\text{tall}}(y): k \geq n \times \text{deg}(y, \text{tall})) = \\
\quad (\text{deg}(x_1, P) - \text{deg}(x_2, P)) = \\
\quad \lambda x_2, \lambda x_1. \exists y, \text{meter}^1_{\text{tall}}(y): \text{deg}(x_1, P) \geq \text{deg}(x_2, P) + n \times \text{deg}(x, \text{tall}) \\
\text{c. n meters shorter:} \quad \iff \quad \lambda x_2, \lambda x_1. \exists y, \text{meter}^1_{\text{tall}}(y): \text{deg}(x_1, \text{short}) \geq \text{deg}(x_2, \text{short}) = \\
\quad + (n \times \text{deg}(x, \text{tall}))
\]

Similarly, with *less* we get:

\[
(21) \quad \text{Degree relations with } \text{less}: \\
\text{a. n meters less:} \quad \iff \quad [\lambda M, \lambda k_2, \lambda k_1. M(k_2 - k_1)](\lambda k. \exists x, \text{meter}^1_{\text{tall}}(x): k \geq n \times \text{deg}(x,\text{tall})) = \\
\]

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We can now work out the truth conditions of comparative statements.

\[ \lambda k_2 \lambda k_1. \exists x, \text{meter}^1_{\text{tall}}(x): (k_2 - k_1) \geq n \times \text{deg}(x, \text{tall}) = \lambda k_2 \lambda k_1. \exists x, \text{meter}^1_{\text{tall}}(x): k_1 \leq k_2 - (n \times \text{deg}(x, \text{tall})) \]

b. n meters less P:
\[ \iff \lambda x_2 \lambda x_1. \exists y, \text{meter}^1_{\text{tall}}(y): \text{deg}(x_1, P) \leq \text{deg}(x_2, P) - (n \times \text{deg}(y, \text{tall})) \]

c. n meters less short:
\[ \iff \lambda x_2 \lambda x_1. \exists y, \text{meter}^1_{\text{tall}}(y): \text{deg}(x_1, \text{short}) \leq \text{deg}(x_2, \text{short}) - (n \times \text{deg}(y, \text{tall})) \]

Than (or as2 in as1 long as2) is a modifier of such degree relations. In phrasal comparatives, when than applies to, say, n meters more, it has no effect (but its meaning is more complex in clausal comparatives).

(22) Than in phrasal comparatives:

a. \[ [\text{Than }]^\text{num} / [\text{as2}]^\text{num} \iff \lambda R. \lambda n_2. \lambda n_1. R(n_1, n_2) \]

b. Than / as2 \[ \iff \lambda R \lambda x_2. \lambda x_1. R(x_1, x_2) \]

c. n meters more than
\[ \iff \lambda R \lambda n_2. \lambda n_1. R(n_1, n_2) \]
\[ (\lambda k_2 \lambda k_1. \exists x, \text{meter}^1_{\text{tall}}(x): k_1 \geq k_2 + (n \times \text{deg}(x, \text{tall}))) = \lambda k_2 \lambda k_1. \exists x, \text{meter}^1_{\text{tall}}(x): k_1 \geq k_2 + (n \times \text{deg}(x, \text{long})) \]

d. n meters more P than
\[ \iff \lambda x_2 \lambda x_1. \exists y, \text{meter}^1_{\text{tall}}(y): \text{deg}(x_1, P) \geq \text{deg}(x_2, P) + (n \times \text{deg}(y, \text{tall})) \]

d. n meters shorter than:
\[ \iff \lambda x_2 \lambda x_1. \exists y, \text{meter}^1_{\text{tall}}(y): \text{deg}(x_1, \text{short}) \geq \text{deg}(x_2, \text{short}) + (n \times \text{deg}(y, \text{tall})) \]

e. Dan is two meter shorter than Sam:
\[ \iff \lambda x_2 \lambda x_1. \exists y, \text{meter}^1_{\text{tall}}(y): \text{deg}(x_1, \text{short}) \geq \text{deg}(x_2, \text{short}) + (2 \times \text{deg}(y, \text{tall}))(\text{Dan})(\text{Sam}) \]
\[ \iff \exists y, \text{meter}^1_{\text{tall}}(y): \text{deg}(\text{Dan, short}) \geq \text{deg}(\text{Sam, short}) + 2 \times \text{deg}(y, \text{tall}) \]

We can now work out the truth conditions of comparative statements.

(23) \[ [[\text{Dan is 2 meters shorter than Sam}]^\text{t}_g] = 1 \text{ iff } \]
\[ \exists d_m \in [[\text{meter}^1_{\text{tall}}]^\text{t}_g \circ \text{deg}^*([[\text{Dan}]^\text{t}_g, \text{short}, \text{t}, \text{g}]) \geq \text{deg}^*([[\text{Sam}]^\text{t}_g, \text{short}, \text{t}, \text{g}) + (2 \times \text{deg}^*(d_m, \text{tall}, \text{t}, \text{g}))} \text{ iff } \]
\[ \exists d_m \in [[\text{meter}^1_{\text{tall}}]^\text{t}_g, (\text{Tran}^*(\text{short}, \text{t}, \text{g}) - f^*([[\text{Dan}]^\text{t}_g, \text{tall}, \text{t}, \text{g})) \geq \text{Tran}^*(\text{short}, \text{t}, \text{g}) - f^*([[\text{Sam}]^\text{t}_g, \text{tall}, \text{t}, \text{g}) + (2 \times f^*(d_m, \text{tall}, \text{t}, \text{g})) \text{ iff } \]
\[ \exists d_m \in [[\text{meter}^1_{\text{tall}}]^\text{t}_g, - f^*([[\text{Dan}]^\text{t}_g, \text{tall}, \text{t}, \text{g}) \geq - f^*([[\text{Sam}]^\text{t}_g, \text{tall}, \text{t}, \text{g}) + (2 \times f^*(d_m, \text{tall}, \text{t}, \text{g})) \text{ iff } \]
\[ \exists d_m \in [[\text{meter}^1_{\text{tall}}]^\text{t}_g, f^*([[\text{Dan}]^\text{t}_g, \text{tall}, \text{t}, \text{g}) \leq f^*([[\text{Sam}]^\text{t}_g, \text{tall}, \text{t}, \text{g}) - (2 \times f^*(d_m, \text{tall}, \text{t}, \text{g})) \text{ iff } \]
\[ f^*([[\text{Dan}]^\text{t}_g, \text{tall}, \text{t}, \text{g}) \leq f^*([[\text{Sam}]^\text{t}_g, \text{tall}, \text{t}, \text{g}) - (2 \times r_{\text{meter}, \text{t}, \text{g}}) \]

(24) \[ [[\text{Dan is 2 meter shorter than Sam}]^\text{t}_g] = 1 \text{ iff } \]
\[ \forall t \in T, t \geq c: f^*([[\text{Dan}]^\text{t}_g, \text{tall}, \text{t}, \text{g}) \leq f^*([[\text{Sam}]^\text{t}_g, \text{tall}, \text{t}, \text{g}) - (2 \times r_{\text{meter}, \text{t}, \text{g}}) \]

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We adequately derive the intuition that *Dan is 2 meters shorter than Sam* is true in a partial context \(c\) iff in any \(t\) extending \(c\), Dan’s height equals at most Sam’s height minus two times the height of a meter unit-object (whatever the numbers associated with these heights are). Crucially, two transformation values cancel one another, so the ratios between the meter unit-objects’ degree and the difference between entities’ degrees in short are known, and they represent adequately the ratio between the meter unit-objects’ height and the height difference between entities.

### 9.3.4 Between-predicate comparisons in predicates with conventional unit names

This analysis of unit-names has an advantage in that it allows a straightforward account for the conditions under which predicates with conventional unit names can co-occur in between-predicate comparisons (like *the sofa is 2 meters longer than its wide*).

The set of objects one meter long and the set of objects one meter wide are different (in any \(t\) and \(g\), \(\{\text{[meter}^1_{\text{long}}]\}_{t,g}\) need not be identical to \(\{\text{[meter}^2_{\text{wide}}]\}_{t,g}\)). However, I propose that we have to postulate that \(r_{\text{meter-long}} = r_{\text{meter-wide}}\) (where for any \(u_P\): \(r_u = \sigma(\{f^*(d,P,t,g): [u^1_P]\}_{t,g}(d) = 1\})\), i.e., that in any \(t\) and \(g\), \(f^*(\text{long},t,g)\) maps meter-long unit objects (entities that are \(\text{meter}^1_{\text{long}}\)) to the same degree to which \(f^*(\text{wide},t,g)\) maps meter-wide unit objects (entities which are \(\text{meter}^1_{\text{wide}}\)). This is precisely the case in real life, because we use the same rulers for length and width. Thus, if we measure the ratio between the length of an entity and the length of a meter unit-object, and we measure the ratio between the width of an entity and the width of a meter unit-object, we can meaningfully compare these numbers. For instance, we can say that the difference between the entity’s width and length equals two times the length (or width) of a meter unit-object. For instance, *The sofa is 2 meter longer than its wide* is true in a context \(c\) iff for any \(t\) above \(c\), and any \(g\):

\[
f^*([[\text{The sofa}]]^t_{tg},\text{long},t,g) - f^*([[\text{The sofa}]]^t_{tg},\text{wide},t,g) = 2 \times r_{\text{meter}}
\]

Predicates such that the degrees of their unit objects are not known to co-vary are not comparable in this sense. For example, *The sofa is 2 meter longer than its heavy* is true in \(c\) and \(g\) iff in any \(t\) in \(T_c\) and \(g\):

\[
f^*([[\text{The sofa}]]^t_{tg},\text{long},t,g) - f^*([[\text{The sofa}]]^t_{tg},\text{heavy},t,g) = 2 \times r_{\text{meter}}
\]

But this condition is hardly ever met, because the ratios between (i) degrees of length minus degrees of weight of entities and (ii) degrees of length of meter unit-objects are different in different total contexts. So this statement is virtually never true, and hence infelicitous.

For example, even if the transformation value is zero in \(c\) in both \(\text{long}\) and \(\text{heavy}\), consider three entities \(d_1\), \(d_2\) and a meter unit-object \(d_m\) such that, say:

- \(f^*(\text{long})\) maps \(d_1\) and \(d_m\) to 4 and 1 in \(t_1\) and \(g\) and to 8 and 2 in \(t_2\) and \(g\),
- \(f^*(\text{wide})\) maps \(d_2\) and \(d_m\) to 2 and 1 in \(t_1\) and \(g\) and to 4 and 2 in \(t_2\) and \(g\), and
- \(f^*(\text{heavy})\) maps \(d_2\) to 3 in \(t_1\) and \(g\) and to 9 in \(t_2\) and \(g\).

The ratio between (i) the difference between \(d_1\)'s degree in \(\text{long}\) and \(d_2\)'s degrees in \(\text{wide}\) and (ii) the meter unit-objects’ degree in \(\text{long}\) is the same number in \(t_1\) and \(t_2\) (because \(d_m\)'s degrees in \(\text{long}\) and \(\text{wide}\) co-vary, so the ratio between \(d_m\)'s degree in \(\text{long}\) and \(d_2\)'s degree in \(\text{wide}\) co-vary):

\[
\deg^*(d_1,\text{long},t_1,g) - \deg^*(d_2,\text{wide},t_1,g) / \deg^*(d_m,\text{long},t_1,g) = (4 - 2) / 1 = 2
\]

\[
\deg^*(d_1,\text{long},t_2,g) - \deg^*(d_2,\text{wide},t_2,g) / \deg^*(d_m,\text{long},t_2,g) = (8 - 4) / 2 = 2
\]
But the ratio between (i) the difference between \(d_1\)'s degree in \(\text{long}\) and \(d_2\)'s degrees in \(\text{heavy}\) and (ii) the meter unit-objects' degree in \(\text{long}\) is a different number in \(t_1\) and \(t_2\) (because \(d_m\)'s degree in \(\text{long}\) and \(d_2\)'s degree in \(\text{heavy}\) do not co-vary):

\[
\text{deg}^+(d_1,\text{long},t_1,g) - \text{deg}^+(d_2,\text{heavy},t_1,g)) / \text{deg}^+(d_m,\text{long},t_1,g) = (4 - 3) / 1 = 1 \\
\text{deg}^+(d_1,\text{long},t_2,g) - \text{deg}^+(d_2,\text{heavy},t_2,g)) / \text{deg}^+(d_m,\text{long},t_2,g) = (8 - 9) / 2 = -1/2
\]

So we cannot truthfully assert that this ratio is, e.g., 2, or any other known number.

Generally:

(25) Between-predicate comparisons in predicates with unit names:

Entities' degrees in two predicates \(P\) and \(Q\) with conventional unit names \(u_P\) and \(u_Q\) are comparable in \(c\) iff \(\forall t \in T_c, \forall g \in G:\)

\[
\sigma(\{\text{deg}^+(d,P,t,g): [u_P^1]^{+}_{t,g}(d) = 1\}) = r_P \\
\sigma(\{\text{deg}^+(d,Q,t,g): [u_Q^1]^{+}_{t,g}(d) = 1\}) = r_Q.
\]

And \(\text{Tran}^+(P,t,g) = \text{Tran}^+(Q,t,g)\)

If the degrees that are assigned to unit objects of two different predicates in total extensions of \(c\) are known to co-vary, the predicates are comparable in \(c\). In most adjective pairs (like \text{heavy} and \text{long}, \text{happy} and \text{long}, etc.) this condition is not met, so they cannot co-occur in between-predicate comparisons. (In 7.4, I present an additional type of circumstances in which between-predicate comparisons are licensed which are not dependent on a common unit, but on the existence of bounds from both sides).\(^5\)

9.3.5 Celsius

Let us now look at the concept \textit{Celsius}. The interpretation of \textit{Celsius} is not generated by the general 'linguistic' rule for the interpretation of unit names proposed in 9.2.3. The mapping of entities that possess heat into temperature in Celsius degrees is such that entities with no heat ("0 Kelvin hot") are "−273 Celsius hot". Entities with heat that corresponds to the heat in 273 Kelvin unit objects ("273 Kelvin hot") are "0 Celsius hot". And in general, for any \(n\), entities "\(n\) Kelvin hot" are "\(n - 273\) Celsius hot": \([n\text{ Celsius hot}]^{+}_{t,g}\) is the characteristic function of the set \{d \in D: (f^+(d,\text{hot},t,g) / r_{\text{Kelvin} - 273} = n\}.

<table>
<thead>
<tr>
<th>Kelvin degree</th>
<th>Celsius degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 - 273 = -273</td>
</tr>
<tr>
<td>1</td>
<td>1 - 273 = -272</td>
</tr>
<tr>
<td>273</td>
<td>273 - 273 = 0</td>
</tr>
<tr>
<td>274</td>
<td>274 - 273 = 1</td>
</tr>
<tr>
<td>275</td>
<td>275 - 273 = 2</td>
</tr>
</tbody>
</table>

\(^5\) It also follows that for any two possible units \(u_1\) and \(u_2\) of ratio predicates (predicates whose transformation value is always set to zero) like \text{long}, there is a real number \(r\), such that for any entity \(d\), \(d\) is mapped to the number \(n\) per \(u_1\) iff it is mapped to \(r \times n\) per unit \(u_2\). For example, as \(r_{\text{kilometer-long}} = (1000 \times r_{\text{meter-long}})\) in any \(t\) and \(g\), the quantity of kilometers of length in any \(d\) equals the quantity of meters of length in \(d\) divided by 1000. So it is predicted that we can easily shift between different units.
So things are 1 degree Celsius iff they are 274 degrees Kelvin. But a box is 1 degree Celsius more than a shelf iff the box is 1 degree Kelvin more than the shelf, not 274 degrees Kelvin more. Thus, Celsius seems to be a 'normal' predicate, not a unit name. One's degree in Celsius is one's degree in Kelvin (the ratio between one's heat and the heat in a Kelvin unit object) – minus 273.

\[(26)\]

For any d, \(\text{deg}^+(d,\text{Celsius hot},t,g) = f^+(d,\text{hot},t,g)/r_{\text{Kelvin},t,g} - 273\)

\[(27)\]

\([\text{The box is 2 degrees Celsius hotter than the shelf}]^+_t,g = 1 \iff \text{deg}^+([\text{the box}]^+_t,g,\text{Celsius hot},t,g) = \text{deg}^+([\text{the shelf}]^+_t,g,\text{Celsius hot},t,g) + 2\]

iff \(\exists d_k \in [\text{Kelvin}^1]_{t,g}, f^+([\text{the box}]^+_t,g,\text{hot},t,g)/f^+(d_k,\text{hot},t,g) - 273 = f^+([\text{the shelf}]^+_t,g,\text{hot},t,g)/f^+(d_k,\text{hot},t,g) - 273 + 2\)

iff \(\exists d_k \in [\text{Kelvin}^1]_{t,g}, f^+([\text{the box}]^+_t,g,\text{hot},t,g) = f^+([\text{the shelf}]^+_t,g,\text{hot},t,g) + 2\)

iff \(f^+([\text{the box}]^+_t,g,\text{hot},t,g) = f^+([\text{the shelf}]^+_t,g,\text{hot},t,g) + 2 \times r_{\text{Kelvin},t,g}\)

The ratio of heat in the box and in a Kelvin unit-object equals the ratio of heat in the shelf and in a Kelvin unit-object plus two (so the box is 2 degrees Kelvin hotter).

Since individuals' Kelvin degrees (the ratios between their degree in hot and the Kelvin unit objects' degree) are known to us (they do not vary through total contexts), so are also their Celsius degrees.

Yet, the numbers that Celsius assigns to entities do not directly represent quantities of heat. In accordance, they are not additive. They do not adequately represent the fact that the heat in two rods together equals the sum of heats in the two separate rods (for any t and g, \(f^+(d_1 \oplus d_2,\text{hot},t,g) = f^+(d_1,\text{hot},t,g) + f^+(d_2,\text{hot},t,g)\)). For example, if rod \(d_1\) and \(d_2\) each contains heat of 2 Kelvin degrees (\(f(d_1,\text{hot},t,g) = f(d_2,\text{hot},t,g) = 2 \times r_{\text{Kelvin}}\)), each falls under "2 – 273 Celsius hot", and the heat contained in both of them together, the heat in 4 Kelvin unit objects (\(f(d_1 \oplus d_2,\text{hot},t,g) = 4 \times r_{\text{Kelvin}}\)), falls under "(4 – 273) Celsius hot". But (2 – 273) + (2 – 273) = (4 – 546) ≠ (4 – 273). Thus, Celsius does not assign \(d_1 \oplus d_2\) the sum of the numbers it assigns to \(d_1\) and \(d_2\). The heat in any entity which is "2 Celsius hot" is not twice the heat in an entity which is "1 Celsius hot". In fact, the handbooks are full with explanations as to why it is senseless to say that "4 Celsius is twice as hot as 2 Celsius". But linguistically, speakers analyze Celsius as a unit name, not a predicate with Kelvin units, so despite these explanations, they cannot help feeling that this sentence is just fine (just like the sentence "4 meters is twice as long as 2 meters"). This further supports my proposal that when speakers use a unit name, they presuppose that the transformation value of the predicate it is a unit of is set to zero. These speakers analyze the statement The box is 2 Celsius hotter than the shelf as if it means: "\(\text{deg}^+([\text{the box}]^+_t,g,\text{hot},t,g) = \text{deg}([\text{the shelf}]^+_t,g,\text{hot},t,g) + 2 \times r_{\text{Celsius}}\)", which is, of course, wrong (the difference is 2× \(r_{\text{Kelvin}}\)).

Perhaps the existence of the concept Celsius gives further support also to the assumption that the "pre-scientific" interpretation of hot (as measuring temperature, not mental states of temperature perception), which is used by speakers in some actual contexts c, is mentally represented as transformed (at least in some total extensions of c). The point of zero is hardly
ever relevant, experienced, or talked about, by speakers that are not-scientists. Thus, for them any choice of a zero is arbitrary (but when a unit name is used, the 'scientific' interpretation must be accommodated; You jump to an extension c’ of c where Tran^+(hot,c’,g) = 0 for any g).

We may also explain the reluctance to combine numerical degree predicates with warm and cold in their positive forms (as in ?today was 25 degrees warm / cold), as reluctance to accommodate the presupposition that the transformation value is set to zero. This brings us to the next section.

9.3.6 Predicates that can never occur with numerical degree predicates

In actual contexts, we never associate specific numbers with entities relative to adjectives like beautiful and happy. For Moltmann (2006) that means that these adjectives are not linked with numerical degrees. On the present analysis, in contrast, it means that they lack conventional unit names. I propose that it is for this reason (and not because predicates like happy do not map entities to numerical degrees) that the degree function is felt to be vague. If the numbers that we associate with entities always reflect the ratios between their degrees and the degree of a known, agreed upon object, then the lack of such conventional objects results in vagueness concerning the mapping of entities to numbers.

Consider, for instance, happy. Maybe for you and for me the type of quantities that happy measures (with respect to a given dimension, say wrt one's career, or love life) are slightly different, but each time one of us uses the predicate (with respect to the given dimension) a quantity of something – a certain type of emotion – is measured. I have proposed that all the functions that are associated with happy in different total extensions of an actual context c (a context that represents the knowledge of a competent speaker) preserve the ordering between entities' degrees, and even the ratios between entities' degrees (i.e. happy is a ratio predicate). However, each one of us is forced to remain lonely with respect to her choice of a unit object for happiness (an entity serving for her as a reference point in a given comparison of emotional states). Emotions are internal states. It is hard to come up with conventions as to which emotional extent should be mapped to degree 1, 2, 3, etc. Even if one speaker maps a certain emotional state to degree 1 in happiness in c and g, no other speaker has access to this emotional state. So there cannot be found an object d such that it would be agreed upon by all the community of speakers that d is mapped to 1. All objects are equally likely to be used as unit objects provided only that in no t above c s.t. f^+(happy,t,g) (for an arbitrary g) maps them to zero (provided that they possess some non-zero amount of P-hood, so to speak). Thus, entities cannot be associated with established numbers (ratios between their quantity of happiness and a given agreed upon quantity of happiness). Established unit names (and numerical degree predicates) characterize predicates whose degrees are based on external (non-mental) states of things (like tall), such that a whole community can have access to the objects which are mapped to 1. For that reason weight can be measured by kilograms, but the physical or emotional states of speakers when they lift objects (their feeling of the objects being heavy, light, etc.) cannot be measured by conventionally established unit names. If a language maps a predicate to the latter type of degrees the predicate will not allow for modification by numerical degree phrases. For that reason, there exists some variation across languages concerning the set of predicates that allow numerical degree modifiers. This does not form evidence for the hypothesis that predicates do not map entities to numerical degrees (contrary to Moltmann's 2006 view). If you think about it, when no unit name is
explicitly mentioned, it is rather meaningless to say that something is tall to degree 456 (456 what? Millimeters? Kilometers? Inches?) In adjectives like happy this is always the situation.

Thus, my analysis takes up the numerical-scale approach, but it explains why the meaning of most of the gradable predicates appears to have characteristics of ordinal scales: They do not have an established unit name such that ratios between degrees can be expressed by using it. Thus, most predicates do not allow for numerical degree modification, and there is much indeterminacy concerning the number set that forms their scale and the number to which they map any given entity.

The proposal that numerical degrees and non-conventional unit objects form part of the interpretation of predicates like happy may be less intuitive than Moltmann's (2006) view, but it clearly has advantages.

First, it allows for an account of the compatibility of happy with ratio modifiers (like twice as P as). For example, statements like: Dan is twice as happy as Sam (or Dan is four times heavier than Sam) show that the ratios between, e.g., happiness degrees, can be treated as meaningful. Crucially, the truth of ratio statements is independent of units. It is a fact that the degree of an entity in a ratio predicate P is n times the degree of another entity in uP-units iff so is the case for any other uQ unit of P (Dan is 3 times taller than Sam in meters iff so is the case in centimeters, inches, etc.), because these ratios are fixed by the additive degree function of tall, not by the unit name.

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\[
[[\text{Dan is twice as happy as Sam}]]_{t,g}^{*} = 1 \iff f^{*}([[\text{Dan}]]_{t,g}^{*}, \text{happy}, t,g) = 2 \times f^{*}([[\text{Sam}]]_{t,g}^{*}, \text{happy}, t,g)
\]

Detailed derivations for ratio statements are given in 9.5. For now it is crucial to note that we could have hardly have assigned such statements an interpretation had predicates like happy not been associated with numerical degrees.

Second, my proposal allows for an account of the compatibility of happy with difference modifiers (like more P than). If one assumes that happy does not map entities to numbers, one needs to give two separate analyses to comparison statements with and without numerical degree predicates. We do not. I propose that in comparison statements with no numerical degree predicate like 2 meters (tall), there is an abstract covert predicate \( [u_{p}^{2}p(n)]_{num} \), equivalent to \( \lambda n. \forall y. u_{p}(y): m = n \times \deg(y, P) \), "n times more than the P degree of a P unit-object". However, in this interpretation, both n and uP are free variables. Let us assume that they both get existentially bound. For example, Dan is happier than Sue is interpreted as "For some n and some unit-set uP, Dan's degree of happiness equals Sue's, plus n times the happiness of an object in [uP]," or, equivalently, "For some n and some d \in D, Dan's degree of happiness equals Sue's, plus n times d's happiness". I further propose that the free variable n can be interpreted as any positive number, but the equative as P as can and should be used whenever n = 0. Thus, when the comparative more is used, n is assumed to be bigger than zero. The interpretation of Dan is happier than Sam, then, boils down to "for some non-zero positive number n and object d with some quantity of happiness\(^6\), Dan's degree of happiness equals Sam's degree of happiness plus n

\(^6\) The definition of unit names requires that the additive degree function give them a non-zero value. Since the function is additive, this means that unit objects will always possess some non-zero quantity of the relevant quality.
times d’s degree of happiness”, which in turn boils down to "the quantity of happiness in Dan is bigger than the quantity of happiness in Sam".\footnote{Alternatively, in analyzing comparison statements with no overt numerical degree predicate like (29) we can follow Landman (2005) and Schwarzschild and Wilkinson (2002) (at least for predicates that lack conventional unit names like happy), and assume that a default numerical predicate, \( \lambda n. n > 0 \) (rather than a numerical degree predicate) is used as the \( M_{t,g} \) argument of the comparative morpheme. This assumption produces the same final truth conditions as ours (e.g. the interpretation of \( \text{Dan is happier than Sam} \) will boil down to "for some non-zero positive number n, Dan's degree of happiness equals Sam's degree of happiness plus n"). At any rate, in the rest of this chapter I will persevere the assumption that in predicates that do have conventional unit names like tall the default predicate is, e.g., \( n \) meters, \( n \) centimeters, or etc., where, again, \( n > 0 \) (the final truth conditions will be the same whatever unit name related to this predicate one uses).}

(29) \[ \text{[Dan is happier than Sue]}^{t,c,g} = 1 \iff \]
\[ \text{[∃n,u\text{\_happy}. Dan is (u\text{\_happy}(n)) happier than Sue]}^{t,c,g} = 1 \iff \]
\[ \forall t \in T, t \geq c: \exists r \in R, r > 0, \exists d \in A \subseteq D, \]
\[ \deg'(\text{[Dan]}^{t,c,g,\text{\_happy},t,g_{n/u}}) = \]
\[ \deg'(\text{[Sue]}^{t,c,g,\text{\_happy},t,g_{n/u}}) + g_{n/u}(n) \times \deg(d,\text{\_happy},t,g_{n/u}) \iff \]
\[ \forall t \in T, t \geq c: \exists r \in R, r > 0, \exists d \in A \subseteq D, \]
\[ f''(\text{[Dan]}^{t,c,g,\text{\_happy},t,g}) = f''(\text{[Sue]}^{t,c,g,\text{\_happy},t,g}) + r \times f''(d,\text{\_happy},t,g) \iff \]

The use of an overt numerical degree predicate is not obligatory even for predicates like long that do map entities to conventional numbers, ratios wrt conventional unit names (for instance, we can say the box is longer than the table). Given this analysis, no meaning intension problem arises in statements that refer to degrees of entities in predicates like happy (contrary to Moltmann’s 2006 intuitions; cf. 3.2.2). In order to speak meaningfully, we do not need to know the happiness degrees (nor the length degrees) of entities, only the ratios between them and the degree of some other object. For the same reason (since speakers need not know the unit, while using ratio modifiers) happy can combine with ratio modifiers like twice or four times.

Finally, our proposal has the advantage that it accounts for the fact that we are able to average over happiness degrees when happy is used as a typicality-dimension (of predicates like people on their wedding days, etc.) If happy is associated with numerical degrees its degree-function can be an argument of a mean function. Moltmann’s (2006) proposal fails to represent that.

9.3.7 Numerical degree predicates in argument position

The present analysis does not suffer from wrong predictions regarding the felicity of statements like Dan is shorter than two meters. In this statement, two meters is a degree argument of the comparative relation, not a part of the relation itself (as in is two meters shorter). Thus, as Landman (2005) has noted, intuitively, two meters in this position should be interpreted as two meters tall. This is problematic for the extent account of the polarity effects (von Stechow 1984; Kennedy 1999), which can only be preserved if two meters can be interpreted as a negative extent in Dan is shorter than two meters, but not in Dan is two meters short (cf. 3.2.4.3.). But there is no principled reason for this difference. On the contrary, it is precisely the fact that two meters \( \emptyset \) is interpreted as two meters tall (and cannot be interpreted as two meters short) that renders the statement Dan is shorter than two meter felicitous (according to my analysis).
Recall that on my analysis, the ('at least') interpretation of \textit{two meters shorter}, and \textit{n meters shorter} (where in the lack of an overt degree predicate, \textit{shorter} is pragmatically enriched), are the following (where \(M_{<e,t>}\) in the interpretation of \textit{shorter} is a variable to be filled by a numerical degree predicate):\(^8\)

\[(30)\]
\begin{enumerate}
  \item Two meters \iff \lambda z. \exists x, \text{meter}^1_{\text{tall}}(x): \deg(z,\text{tall}) \geq 2 \times \deg(x,\text{tall})
  \item Shorter \iff \lambda M. \lambda x_2. \lambda x_1. M(\deg(x_1,\text{short}) - \deg(x_2,\text{short}))
  \item n meters shorter \iff \\
    \lambda x_2. \lambda x_1. \exists x, \text{meter}^1_{\text{tall}}(x): \deg(x_1,\text{short}) - \deg(x_2,\text{short}) \geq n \times \deg(x,\text{tall})
\end{enumerate}

On this proposal, degrees are numbers, \(n\), not tuples \(<n,\text{meter},\text{height}>\), so degree predicates like \textit{two meters} are never interpreted as degree denoting terms. When they form arguments of degree relations they type shift to a generalized quantifier meaning (to the set of properties \(M\) that anything \(n\) meters tall has).

\[(31)\]
\begin{enumerate}
  \item Numerical degree predicates in argument position:
    \begin{enumerate}
      \item \hspace{1cm} \(\lambda z. \exists x, \text{meter}^1_{\text{tall}}(x): \deg(z,\text{tall}) \geq 2 \times \deg(x,\text{tall})\) = \\
      \hspace{1cm} \lambda M. \forall z, s.t. (\exists x, \text{meter}^1_{\text{tall}}(x): \deg(z,\text{tall}) \geq 2 \times \deg(x,\text{tall})): M(z)
    \end{enumerate}
  \item \(\lambda z. \exists x, \text{meter}^1_{\text{tall}}(x): \deg(x_1,\text{short}) - \deg(x_2,\text{short}) \geq n \times \deg(x,\text{tall})\) = \\
    \hspace{1cm} \lambda GQ. \lambda x_1. GQ(\lambda x_2. \exists y, \text{meter}(y): \deg(x_1,\text{short}) - \deg(x_2,\text{short}) \geq n \times \deg(y,\text{long}))
\end{enumerate}

When an expression that denotes a two place entity relation has to combine with a generalized quantifier, semanticists often assume that it type shifts to a function from a generalized quantifier meaning to an entity predicate. Similarly, the comparative morpheme turns into a function from generalized quantifier meanings to degree predicates.

\begin{enumerate}
  \item \(\lambda x_2. \lambda x_1. \exists y, \text{meter}^1_{\text{tall}}(y): \deg(x_1,\text{short}) - \deg(x_2,\text{short}) \geq n \times \deg(y,\text{tall})\) = \\
    \hspace{1cm} \lambda GQ. \lambda x_1. GQ(\lambda x_2. \exists y, \text{meter}(y): \deg(x_1,\text{short}) - \deg(x_2,\text{short}) \geq n \times \deg(y,\text{long}))(z)
\end{enumerate}

When \(\text{TS}(\text{meter}^1_{\text{tall}}(2))\) and \(\text{TS}(\text{shorter than 2 meters})\) combine we get a degree predicate. The meaning of the predicate "\(n\) meters shorter than 2 meters" can be paraphrased as: "the set of entities such that the difference between their degree in \textit{short} and the degree of anything that is two meter long in \textit{short}, is \(n\) times the degree in \textit{tall} of a meter unit object" (where \(n\) is bigger than zero). This boils down to "the set of entities that are shorter than anything that is two meters tall".

Again, if all the occurrences of '\(\geq\)' and '\(<\)' in the derivations below are replaced with '=' we get the 'exactly' interpretation.

\(^8\) Again, if all the occurrences of '\(\geq\)' and '\(<\)' in the derivations below are replaced with '=' we get the 'exactly' interpretation.

\(^{264}\)
\(\lambda x. \forall z, \text{s.t. } (\exists x, \text{meter}^1_{\text{tall}}(x) \land \text{deg}(z,\text{tall}) \geq 2 \times \text{deg}(x,\text{tall})):
\exists y, \text{meter}^1_{\text{tall}}(y), \text{deg}(x_1,\text{short}) - \text{deg}(z,\text{short}) \geq n \times \text{deg}(y,\text{tall}) = 0\)

Since all the meter object units are equally tall, this boils down to:

\(\lambda x. \exists y, \text{meter}^1_{\text{tall}}(y), \forall z \text{s.t. } \text{deg}(z,\text{long}) \geq 2 \times \text{deg}(y,\text{tall}):
\text{deg}(x,\text{short}) - \text{deg}(z,\text{short}) \geq n \times \text{deg}(y,\text{tall}) = 0\)

We can now apply this predicate to Dan:

d. Dan is \(n\) meters shorter than 2 meters \(\iff\)
\(\lambda x. \exists y, \text{meter}^1_{\text{tall}}(y), \forall z \text{s.t. } \text{deg}(z,\text{tall}) \geq 2 \times \text{deg}(y,\text{tall}):
\text{deg}(x,\text{short}) - \text{deg}(z,\text{short}) \geq (n \times \text{deg}(y,\text{tall}))(\text{Dan}) = 0\)

Let us interpret a statement with such a predicate:

\[(32) \quad [(\text{Dan is } (n \text{ meters}) \text{ shorter than two meters } (\text{tall}))]^{*}_{tg} = 1 \text{ iff}
[(\exists y, \text{meter}^1_{\text{tall}}(y), \forall z \text{s.t. } \text{deg}(z,\text{tall}) \geq 2 \times \text{deg}(y,\text{tall})): \text{deg}(\text{Dan},\text{short}) - \text{deg}(z,\text{short}) \geq (n \times \text{deg}(y,\text{tall}))(\text{Dan}) = 0]^{*}_{tg} = 1 \text{ iff}
\exists d_m \in [\text{meter}^1_{\text{tall}}]^*_{tg}, \forall d_z \in D, \text{deg}^+(d_z,\text{tall},t,g) \geq 2 \times \text{deg}^+(d_m,\text{tall},t,g):
\text{deg}^+([(\text{Dan})]^*_{tg},\text{short},t,g) - \text{deg}^+(d_z,\text{short},t,g) \geq g(n) \times \text{deg}(d_m,\text{tall},t,g) \text{ iff}
\exists d_m \in [\text{meter}^1_{\text{tall}}]^*_{tg}, \forall d_z \in D, \text{f}^+(d_z,\text{tall},t,g) \geq 2 \times \text{f}^+(d_m,\text{tall},t,g):
\text{f}^+([(\text{Dan})]^*_{tg},\text{short},t,g) = \text{f}^+(d_z,\text{short},t,g) \text{ iff}
\exists d_m \in [\text{meter}^1_{\text{tall}}]^*_{tg}, \forall d_z \in D, \text{f}^+(d_z,\text{tall},t,g) \leq (2 - g(n)) \times \text{f}^+(d_m,\text{tall},t,g) \text{ iff}
\exists d_m \in [\text{meter}^1_{\text{tall}}]^*_{tg}, \text{f}^+([(\text{Dan})]^*_{tg},\text{short},t,g) \leq (2 - g(n)) \times \text{f}^+(d_m,\text{tall},t,g) \text{ iff}
\exists d_m \in [\text{meter}^1_{\text{tall}}]^*_{tg}, \text{f}^+([(\text{Dan})]^*_{tg},\text{short},t,g) \leq (2 - r) \times \text{f}^+(d_m,\text{tall},t,g) \text{ iff}\]

With discourse existential closure, and with the assumption that \(g(n)\) is larger than zero (because otherwise we should have used \(\text{as}\) rather than \(\text{more}\)), the interpretation boils down to "Dan’s height is smaller than 2 meters", as desired.

\[(33) \quad [(\exists n: \text{Dan is } (n \text{ meters}) \text{ shorter than two meters } (\text{tall}))]_{tg} = 1 \text{ iff}
\exists r \in R, r > 0, g(n) = r:
\exists d_m \in [\text{meter}^1_{\text{tall}}]^*_{tg}, \text{f}^+([(\text{Dan})]^*_{tg},\text{short},t,g) \leq (2 - g(n)) \times \text{f}^+(d_m,\text{tall},t,g) \text{ iff}
\exists r \in R, r > 0: \exists d_m \in [\text{meter}^1]^*_{tg}, \text{f}^+([(\text{Dan})]^*_{tg},\text{short},t,g) \leq (2 - r) \times \text{f}^+(d_m,\text{tall},t,g) \text{ iff}\]

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\[\exists d_m \in [\text{meter}^1\text{tall}]]_{t.g}^*, f^r([\text{Dan}])_{t.g}^*\text{tall}, t.g < 2 \times f^r(d_m, \text{tall}, t.g) \iff\]
\[f^r([\text{Dan}])_{t.g}^*, \text{tall}, t.g < 2 \times r_{\text{meter}, t.g}\]

### 9.3.8 Complex numerical degree relations

Grammar generates infinitely many complex degree relations, like 2 meter more than 4 meters less than 5 meter (shorter than …) and 2 meter more than 3 times 4 meter (longer than …) In this section I show that with the semantics that I have assigned to comparative morphemes and to numerical degree predicates, we can indeed generate these complex relations (though we may not be able to process them easily). Relations with ratio morphemes are discussed in 9.5.

We have seen that when we feed a type-shifted comparative-morpheme meaning with a type-shifted degree predicate, they produce a degree predicate. For example, the predicate 2 meters more than 5 meters denotes the set of degrees that are \((5 + 2)\) times as big as the meter's degree. The predicate 2 meters less than 5 meters denotes the set of degrees that are \((5 - 2)\) times smaller than the meter's degree:

\[
(34) \quad \text{Complex numerical degree relations}
\]

a. \(T^s\text{n meter}^2\long\) more than \((T^s\text{m meters}^2\long) \iff\)
\[
[\lambda GQ, \lambda k_1, GQ(\lambda k_2. \exists y: \text{meter}^1\long(y): k_1 \geq k_2 + (n \times \text{deg}(y, long))]\]
\[
(\lambda M. \forall k, \text{s.t.} (\exists x, \text{meter}^1\long(x): k \geq m \times \text{deg}(x, long)): M(k)) =\]
\[
\lambda k_1, [\lambda M. \forall k, \text{s.t.} (\exists x, \text{meter}^1\long(x): k \geq m \times \text{deg}(x, long))]: M(k)]\]
\[
(\lambda k_2. \exists y: \text{meter}^1\long(y): k_1 \geq k_2 + (n \times \text{deg}(y, long))) =\]
\[
\lambda k_1. \exists x, \text{meter}^1\long(x), k_1 \geq (m \times \text{deg}(x, long)) + (n \times \text{deg}(x, long)) = \]
\[
\lambda k_1. \exists x, \text{meter}^1\long(x), (k_1 \geq (m + n) \times \text{deg}(x, long))\]

b. \(T^s\text{n meter}^2\long\) less than \((T^s\text{m meters}^2\long) \iff\)
\[
[\lambda GQ, \lambda k_1, GQ(\lambda k_2. \exists y: \text{meter}^1\long(x): k_1 \leq k_2 - (n \times \text{deg}(y, long)))]\]
\[
(\lambda M. \forall k, \text{s.t.} (\exists x, \text{meter}^1\long(x): k \geq m \times \text{deg}(x, long)): M(k)) =\]
\[
\lambda k_1, [\lambda M. \forall k, \text{s.t.} (\exists x, \text{meter}^1\long(x): k \geq m \times \text{deg}(x, long))]: M(k)]\]
\[
(\lambda k_2. \forall k, \text{s.t.} (\exists x, \text{meter}^1\long(x): k \leq m \times \text{deg}(x, long)) \iff M(k)]\]
\[
(\lambda k_3. \exists y, \text{meter}^1\long(y): k_1 \leq k_2 - (n \times \text{deg}(y, long)) =\]
\[
\lambda k_1, \exists x, \text{meter}^1\long(x): k_1 \leq (m \times \text{deg}(x, long)) - (n \times \text{deg}(x, long)) =\]
\[
\lambda k_1, \exists x, \text{meter}^1\long(x): k_1 \leq (m - n) \times \text{deg}(x, long)\]

Such a degree predicate can combine with comparative morphemes to produce a degree relation. For example, the relation 2 meters more than 5 meters more holds of a degree pair \(<k_1, k_2>\) iff the difference between \(k_1\) and \(k_2\) is bigger than \((5 + 2)\) times the meter unit-objects’ degree. The predicate 2 meters less than 5 meters more \(d\) holds of a degree pair \(<k_1, k_2>\) iff the difference between \(k_1\) and \(k_2\) is smaller than \((5 - 2)\) times the meter unit-objects’ degree, etc.

c. \(n \text{ meter}^1\long\) more than \(m \text{ meters}^2\long\) (more) \iff\)
\[
(\lambda k_1. \exists x, \text{meter}^1\long(x): k_1 \geq (m + n) \times \text{deg}(x, long))\]
\[
[\lambda M. \lambda k_2, \lambda k_1. M(k_1 - k_2)] =\]

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With this, Landman explains the fact that NPIs are licensed in clausal, but not phrasal, interpretation of positive and negative predicates in clausal comparative statements, Landman comparatives (for details see Landman 2005). In order to produce the right results, we can start again.

\[ \lambda k_2, \lambda k_1. [\lambda k_1. \exists x, \text{meter}^1_{\text{long}}(x): k_1 \geq (m + n) \times \deg(x, \text{long})](k_1 - k_2) = \]
\[ \lambda k_2, \lambda k_1. [\lambda k_1. \exists x, \text{meter}^1_{\text{long}}(x): (k_1 - k_2) \geq (m + n) \times \deg(x, \text{long})] = \]
\[ \lambda k_2, \lambda k_1. [\lambda k_1, \exists x, \text{meter}^1_{\text{long}}(x): k_1 \geq k_2 + (m + n) \times \deg(x, \text{long})] = \]

\begin{enumerate}
\item n \text{ meter}^2_{\text{long}} \text{ less than m meters}^2_{\text{long}} \text{ (more)} \iff \\
\quad (\lambda k_1, \exists x, \text{meter}^1_{\text{long}}(x): k_1 \leq (m + n) \times \deg(x, \text{long})) \\
\quad [\lambda M. \lambda k_2, \lambda k_1. M(k_1 - k_2)] = \]
\item n \text{ meter}^2_{\text{long}} \text{ more than m meters}^2_{\text{long}} \text{ (less)} \iff \\
\quad (\lambda k_1, \exists x, \text{meter}^1_{\text{long}}(x): k_1 \leq (m + n) \times \deg(x, \text{long})) \\
\quad [\lambda M. \lambda k_2, \lambda k_1. M(k_1 - k_2)] = \]
\end{enumerate}

We can feed such a relation with a degree (or a type-shifted degree predicate, a degree generalized-quantifier, as described above), to produce a predicate. Then this predicate can combine with a comparative morpheme to produce a relation, and we can start again.

9.4 Direct consequences: Clausal comparatives

9.4.1 A supremum theory without including a supremum in a predicate interpretation

In Landman (2005), the interpretation of clausal comparatives is mediated by two non-trivial ingredients; a supremum operation and a zero relation (cf. 3.2.4).

\begin{enumerate}
\item (35) Parts of the interpretation of \textit{than}$_{\text{clausal}}$ in Landman (2005):
\begin{enumerate}
\item A supremum operation, $\cup_R$
\item A zero relation, $R_0$
\end{enumerate}
\end{enumerate}

With this, Landman explains the fact that NPIs are licensed in clausal, but not phrasal, comparatives (for details see Landman 2005). In order to produce the right results as to the interpretation of positive and negative predicates in clausal comparative statements, Landman (2005) postulates that the interpretation of positive and negative predicates includes:

\begin{enumerate}
\item (36) Some parts of the interpretation of predicates in Landman (2005):
\begin{enumerate}
\item A degree-relation, $>_{p} \ (\lambda \delta_2, \lambda \delta_1, \delta_1 > \delta_2 \text{ in positive predicates}; \text{this relation determines for any two P degrees which one is bigger relative to P})$
\end{enumerate}
\end{enumerate}

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b. A difference operation, \( \neg_p (\lambda \delta_2 \cdot \lambda \delta_1 \cdot \delta_2 - \delta_1 \), in positive predicates.

c. A supremum operation, \( \cup_p \) (an operation from a set to its minimum or lower bound, in positive predicates).

In Landman (2005), in negative predicates, these relations and operation are conversed compared to their positive antonym. E.g., the relation \( \succ_{\text{short}} \) reverses the ordering between degrees that is given by \( \succ_{\text{tall}} \) (it is the relation \( \lambda \delta_2, \lambda \delta_1, \delta_1 \ll \delta_2 \); The operation \( \neg_{\text{short}} \) is a reversed difference operation \( \lambda \delta_2, \lambda \delta_1, \delta_1 - \delta_2 \), and the supremum operation \( \cup_{\text{short}} \) is the infimum operation \( \cap_{\text{tall}} \) (an operation from a set to its maximum or upper bound; cf. 3.2.4.5 and 3.2.4.7). The definition of the ingredients in the interpretation of a clausal comparative statement in Landman (2005) (cf. (35)) is sensitive to whether the predicative argument is positive or negative.

In this section I show that, given our analysis of negative predicates as denoting reversed degree functions, we can adopt Landman's (2005) analysis of clausal comparatives without adding any relation or operation to the interpretation of predicates. So our theory allows an explanation of the distribution of NPIs in comparatives for free (without adding the ingredients in (36a-c) to the interpretation of predicates).

Let us employ the following definitions for the parts of the interpretation of \( \text{than}_{\text{clausal}} \) (which do not use anything in the interpretation of the predicative argument, e.g. \( \text{tall} \) or \( \text{short} \)). For each numerical degree relation \( R \) (say \( n \) meters more than \( m \) meters more), the pair \( \langle \cup_R, R_0 \rangle \) consisting of the supremum of \( R \), \( \cup_R \), and the zero relation of \( R \), \( R_0 \) is defined as follows.

\[(37) \quad \text{The supremum and zero relation in the meaning of } \text{than}_{\text{clausal}}:\]

a. \( \langle \cup_R, R_0 \rangle = \langle \cup, > \rangle \) (where \( \cup \) is 'minimum or lower bound' and \( > \) is 'bigger') iff \( R \) contains an even number of negative comparative morphemes (say, \textit{less}).

b. \( \langle \cup_R, R_0 \rangle = \langle \cap, < \rangle \) (where \( \cap \) is 'maximum or upper bound' and \( < \) is 'smaller'), otherwise.

For instance, the supremum of \( m \) meters more than \( n \) meters more tall is \( \cup \), because there is an even number of occurrences of \textit{less} in this relation (zero). The supremum of \( m \) meters less than \( n \) meters less short is \( \cup \) for the same reason (two occurrences of \textit{less}). The supremum of \( m \) meters more than \( n \) meters less short is \( \cap \), because there is an odd number of occurrences of \textit{less} in this relation (one), etc.

Principle (36) is more elegant than the one employed in Landman's (2005) original supremum theory in that no expression affects the supremum in the meaning of the clausal morpheme \textit{than} except for negative (reversed) morphemes like \textit{less} (that denotes a reversed difference function), or \textit{at most} (that denotes a negative comparative relation; a relation that is reversed wrt the relation \textit{at least}). The difference morphemes \textit{more} and \textit{less} are indeed tightly connected to \textit{than} so it is highly reasonable that they affect its interpretation. The relational morpheme \textit{at most} (as in \textit{at most two meters more}) explicitly expresses a comparison relation, so, again, it is highly reasonable that it affects the interpretation of \textit{than} (the definition of its zero relation, and through it, of the supremum relation)\(^9\). Landman's theory has a problem of explanatory adequacy, because the interpretation of \textit{than}_{\text{clausal}} in his theory is sensitive to features in the interpretation of the

---

\(^9\) I presuppose that the things forming the supremum and zero relation in a given comparison statement always co-vary. This is based on the fact that in the long list of detailed examples reviewed in Landman 2005 they always do. So unless other examples are found, there is no need for defining the zero relation separately.
predicate in the comparison statement (e.g., tall or short). This is both unintuitive and complicated. Furthermore, the supremum operation in predicates’ interpretation is not independently motivated. In addition, the supremums of positive predicates and their negative antonyms come out reversed by separate stipulation, and not derived from or linked to another feature of the antonym. In my theory, the fact that the supremum of R turns into the converse operation in derivations with negative predicates (as demonstrated below) is derived from the fact that negative predicates are linked with reversed degree functions (functions that reverse the entity ordering). The association of negative predicates with reversed functions has many positive outcomes independent of the analysis of clausal comparatives, as explained at length in chapter 7 and in earlier sections of this chapter. Finally, in my theory, the supremum (like any other operation or relation) operates on a set of numbers, not degree tuples, which is again, both simpler and more explanatory (the contribution of the unit name is better understood, as demonstrated in earlier sections of this chapter).

Following Landman (2005), I assume that the interpretation of a clausal comparative predicate has the form "is R P than GQ is Q", i.e. it is built from a numerical degree relation R (everything in the matrix clause above the subject excluding the predicate P and the than-clause), a predicate P (the predicate in the matrix clause, say tall or short), a generalized quantifier GQ (the noun phrase in the than-clause, say every boy, two meters, John, etc.), and a predicate Q (the predicate in the than-clause). Than takes as arguments a relation R, a generalized quantifier GQ, and two predicates P and Q, and it returns the comparative predicate: \( \lambda x_1.R_0(deg(x_1,P), \cup(\lambda k.GQ(\lambda x_2.R(k,deg(x_2,Q)))) \). This predicate holds of entities such that the R\(_0\) relation holds between their degree in P and the supremum of the number-set \( \lambda k.GQ(\lambda x_2.R(k,deg(x_2,Q))) \). This set contains all the numbers k such that the relation R holds between them and GQ's (e.g. every boy's) degree in Q.

(38) Clausal comparatives:
   a. than\(_{\text{clausal}} \leftrightarrow \lambda Q.\lambda GQ.\lambda P.\lambda R. \lambda x_1.R_0(deg(x_1,P), \cup(\lambda k.GQ(\lambda x_2.R(k,deg(x_2,Q))))\))

For example, in Dan is more than 2 meters taller than every boy is (tall), we feed this function with the matrix predicate (tall), the predicate of the than-clause (tall) and the GQ meaning of every boy is:

b. every boy is \( \leftrightarrow \lambda P.\forall y, \text{boy}(y): P(y) \)
c. than (every boy is ,tall, tall) \( \leftrightarrow \lambda R.\lambda x.R_0(deg(x,tall), \cup_R(\lambda k.\forall y, \text{boy}(y): P(y))(\lambda y,R(k,deg(y,tall)))) = \lambda R.\lambda x.R_0(deg(x,tall), \cup_R(\lambda k.\forall y, \text{boy}(y): R(k,deg(y,tall)))) \)

Next we feed this with the relation (n meters) more than 2 meters more, \( \lambda k_2.\lambda k_1. \exists x, \text{meter}^1_{\text{tall}}(x), k_1 \geq k_2 + (2 + n) \times \text{deg}(x,\text{tall}) \), whose zero-relation is \( \lambda k_2.\lambda k_1. k_1 > k_2 \), and whose supremum is \( \cup \):

d. n meter more than 2 meters more tall than every boy is tall \( \leftrightarrow \lambda R.\lambda x.R_0(deg(x,tall), \cup_R(\lambda k.\forall y, \text{boy}(y): R(k,deg(y,tall)))) \)

---

10 Note also that I add to the interpretation of predicates a transformation value, but this is done for other purposes (such as explaining the distribution of unit names and degree predicates), and is not needed for deriving truth conditions of clausal comparatives.

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Finally, we feed this with the matrix subject, Dan.

e. Dan is (n meters) more than 2 meters more tall than every boy is tall ⇔
\[ \text{deg}(\text{Dan}, \text{tall}) > \bigcup(\lambda k. \forall y, \text{boy}(y): \exists x, \text{meter}^1_{\text{tall}}(x), k \geq \text{deg}(y, \text{tall}) + (2 + n) \times \text{deg}(x, \text{tall}) ) \]

f. \[ ((\exists n: \text{Dan is (n meters) more than 2 meters more tall than every boy is})[_{t,g}] = 1 \]
iff \[ \exists r \in R, r > 0: \text{deg}^{+}([[\text{Dan}]]^{+}_{tg}, \text{tall}, t, g) > \bigcup\{(k \in R: \forall b \in [[\text{boy}]]^{+}_{tg}, \exists d_m \in [[\text{meter}^1_{\text{tall}}]]^{+}_{tg}; k \geq \text{deg}^{+}(b, \text{tall}, t, g) + (2 + r) \times \text{deg}^{+}(d_m, \text{tall}, t, g) \} \]
iff \[ f^{+}([[\text{Dan}]]^{+}_{tg}, \text{tall}, t, g) > \bigcup\{(k \in R: \forall b \in [[\text{boy}]]^{+}_{tg}, k < f^{+}(b, \text{tall}, t, g) + 2 \times r_{\text{meter}, t} \} \]
Dan's height is bigger than the supremum (minimum or lower bound) of the set of degrees bigger than all the boys' heights plus 2 meters, i.e.
iff \[ \forall b \in [[\text{boy}]]:: f^{+}([[\text{Dan}]]^{+}_{tg}, \text{tall}, t, g) > \text{tall} f^{+}(b, \text{tall}, t, g) + 2 \times r_{\text{meter}, t} \]
Dan's height is bigger than every boy's height plus 2 meters.

The final truth conditions require that Dan's height be bigger than the supremum (minimum or lower bound) of the set of degrees bigger than every boy's height in more than 2 times the meter unit-objects' height (as desired). For example, if the tallest boy is 2 meter tall (we are discussing giant boys here) than the supremum (minimum or lower bound) of the set of degrees bigger than all the boys' heights plus 2 meters (i.e. the lower bound of the set λn. 4 < n) is 4. Thus, Dan is required to be taller than 4 meters (i.e. more than 2 meters taller than every boy), as desired.

If we replace the relation with (n meters) less than 2 meters more, \[ \lambda k_2. \lambda k_1. \exists x, \text{meter}^1_{\text{tall}}(x), k_1 \leq k_2 + (2 - n) \times \text{deg}(x, \text{tall}), \]
whose zero-relation is \[ \lambda k_2. \lambda k_1. k_1 < k_2 \]
and supremum is \( \cap \), we get:

g. \[ ((\exists n: \text{Dan is (n meters) less than 2 meters more tall than every boy is})[_{t,g}] = 1 \]
iff \[ \exists r \in R, r > 0: \text{deg}^{+}([[\text{Dan}]]^{+}_{tg}, \text{tall}, t, g) < \bigcap\{(k \in R: \forall b \in [[\text{boy}]]^{+}_{tg}, \exists d_m \in [[\text{meter}^1_{\text{tall}}]]^{+}_{tg}; k \leq \text{deg}^{+}(b, \text{tall}, t, g) + (2 - r) \times \text{deg}^{+}(d_m, \text{tall}, t, g) \} \]
iff \[ f^{+}([[\text{Dan}]]^{+}_{tg}, \text{tall}, t, g) < \bigcap\{(k \in R: \forall b \in [[\text{boy}]]^{+}_{tg}, k < f^{+}(b, \text{tall}, t, g) + 2 \times r_{\text{meter}, t} \} \]
Dan's height is smaller than the infimum (maximum or upper bound) of the set of degrees smaller than all the boys' heights plus 2 meters, i.e.
iff \[ \forall b \in [[\text{boy}]]:: f^{+}([[\text{Dan}]]^{+}_{tg}, \text{tall}, t, g) < \text{tall} f^{+}(b, \text{tall}, t, g) + 2 \times r_{\text{meter}, t} \]
Dan's height is smaller than every boy's height plus 2 meters.
For example, if the shortest boy is 2 meter tall than the infimum (maximum or upper bound) of the set of degrees smaller than all the boys' heights plus 2 meters (i.e. the upper bound of the set $\lambda n. n < 4$) is 4. Thus, Dan is required to be shorter than 4 meters, as desired (Dan is less than two meters taller than the shortest boy, and surely less than 2 meters taller than the taller boys).

If we replace the relation with (n meters) more than 2 meters less, $\lambda k_2. \lambda k_1. \exists x, \text{meter}^t_{\text{tall}}(x), k_1 \leq k_2 - (2 + n) \times \text{deg}(x, \text{tall})$, whose zero-relation is $\lambda k_2. \lambda k_1, k_1 < k_2$, and supremum is $\cap$, we get:

$$\exists n: \text{Dan is (n meters) more than 2 meters less tall than every boy is}$$

If we replace the relation with (n meters) less than 2 meter less, $\lambda k_2. \lambda k_1. \exists x, \text{meter}^t_{\text{tall}}(x), k_1 \geq k_2 - (2 - n) \times \text{deg}(x, \text{tall})$, whose zero-relation is $\lambda k_2. \lambda k_1, k_1 > k_2$, and supremum is $\cup$, we get:

$$\exists n: \text{Dan is (n meters) less than 2 meters less tall than every boy is}$$

For example, if the tallest boy is 4 meters tall than the supremum (minimum or lower bound) of the set of degrees bigger than all the boys' heights minus 2 meters (i.e. the lower bound of the set $\lambda n. n > 2$) is 2. Thus, Dan is required to be taller than 2 meters, as desired (Dan is more than two meters less tall than the tallest boy, and surely more than 2 meters less tall than the shorter boys).

If we replace the relation with (n meters) less than 2 meter less, $\lambda k_2. \lambda k_1. \exists x, \text{meter}^t_{\text{tall}}(x), k_1 \geq k_2 - (2 - n) \times \text{deg}(x, \text{tall})$, whose zero-relation is $\lambda k_2. \lambda k_1, k_1 > k_2$, and supremum is $\cup$, we get:

$$\exists n: \text{Dan is (n meters) less than 2 meters less tall than every boy is}$$

For example, if the tallest boy is 4 meters tall than the supremum (minimum or lower bound) of the set of degrees bigger than all the boys' heights minus 2 meters (i.e. the lower bound of the set $\lambda n. n > 2$) is 2. Thus, Dan is required to be taller than 2 meters, as desired (Dan is less than two meters less tall than the tallest boy, and surely less than 2 meters less tall than the shorter boys).
9.4.2 Comparisons with negative predicates

What happens with negative predicates like short? If we feed \([\text{than}]\)\textsubscript{t,g} with two negative predicates like short and the GQ meaning of every boy is, we get:

\[
(39) \quad \text{Clausal comparatives with negative predicates:}
\]

a. \(\text{than (every boy is, short, short)} \iff \lambda x. R(x, \text{short}, \text{short})\)

\[
= \lambda x. \forall b. (\text{short}(b) \land \text{short}(b))
\]

\[
\land \lambda y. (\text{short}(y) \land \text{short}(y))
\]

b. \(n \text{ meters more than 2 meters more short than every boy is tall} \iff \lambda x. (\text{short}(x) \land \text{short}(x))\)

\[
\land \lambda y. (\text{short}(y) \land \text{short}(y))
\]

\[
\land \lambda z. (\text{short}(z) \land \text{short}(z))
\]

\[
\land \lambda w. (\text{short}(w) \land \text{short}(w))
\]

\[
\land \lambda v. (\text{short}(v) \land \text{short}(v))
\]

\[
\land \lambda u. (\text{short}(u) \land \text{short}(u))
\]

The difference between tall and short is that short reverses the additive degree function of tall and transforms its values by a constant \(\text{Tran}^+\). Thus, we get:

\[
c. \quad \text{[(\exists n: Dan is (n meters) more than 2 meters more short than every boy is)]}_{t,g} = 1
\]

\[
\text{iff } \exists r \in R, r > 0: \quad \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g) > \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g)
\]

\[
\lor (\forall b \in [\text{boy}]_{t,g}, \exists d \in [\text{meter}_{t}]_{t,g}, k \geq \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g) + (2 + r) \times \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g))
\]

\[
\text{iff } \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g) > \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g)
\]

\[
\lor (\forall b \in [\text{boy}]_{t,g}, k > \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g) + (2 + r) \times \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g))
\]

\[
\text{iff } \text{Tran}^+([\text{Dan}]_{t,g}, \text{short}, t, g) - f^+([\text{Dan}]_{t,g}, \text{short}, t, g) > \text{Tran}^+([\text{Dan}]_{t,g}, \text{short}, t, g) - f^+([\text{Dan}]_{t,g}, \text{short}, t, g)
\]

\[
\lor (\forall b \in [\text{boy}]_{t,g}, \text{Tran}^+([\text{Dan}]_{t,g}, \text{short}, t, g) - k < f^+([\text{Dan}]_{t,g}, \text{short}, t, g) - (2 + r) \times \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g))
\]

Now we have several steps of computations. First:

\[
\text{iff } \text{Tran}^+([\text{Dan}]_{t,g}, \text{short}, t, g) - f^+([\text{Dan}]_{t,g}, \text{short}, t, g) > \text{Tran}^+([\text{Dan}]_{t,g}, \text{short}, t, g) - k \in R, f^+([\text{Dan}]_{t,g}, \text{short}, t, g) - (2 + r) \times \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g))
\]

In order to see that we can make this step, imagine that \(\text{Tran}^+([\text{Dan}]_{t,g}, \text{short}, t, g) = 4\), that \(r_{\text{meter},t} = 1\), and that the shortest boy is 4 meters tall:

\[
\{k \in R: \forall b \in [\text{boy}]_{t,g}, \text{Tran}^+([\text{Dan}]_{t,g}, \text{short}, t, g) - k < f^+([\text{Dan}]_{t,g}, \text{short}, t, g) - (2 + r) \times \text{deg}^+([\text{Dan}]_{t,g}, \text{short}, t, g))
\]

\[
\{k \in R: 4 - k < 2 \} = \{k \in R: 4 < k \} = \{4 - k: k \in R, k < 2 \} =
\]
\{\text{Tran}^+(\text{short},t,g) - k: k \in \mathbb{R}, \forall b \in \{\text{boy}\}_{t,g}^{+}, k < f^+(b,\text{tall},t,g) - 2 \times r_{\text{meter},1}\}\}.

(The set of numbers that equal 4 minus a real that is smaller than 2 \{(4 - k: k \in \mathbb{R}, k < 2 \}) is precisely the set of reals bigger than 2).

Second, the supremum (minimum or lower bound) of the set of numbers (\text{Tran}^+(\text{short},t,g) - k) such that k satisfies the given constraint equals \text{Tran}^+(\text{short},t,g) minus the infimum (maximum or upper bound) of the set of numbers \(k\) that satisfy the given constraint (since to get the lower bound of the set of differences (\text{Tran}^+(\text{short},t,g) - k) you have to subtract the upper bound of the set of numbers \(k\)). Therefore, we get

\[
\text{iff } \quad \text{Tran}^+(\text{short},t,g) - f^+([\text{Dan}]^+_{t,g}, \text{tall},t,g) > \text{Tran}^+(\text{short},t,g) -
\cap(\{ k: k \in \mathbb{R}, \forall b \in \{\text{boy}\}_t^+, k < f^+(b,\text{tall},t,g) - 2 \times r_{\text{meter},1}\})
\]

For example, \(\cup(\{4 - k: k \in \mathbb{R}, k < 2 \}) = 4 - \cap((k: k \in \mathbb{R}, k < 2))\) (the lower bound of the set of numbers (4 - k) where k is a real smaller than 2 (i.e. the number 2) equals 4 minus the upper bound of the set of reals smaller than 2 (the number 2)).

And then it is easy to go to:

\[
\text{iff } \quad - f^+[\{\text{Dan}\}_{t,g}^+, \text{tall},t,g] > \cap(\{ k: k \in \mathbb{R}, \forall b \in [\text{boy}]_{t,g}^+, k < f^+(b,\text{tall},t,g) - 2 \times r_{\text{meter},1}\})
\]

Dan's height is smaller than the infimum (maximum or upper bound) of the set of degrees smaller than all the boys' heights minus 2 meters, i.e.

\[
\forall b \in [\text{boy}], f^+[\{\text{Dan}\}_{t,g}^+, \text{tall},t,g] < f^+(b,\text{tall},t,g) - 2 \times r_{\text{meter},1}
\]

We get that \(\text{Dan is more than 2 meters shorter than every boy}\) is true iff Dan's height is smaller than every boy's height minus 2 meters, as desired.

If we replace the relation with \((n \text{ meters}) \text{ less than 2 meters more}, \lambda k_2, \lambda k_1, \exists x, \text{meter}^1_{\text{tall}}(x), k_1 \leq k_2 + (2 - n) \times \text{deg}(x, \text{tall}), \) whose zero-relation is \(\lambda k_2, \lambda k_1, k_1 < k_2 \) and supremum is \(\cap, \) we get:

\[
\text{iff } \quad \exists x, \text{meter}^1_{\text{tall}}(x), k_1 \leq \deg(b,\text{short},t,g) + (2 - r) \times \text{deg}(d_m,\text{tall},t,g))
\]

We get that \((\exists n: \text{Dan is (n meters) less than 2 meters more short than every boy is})_{t,g}^+ = 1\)

\[
\text{iff } \quad \exists x, \text{meter}^1_{\text{tall}}(x), k_1 \leq \deg(b,\text{short},t,g) + (2 - r) \times \text{deg}(d_m,\text{tall},t,g))
\]

\[
\text{iff } \quad \exists x, \text{meter}^1_{\text{tall}}(x), k_1 \leq \deg(b,\text{short},t,g) + (2 - r) \times \text{deg}(d_m,\text{tall},t,g))
\]

\[
\text{iff } \quad \exists x, \text{meter}^1_{\text{tall}}(x), k_1 \leq \deg(b,\text{short},t,g) + (2 - r) \times \text{deg}(d_m,\text{tall},t,g))
\]
We get that \textit{Dan is less than 2 meters shorter than every boy is} is true iff Dan's height is bigger than every boy's height minus 2 meters, as desired.

If we replace the relation with \textit{(n meters) more than 2 meters less short than}, \(\lambda k_2, \lambda k_1. \exists x, \text{meter}^1_{\text{tall}}(x), k_1 \leq k_2 - (2 + n) \times \deg(x, \text{tall})\), whose zero-relation is \(\lambda k_2, \lambda k_1, k_1 < k_2\) and supremum is \(\lor\), we get:

e. \[[\exists n: \text{Dan is (n meters) more than 2 meters less short than every boy is}]]_{t,g} = 1\]
   iff \(\exists r \in R, r > 0: \deg^+([[[\text{Dan}]]]_{t,g}^+, \text{short}, t,g) < \)
   \(\land \{ \{ k \in R: \forall b \in [[\text{boy}]]_{t,g}^+, \exists d \in [\text{meter}^1_{\text{tall}}]_{t,g}: k \leq \deg^+(b, \text{short}, t,g) - (r + 2) \times \deg^+(d_m, \text{tall}, t,g) \}\}\)
   iff \(\deg^+([[[\text{Dan}]]]_{t,g}^+, \text{short}, t,g) < \)
   \(\land \{ \{ k \in R: \forall b \in [[\text{boy}]]_{t,g}^+, k < \deg^+(b, \text{short}, t,g) - 2 \times r_{\text{meter}, t} \}\}\)
   iff \(\text{Trans}^+(\text{short}, t,g) - \text{f}^+([[[\text{Dan}]]]_{t,g}^+, \text{tall}, t,g) < \)
   \(\land \{ \{ k \in R: \forall b \in [[\text{boy}]]_{t,g}^+, k < \text{Trans}^+(\text{short}, t,g) - \text{f}^+(b, \text{tall}, t,g) - 2 \times r_{\text{meter}, t} \}\}\)
   iff \(\text{Trans}^+(\text{short}, t,g) - \text{f}^+([[[\text{Dan}]]]_{t,g}^+, \text{tall}, t,g) < \)
   \(\land \{ \{ \text{Trans}^+(\text{short}, t,g) - k < \text{Trans}^+(\text{short}, t,g) - k > \text{f}^+(b, \text{tall}, t,g) + 2 \times r_{\text{meter}, t} \}\}\)
   iff \(\text{Trans}^+(\text{short}, t,g) - \text{f}^+([[[\text{Dan}]]]_{t,g}^+, \text{tall}, t,g) < \)
   \(\land \{ \{ \text{Trans}^+(\text{short}, t,g) - k < \text{Trans}^+(\text{short}, t,g) - k > \text{Trans}^+(\text{short}, t,g) - k < \text{Trans}^+(\text{short}, t,g) - \text{f}^+(b, \text{tall}, t,g) + 2 \times r_{\text{meter}, t} \}\}\)
   iff \(\text{Trans}^+(\text{short}, t,g) - \text{f}^+([[[\text{Dan}]]]_{t,g}^+, \text{tall}, t,g) < \)
   \(\land \{ \{ \text{Trans}^+(\text{short}, t,g) - k < \text{Trans}^+(\text{short}, t,g) - \text{f}^+([[[\text{Dan}]]]_{t,g}^+, \text{tall}, t,g) + 2 \times r_{\text{meter}, t} \}\}\)
   iff \(\text{Trans}^+(\text{short}, t,g) - \text{f}^+([[[\text{Dan}]]]_{t,g}^+, \text{tall}, t,g) < \)

We get that \textit{Dan is more than 2 meters less short than every boy is} is true iff Dan's height is bigger than every boy's height plus 2 meters, as desired.

If we replace the relation with \textit{(n meters) less than 2 meters less short than}, \(\lambda k_2, \lambda k_1. \exists x, \text{meter}^1_{\text{tall}}(x), k_1 \geq k_2 - (2 + n) \times \deg(x, \text{tall})\), whose zero-relation is \(\lambda k_2, \lambda k_1, k_1 > k_2\) and supremum is \(\lor\), we get:

f. \[[\exists n: \text{Dan is (n meters) less than 2 meters less short than every boy is}]]_{t,g} = 1\]
   iff \(\exists r \in R, r > 0: \deg([[[\text{Dan}]]]_{t,g}^+, \text{short}, t,g) > \)
   \(\lor \{ \{ k \in R: \forall b \in [[\text{boy}]]_{t,g}^+, \exists d \in [\text{meter}^1_{\text{tall}}]_{t,g}: k \geq \deg^+(b, \text{short}, t,g) + (r - 2) \times \deg^+(d_m, \text{tall}, t,g) \}\}\)
   iff \(\deg([[[\text{Dan}]]]_{t,g}^+, \text{short}, t,g) > \)
   \(\lor \{ \{ k \in R: \forall b \in [[\text{boy}]]_{t,g}^+, k > \deg^+(b, \text{short}, t,g) - 2 \times r_{\text{meter}, t} \}\}\)
   iff \((\text{Trans}^+(\text{short}, t,g) - f([[[\text{Dan}]]]_{t,g}^+, \text{tall}, t,g) > \)

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\[ \cup \{ k \in R : \forall b \in [[\text{boy}]]_{t,g} : k > \text{Tran}^* (\text{short},t,g) - f^*(b,\text{tall},t,g) - 2 \times \text{r}_{\text{meter},t} \} \]

iff \[ f^*([[\text{Dan}]]^*_{t,g}, \text{tall},t,g) < \]

\[ \cap \{ k : k \in R, \forall b \in [[\text{boy}]]^*_{t,g}, k < f^*(b,\text{tall},t,g) + 2 \times \text{r}_{\text{meter},t} \} \]

Dan's height is smaller than the infimum (maximum or upper bound) of the set of degrees smaller than all the boys' heights plus 2 meters, i.e.

iff \[ \forall b \in [[\text{boy}]]^*_{t,g} : f^*([[\text{Dan}]]^*_{t,g}, \text{tall},t,g) < f^*(b,\text{tall},t,g) + 2 \times \text{r}_{\text{meter},t} \]

So we get that \textit{Dan is less than 2 meter less short than every boy is} iff Dan's height is smaller than every boy's height plus 2 meters, as desired.

In sum, in virtue of the postulate that negative predicates denote reversed functions, we derive the same predictions as the original supremum theory (Landman 2005), but for free (without enriching the interpretation of predicates with supremum and difference operations). This gives further support to the reversed-function theory.

### 9.4.3 Cross polar anomalies and normalies

Another virtue of our proposal is that we predict the intuition that cross polar comparisons like \textit{The ladder is taller than the house is short} are anomalies. This prediction is derived in virtue of the fact that we assume negative predicates like \textit{short} to have a transformation value, which varies between total contexts. As a result, when we try to compare degrees in a negative predicate \( Q \) (e.g., \textit{short}) with degrees in another predicate \( P \) (e.g., \textit{tall}) whose transformation value does not co-vary with that of \( Q \), the transformation value does not get canceled out (so it plays a crucial role in the truth conditions), and in our partial context \( c \), they can never be verified.

\[ \text{(40) Cross polar anomalies} \]

\[ [[\text{Dan is (not tall) taller than Sam is short / isn't tall}]]_{c,g} = 1 \]

iff \[ \forall t \in T, t \geq c, \exists r \in R, r > 0: \]

\[ \text{deg}^* ([[\text{Dan}]]^*_{t,g}, \text{tall},t,g) > \]

\[ \cup \{ k \in R : \exists d_m \in [[\text{u}]]^*_{t,g}, \exists r \in R, r > 0: \]

\[ k > (\text{Tran}^* (\text{short},t,g) - f^*([[\text{Sam}]]^*_{t,g}, \text{tall},t,g)) + r \times (f^*(d_m,\text{tall},t,g) + (\text{Tran}^* (\text{tall},t,g))) \}

iff \[ \forall t \in T, t \geq c, \exists r \in R: \]

\[ f^*([[\text{Dan}]]^*_{t,g}, \text{tall},t,g) > \]

\[ \cup \{ k \in R : \exists d_m \in [[\text{u}]]^*_{t,g}, \exists r \in R, r > 0: \]

\[ k > (\text{Tran}^* (\text{short},t,g) - f^*([[\text{Sam}]]^*_{t,g}, \text{tall},t,g)) + r \times (f^*(d_m,\text{tall},t,g) + (\text{Tran}^* (\text{tall},t,g))) \}

iff \[ \forall t \in T, t \geq c, \exists r \in R: \]

\[ f^*([[\text{Dan}]]^*_{t,g}, \text{tall},t,g) > \]

\[ \cup \{ k \in R : k > \text{Tran}^* (\text{short},t,g) - f^*([[\text{Sam}]]^*_{t,g}, \text{tall},t,g) \}

iff \[ \forall t \in T, t \geq c, \exists r \in R: \]

\[ f^*([[\text{Dan}]]^*_{t,g}, \text{tall},t,g) > \]

\[ \cup \{ k \in R : k > \text{Tran}^* (\text{short},t,g) - f^*([[\text{Sam}]]^*_{t,g}, \text{tall},t,g) \}

iff \[ \forall t \in T, t \geq c, \exists r \in R: \]

\[ f^*([[\text{Dan}]]^*_{t,g}, \text{tall},t,g) > \]

\[ \cup \{ \text{Tran}^* (\text{short},t,g) - k : k \in R, k < f^*([[\text{Sam}]]^*_{t,g}, \text{tall},t,g) \} \]
iff \( \forall t \in T, t \geq c, f^*([[Dan]]^+_t,g, tall,t,g) > \)
\(\text{Tran}^*(\text{short},t,g) - \bigcap \{ k \in R : k < f^*([[Sam]]^+_t,g, tall,t,g) \} \))

Polar nomalies like *The ladder is (10 cm) shorter than the house is high* exist, though they are
marginal (Landman 2005; Buring 2007). I propose that in such examples we actually compare
two degrees of *short*. We treat the sentence as a within-predicate comparison statement, and if we
find an excuse for the occurrence of *high* at the end of the sentence, the derivation does not crash.
The mention of *tall* or *high* in the *than*-clause can be taken to echo the fact that the degree
function of these predicates is part of the interpretation of *shorter*, but it does not prevent the
reversing of the *tall*-degrees (unless *tall* or *high* are stressed). In fact, the use of the direct
antonym *tall* in the *than*-clause is less acceptable than the use of *high*, because the use of *high*
can be viewed as adding content to the sentence. For example, it’s raison d’etre in the sentence may be
that it cancels the presupposition of *tall* that the ordered entity (the house) is human. Crucially,
the predicate *short* in cross polar anomalies (*Dan is taller than Sam is short*), cannot be taken as
echoing the fact that its degree function is part of the interpretation of *taller*, because it is not.
And if *short* may have a raison d’etre in this sentence (if it can add anything new to the
interpretation) then it is precisely the reversal of Sam’s (the internal argument’s) degree. As we
cannot cancel this reversal, the derivation crashes.

\[
(41) \text{Cross polar nomalies} \\
[[\text{The ladder is shorter than the house is high}]]_{t,g} = 1 \text{ iff} \\
\text{iff } \exists r \in R, r > 0: \quad \deg^*(([[\text{the ladder}]]^+_t,g, short,t,g)) > \\
\cup \{ \{ k \in R : \exists d_m \in [[u_{\text{high}}]]^+_t,g; \}
\quad k > \deg^*(([[\text{the house}]]^+_t,g, short,t,g) + r \times \deg^*(d_m, high,t,g)) \} \\
\text{iff } f^*(([[\text{the ladder}]]^+_t,g, high,t,g)) < \bigcap \{ \{ k \in R, k < f^*(([[\text{the house}]]^+_t,g, high,t,g) \}
\text{iff } f^*(([[\text{the ladder}]]^+_t,g, high,t,g) < f^*(([[\text{the house}]]^+_t,g, high,t,g) \\
\]
iff \ f([\text{The box}], \text{long}, \text{t}, \text{g}) > f^*[([\text{The shelf}])^*_{\text{t}\text{g}}, \text{wide}, \text{t}, \text{g}) + 2 \times \text{r}_{\text{meter}, \text{t}}

As discussed earlier, this comparison is possible in virtue of the convention that $r_{\text{meter-long}} = r_{\text{meter-width}}$. Also comparisons of deviation are of this sort (Kennedy 1999). For example, the final degree functions of both fast and slow in *My watch is faster than your watch is slow* are interpreted as denoting the distance of time points from the current time. The degree functions of both the negative and the positive adjective are converted to denote distance from a certain midpoint (and they share the unit).

9.5 Direct consequences:

The distribution of difference and ratio modifiers

An important advantage of my analysis is that it accounts for at least one factor that affects the distribution of ratio morphemes like *four times* in *four times heavier* or *twice* in *twice as tall*, and for the main factor that affects the distribution of difference modifiers like *more P than*.

9.5.1 The distribution of ratio modifiers

Consider ratio statements like *80 meters is four times more than 20 meters, Dan is four times heavier than 3 kg, The shelf is twice as long as the wall, etc.*\(^{11}\)

The ratio modifiers *4 times* and *twice* have the multiplication operation, \(\lambda n_2.\lambda n_1. n_1 = 2 \times n_2\), as part of their interpretation. These modifiers cannot combine directly with predicates (as in: *Dan is twice heavy Mary*). A comparative or equative morpheme must mediate the combination. Thus, I take ratio modifiers like \(n \text{ times}\) to be functions from a comparative-morpheme interpretation \(C\) to a degree relation (\(C\) is fed with a numerical predicate, \(\lambda k.\lambda k = 0\), so it turns into a relation)\(^{12}\).

The relation is such that its second argument is multiplied by \(n\).

\[
\text{(43) Ratio modifiers:}
\]

a. Let us add to the language the following expressions: *twice, 2 times, 3 times, ..., n times*

b. Twice \(\Leftrightarrow \lambda C.\lambda k_2.\lambda k_1. [C(\lambda k. k = 0)](2 \times k_2)(k_1)\)

c. Four times: \(\Leftrightarrow \lambda C.\lambda k_2.\lambda k_1. [C(\lambda k. k = 0)](4 \times k_2)(k_1)\)

When a ratio modifier like *four times* combines with a difference modifier like *more*, the result is a degree relation that gives any two degrees truth value 1 iff the former is 4 times the latter.

d. Four times more than \(\Leftrightarrow \)

\[
[\lambda C.\lambda k_2.\lambda k_1. [C(\lambda k. k = 0)](4 \times k_2)(k_1)](\lambda M.\lambda k_2.\lambda k_1. M(k_1 - k_2)) = \lambda k_2.\lambda k_1. [\lambda M.\lambda k_2.\lambda k_1. M(k_1 - k_2)](\lambda k. k = 0)](4 \times k_2)(k_1) =
\]

\(^{11}\) In English, ratio morphemes combine more easily with equatives (as in *twice as heavy*) than with comparative morphemes (as in *twice heavier*). The situation is different in languages like Hebrew (and, in practice, even in English both are being used).

\(^{12}\) By feeding \(C\) with a numerical predicate, \(\lambda k.\lambda k = 0\), we get an 'exactly' relation. By feeding \(C\) with a numerical predicate, \(\lambda k.\lambda k \geq 0\), we get an 'at least' relation. If \(\lambda k.\lambda k = 0\) is the default, this may explain the fact that ratio morphemes combine more easily with equatives (as in *twice as heavy*) than with comparative morphemes (as in *twice heavier*).
\[ \lambda k_2, \lambda k_1. (\lambda k_2, \lambda k_1. [\lambda k. k = 0 (k_1 - k_2)]) (4 \times k_2)(k_1) = \\
\lambda k_2, \lambda k_1. [\lambda k_2, \lambda k_1.(k_1 - k_2) = 0](4 \times k_2)(k_1) = \\
\lambda k_2, \lambda k_1. (k_1 - (4 \times k_2)) = 0 = \\
\lambda k_2, \lambda k_1. k_1 = 4 \times k_2 \]

Ratio modifiers can shift to entity type:

e. \lambda P. Four times P \iff \\
[\lambda P. \lambda C. \lambda x_1, \lambda x_2. [C(\lambda k. k = 0)](4 \times \text{deg}(x_2, P))(\text{deg}(x_1, P))] 

f. Four times taller than \iff \\
[\lambda C. \lambda x_1, \lambda x_2. [C(\lambda k. k = 0)](4 \times \text{deg}(x_2, \text{tall}))(\text{deg}(x_1, \text{tall})) = \\
(\lambda M. \lambda k_2, \lambda k_1. M(k_1 - k_2)) = \\
\lambda x_2, \lambda x_1. [(\lambda M. \lambda k_2, \lambda k_1. M(k_1 - k_2)(\lambda k. k = 0)](4 \times \text{deg}(x_2, \text{tall}))(\text{deg}(x_1, \text{tall})) = \\
\lambda x_2, \lambda x_1. [\lambda k. k = 0](\text{deg}(x_1, \text{tall}) - 4 \times \text{deg}(x_2, \text{tall})) = \\
\lambda x_2, \lambda x_1. \left( \text{deg}(x_1, \text{tall}) = 4 \times \text{deg}(x_2, \text{tall}) \right) ](\text{Sam})(\text{Dan}) = \\
\text{deg}(\text{Dan}, \text{tall}) = 4 \times \text{deg}(\text{Sam}, \text{tall}) 

g. Dan is four times taller than Sam \iff \\
[\lambda x_2, \lambda x_1. [(\text{deg}(x_1, \text{tall}) = 4 \times \text{deg}(x_2, \text{tall}))]](\text{Sam})(\text{Dan}) = \\
\text{deg}(\text{Dan}, \text{tall}) = 4 \times \text{deg}(\text{Sam}, \text{tall}) 

The statistics handbooks explain that in interval-scale properties (where the zero is transformed), ratios between numbers on the scale are not meaningful, so operations such as multiplication and division cannot be carried out. In accordance with this explanation, we have seen in 2.1 that ratio modifiers (such as two times more P) are freely licensed with ratio adjectives (like tall), but they are usually not freely licensed with negative adjectives, namely interval adjectives like short (adjectives whose degrees are transformed by an unknown value). The contrast in felicity between positive and negative antonyms is demonstrated in the following examples.

(44) 

a. Dan is twice as tall as Sam 
b. ? Dan is twice as short as Sam 
c. The table is twice as long as the sofa 
d. ? The table is twice as short as the sofa 
e. The table is twice as big as the chair 
f. ? The table is twice as small as the chair 
g. Dan is twice as fast as Sam 
h. ? Dan is twice as slow as Sam

When Google searching for entries of the form twice as ADJ as and half as ADJ as with reversed (negative) predicates, entries can be found, but the number of entries is almost always significantly smaller, compared to the non-reversed (positive) predicates. The number of uses of twice as (and half as) with adjectives and their antonyms (and the ratio between the two numbers) are found in table 1 in 2.1. In short, in 75% of the cases (12 of the 16 adjective pairs I was searching for), the use of twice and half is considerably more frequent in positive adjectives than in their negative antonyms (long / short; tall / short; fast / slow; big / small; true / false; safe /
unsafe; healthy / sick; good / bad; happy / unhappy; likely / unlikely; similar / different; similar / dissimilar.

We can now explain these felicity contrasts and distributional facts. Consider for instance two entities such that \( d_2 \) has a double length compared to \( d_1 \). In each \( t \), the final degree function of a ratio predicate like \( \text{tall} \) maps \( d_1 \) to some number \( n \) (say, 2) and \( d_2 \) to two times that number, \( 2n \) (say, 4), (this follows from our assumption that the degree function of \( \text{tall} \) is additive; it always adequately represents quantities of length). \( \text{short} \) reverses the degrees. In the given situation, \( \text{short} \) maps \( d_1 \) to \( n' = \text{Tran}^\star(\text{short},t,g) – n \) (e.g., \( \text{Tran}^\star(\text{short},t,g) – 2 \)), and \( d_2 \) to \( m' = \text{Tran}^\star(\text{short},t,g) – 2n \) (e.g., \( \text{Tran}^\star(\text{short},t,g) – 4 \)). But \( m' \) is not two times \( n' \), unless \( \text{Tran}^\star(\text{short},t,g) \) is set to zero. Thus, \( d_2 \) has a double length compared to \( d_1 \) iff \( d_2 \) is twice as tall, but not iff \( d_1 \) is twice as short. As a consequence, twice as short is less acceptable than twice as tall.

I propose that ratio modifiers are freely licensed in an actual context \( c \) iff the transformation parameter is set to zero in \( c \), i.e. only with ratio predicates. With interval predicates, the existence of total contexts above \( c \) in which the transformation parameter is not set to zero reduces the felicity of ratio modifiers (it reduces our willingness to accommodate the presupposition that the transformation value is zero). Thus, the felicity of twice as short as is degraded compared to twice as tall.

Thus, ratio statements with ratio predicates can have definite truth values in partial contexts:

\[
(45) \quad \forall P \in \text{CONCEPT}^1:
\begin{align*}
&\text{a. With ratio predicates where } \text{Tran}^\star(P,c,g) = 0:\n&\quad \forall d_1, d_n \in D, \exists n \in \mathbb{R}:
&\quad \forall t \in T_c: \deg^\star(d_n, F, t, g) = n \times \deg^\star(d_1, F, t, g) \\
&\text{b. With interval predicates where } \text{Tran}^\star(P,c,g) \text{ is unknown, this is not the case:} \\
&\quad \forall d_1, d_n \in D, \text{ s.t. } d_1 \text{ and } d_n \text{ are not equally tall:} \\
&\quad \forall t_1, t_2 \in T_c: \deg^\star(d_n, P, t_1, g) / \deg^\star(d_1, P, t_1, g) \neq \deg^\star(d_n, P, t_2, g) / \deg^\star(d_1, P, t_2, g)
\end{align*}
\]

In any \( t \) and \( g \), \( f^\star(\text{tall}, t, g) \) is additive, so the truth conditions adequately predict the intuition that \( \text{Dan is 4 times taller than Sam} \) is judged true whenever Dan's height is four times Sam's.

But ratio statements with interval predicates cannot adequately represent such facts:

\[
(46) \quad [[\text{Dan is 4 times taller than Sam}]]_{t,g} = 1
\]
\[
\forall t \in T, t \geq c: [[\deg(\text{Dan,tall}) = 4 \times \text{deg(Sam,tall)} ]]_{t,g} = 1 \quad \text{iff} \\
\forall t \in T, t \geq c: \deg^\star([[\text{Dan}]]_{t,g}, \text{tall}, t, g) = 4 \times \deg^\star([[\text{Sam}]]_{t,g}, \text{tall}, t, g) \quad \text{iff} \\
\forall t \in T, t \geq c: f^\star([[\text{Dan}]]_{t,g}, \text{tall}, t, g) = 4 \times f^\star([[\text{Sam}]]_{t,g}, \text{tall}, t, g)
\]

In any \( t \) and \( g \), \( f^\star(\text{tall}, t, g) \) is additive, so the truth conditions adequately predict the intuition that \( \text{Dan is 4 times longer than Sam} \) is judged true whenever Dan's height is four times Sam's.
But in any \(t\) and \(g\), \(f^t(\text{tall},t,g)\) is additive (it adequately represents the ratios between quantities of height), so for some number \(n\), in any \(t\), \(f^t(\text{Sam},\text{tall},t,g) = n \times f^t(\text{Dan},\text{tall},t,g)\). Thus, for some \(n\):

\[
\forall t \in T, t \geq c: 4n \times f^t([\text{Dan}]+_g\text{tall},t,g) = 3 \times \text{Tran}^t(\text{short},t,g) + f^t([\text{Dan}]+_g\text{tall},t,g)
\]

These truth conditions are no good. First, they never give us truth value 1, because \(\text{Tran}^t(\text{short},t,g)\) varies in different total contexts, and there is no basis for thinking that there are no contexts in which it exceeds \((4n - 1)\) times Dan's degree of height. Second, intuitively, the truth conditions should not require that Dan's degree be larger than any number. They should only require that the ratio between it and Sam's degree be 4. We see that the ratio modifiers cannot appropriately apply to predicates whose degree function assigns transformed values. The larger the proportion of total contexts in which the values are transformed, the worse the use of ratio modifiers should be (because the harder it becomes to accommodate the assumption that the values are not transformed).

### 9.5.2 Open questions and observations concerning the distribution of ratio modifiers

There are still many open questions and some possible answers to them.

First, perhaps we can explain the uses of ratio modifiers with negative predicates as uses in which they are interpreted as ratio predicates. For instance, if we interpret \textit{cold} as measuring the extent of our mental experience when we touch something cold, then \textit{cold} may well be treated as a ratio predicate (denoting a quantity function). In uses of \textit{cold} with unit names like Kelvin the interpretation is different (and then it is an interval predicate). As we usually do not know exactly how one Kelvin unit feels, we are likely to measure our experiences with heat using reference points which are only available to us (mental states), and vice versa. For the same reasons, when using conventional unit names, it is more likely that we measure heat, not heat-perception. This ambiguity in word meaning may produce the apparent exceptions to the predicted pattern. In order to confirm this hypothesis, more attention should be paid to each given example of negative predicates with ratio modifiers. I leave this interesting task for future research.

Second, we can now make more precise the semantics of adjectives like \textit{healthy wrt blood pressure} (bp), and explain their felicity in ratio statements. Intuitively, their additive degree function may be based on known numerical degrees (the ratios between entities' bp degrees and the bp degree in some unit of bp, \(r_{bp} = \sigma(\{f^t(d_{bp},bp,t,g): d_{bp} \in \{u_{bp}\}+_g\})\)). We can state that for any \(g\) and \(t\)

\[
\forall d \in D: f^t(d,\text{healthy wrt bp},t,g) = -| f^t(d,\text{bp},t,g)/r_{bp} - \text{Value}^t(\text{healthy wrt bp},t,g) |.
\]

The ratio, \(f^t(d,\text{bp},t,g)/r_{bp}\), is a known number. It does not vary between total extensions of actual contexts. (If I do not know someone's ratio that is only because I do not know precisely which individual it is). It follows that \textit{healthy wrt bp} is a ratio predicate iff \(\text{Tran}^t(\text{healthy wrt bp},c,g)\) is set to zero and \(\text{Value}^t(\text{healthy wrt bp},c,g)\) is a known number (i.e. \(f^t(d,\text{healthy wrt bp},c,g)\) is completely given). In each use of such a predicate with comparison or ratio modifiers the parameter \(\text{Value}^t(\text{healthy wrt bp},c,g)\) must be a known number (otherwise we have no basis for saying that something is more or less close to it, or is \(n\) times further away from it, than something else). When the selected value is a known number, and the transformation value is set to zero, we know the ratios between the entities' degrees in \textit{healthy wrt bp}. For example, consider
5 entities $d_1$...$d_5$ such that their bp degrees in $u_{1_{bp}}$ units are 1...5, respectively, and such that $\text{Value}^*(\text{healthy wrt bp}, c, g) = 3$. Their final degrees in $bp$, $\text{healthy wrt bp}$, and $\text{sick wrt bp}$ are:

<table>
<thead>
<tr>
<th></th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$d_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bp</strong></td>
<td>$f^*(d, bp, t, g) / h_{bp}$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>Healthy wrt bp</strong></td>
<td>$f^<em>(d, \text{healthy wrt bp}, c, g)$ – $\text{Tran}^</em>(\text{healthy wrt bp}, c, g)$</td>
<td>$-</td>
<td>1 – 3</td>
<td>$</td>
<td>$-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= - 2$</td>
<td>$= - 1$</td>
<td>$= 0$</td>
<td>$= - 1$</td>
</tr>
<tr>
<td><strong>Sick wrt bp</strong></td>
<td>$\text{Tran}^<em>(\text{healthy wrt bp}, c, g)$ – $f^</em>(d, \text{healthy wrt bp}, c, g)$</td>
<td>$</td>
<td>1 – 3</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 2$</td>
<td>$= 1$</td>
<td>$= 0$</td>
<td>$= 1$</td>
</tr>
</tbody>
</table>

We see that we know the ratios between the entities' final degrees in $\text{healthy wrt bp}$. For example, $\deg^*(d_3, \text{healthy wrt bp}, c, g) / \deg^*(d_4, \text{healthy wrt bp}, c, g) = 2/1$ and $\deg^*(d_1, \text{healthy wrt bp}, c, g) / \deg^*(d_4, \text{healthy wrt bp}, c, g) = 2/1$. So given a known selected value, this predicate should be licensed in ratio statements (if it is a ratio predicate, namely iff we are willing to accommodate the presupposition that the transformation value is set to zero). In fact, it makes sense to say that someone is *twice as healthy* (wrt bp) iff you presume to know the ideal bp level. Otherwise you cannot even tell the entities' ordering (who is closer to the ideal point).

Perhaps speakers that are not doctors assign entities $f^*(\text{healthy wrt bp}, c, g)$ degrees directly, without specifying the $f^*(bp, c, g)$ function and the ideal bp value of $\text{healthy}$. These speakers are likely wrong as they can only base their interpretation of this adjective on learning orders and typical dimensions of healthy people (cf. chapters 7-8). According to my proposal, if one learns the medical facts (that are based on medical means for measuring bp and conventional norms concerning healthy bp degrees), one needs to correct one's earlier knowledge (as in the example of early exposure to ostriches; cf. 8.3). Future research should determine whether this scenario is close enough to the facts (if not, that would form a counterexample to the learning principle and/or to the interpretation that I have assigned to distance-predicates).

Third, I have found no uses at all of the predicates *typical* and *normal* in ratio statements with *twice as P as* and *half*, even though the selected values of these adjectives can be fixed to be those of the nominal concept that forms their of argument (for instance, the selected value of the concept *bird* in *size*). The reasons for this distributional gap await future research.

Fourth, in 25% of the cases (2 of the 16 adjective pairs I have searched for), the use of *twice* and *half* is less frequent in the positive adjectives than in their negative antonyms (*intelligent* / *stupid: safe* / *dangerous*). This may suggest that *dangerous* (unlike *unsafe*) may be a ratio predicate (that maps all entities with no danger in them to zero), while *safe*, may sometimes not map entities with no safety at all in them to zero (and the same for *intelligent* and *stupid*). Perhaps we measure *danger* (or *stupidity*) by the distance from some ideal (*dangerous* and *stupid* are negative, because distance functions involve reversing as discussed in chapters 5-7), and *safe* and *intelligent* are reversed twice so they are positive, but not ratio). In fact, *unsafe*, which cannot be interpreted as positive (unlike *dangerous*, it is negative by its morphology, which in the current proposal means that it is necessarily reversed) is much less acceptable (and more rarely used) than *safe* in ratio statements (i.e. in much fewer cases we are willing to accommodate the presupposition that its transformation value is zero). Future research should address the task of empirically identifying the group of adjective pairs which exhibit this reversed pattern, and supporting my (or another) semantic analysis (where they will be both negative and ratio).
Fifth, in yet another 2 pairs, the pattern with *twice* and with *half* are opposite (*beautiful* / *ugly*; *right* / *wrong*), a fact which my proposal cannot not explain.

Sixth, *half* as *P as* is generally less frequently used, perhaps because fractions are harder to process, or for other additional reasons.

In sum, the transformation value seems to be but one of several factors affecting the distribution of ratio-modifiers. Still, this factor plays an important role.

### 9.5.3 Complex ratio predicates

Grammar generates infinitely many complex ratio relations (like *2 meters more than 3 times 4 meters*). In this section I show that with the proposed semantics of ratio morphemes we can indeed generate these complex relations (though they are even harder to process than complex difference relations, perhaps because their semantics involves both difference and ratio modifiers).

We can feed (type shifted) degree predicates like *2 meters* into (type-shifted) ratio-relations (like *λ k2.λ k1. k1 = (4 × k2)*), to produce degree predicates like *four times more than 2 meters* (that denote degrees that are at least 8 meters):

\[\lambda k1. \exists x, \text{meter}_{\text{tall}}(x), k1 = 2 \times \text{deg}(x,\text{long})\]

These predicates can combine with comparative morphemes in the usual way, to produce relations like *four times more than 2 meters more*, and so on and so forth.

### 9.5.4 The distribution of difference modifiers

Let us recapitulate. On my proposal, nominal concepts are linked with mean functions (their additive degree functions map entities to their mean in a set of dimensions; cf. 4.1 and 7.4). We saw in 2.1 that comparatives cannot occur with bare nouns in within-predicate comparisons. Adjectives can be multi-dimensional too, but they do occur (bare) in within-predicate comparisons. In this section I propose a completion for my explanation of these facts.
Consider the predicate fat (or heavy, warm, etc). In some contexts, fat is measured by weight. In these contexts, it can be measured by conventional unit names like kilograms. In other contexts, fat is measured by some combination of weight and height. In some of these other contexts (say, when your doctor tells you that you need to diet), there are conventions as to how these two dimensions combine (the doctor uses some conventional formula, say – she divides your weight by your height). In such contexts, fat is one dimensional (it is ordered by the unique dimension weight divided by height). In other contexts (for instance, when we try to decide which one in our family is fatter), there are no such conventions, and many more dimensions may count (like one's waist line, general look, or physical conditions). In such contexts, we know that weight and (lack of) height raise one's degree in fat, but there are many ways they might be weighed. In such uses, fat (or fat person) is multi-dimensional.

But, we have seen in 7.5, that multi-dimensional adjectives can always turn one-dimensional (by accommodation of a wrt phrase), whereas nouns are inherently multi-dimensional (they cannot combine with wrt phrases that reduce dimension sets to singletons). When adjectives occur in the comparative, we compare degrees along one dimension at a time. Recall that my general-judgment questionnaire shows that when two people are such that one weighs more but he is also taller, speakers are reluctant to use the comparative. They say that neither one is fatter, neither is less fat, and they are not equally fat, i.e. they actually say that it is inappropriate to use the comparative in such circumstances. Only when the two entities are in equal relations on all the potential dimensions (the same person weighs more, is shorter, looks worse, etc.) do people say that he is fatter, i.e. they actually say that for every dimensions F of fat, he is fatter wrt to F. We see that the comparative (in within-predicate comparisons) is licensed iff the predicate is one-dimensional.

Why can't we represent nominal concepts like bird as one-dimensional, ordered along dimensions equivalent to bird? We have seen that comparatives have the difference operation as part of their meaning. Books in statistics tell us that difference operations cannot apply to degrees in ordinal scales, where the differences between degrees are not meaningful (not all the differences between adjacent degrees are identical). I do assume that in any total context nouns are linked with (at least) interval scales (say, the real interval [0,1]). I propose that the reason that difference modifiers cannot occur with nominal concepts is that in the contexts representing our partial knowledge, it is not only the case that we do not know the precise function that is linked with them, but also, the different possible functions that are linked with a given nominal concept in different total extensions do not yield the same ratios between the differences between the degrees of entity pairs. There is no number r such that we can tell that the difference between the degrees of any two entities, equals r times a third degree (a 'unit'), in any total context.

\[ (49) \] In nominal predicates P, the differences between values that the final degree function assigns to entities vary across total contexts:
\[
\forall d_{u1}, d_{u2} \in [\neg (=P)], \forall d_{t1}, d_{t2} \in [\neg (=P)],
\neg \exists e \in E, \forall t \in T, \forall g \in G: \quad \deg^+(d_{t1}, P, t, g) - \deg^+(d_{t2}, P, t, g) = r \times (\deg^+(d_{u1}, P, t, g) - \deg^+(d_{u2}, P, t, g))
\]

In other words, for any two entities, \(d_{u1}, d_{u2}\) whose degrees are known not to be equal, and any other two entities \(d_{t1}, d_{t2}\) whose degrees are known not to be equal, there are two total extensions \(t_{1}, t_{2}\) of \(c\) s.t.:
\[
(\deg^+(d_{t1}, P, t_{1}, g) - \deg^+(d_{t2}, P, t_{2}, g)) / (\deg^+(d_{u1}, P, t_{1}, g) - \deg^+(d_{u2}, P, t_{1}, g)) \neq (\deg^+(d_{t1}, P, t_{2}, g) - \deg^+(d_{t2}, P, t_{2}, g)) / (\deg^+(d_{u1}, P, t_{2}, g) - \deg^+(d_{u2}, P, t_{2}, g))
\]
I propose that having such a number \( r \) is a pre-condition for the use of comparatives. Given the analysis of comparison statements as stating that the difference between two entities equals \( n \) times the degree of a third entity (a unit-object), that is the minimal requirement for the licensing of comparatives, and the nominal functions do not meet it. In that respect, the nominal functions are similar to ordinal scale properties.

Let me illustrate this with the example \#Tweety is more a bird than Tan. We may feel that this statement is true, but still feel that it is not felicitous. Why? I have proposed that in comparison statements with no numerical degree predicate, a covert predicate of the form "\( u^2_{\text{bird}}(n) \)" enters the interpretation. Thus we get the statement: For some \( n \), Tweety is \( u^2_{\text{bird}}(n) \) more a bird than Tan. The semantics requires that in any \( t \) above \( c \), Tweety's degree will be bigger than Tan's by some number \( n \).

\[
\begin{align*}
(50) & \quad \text{a. Semantics:} \\
& \quad [\exists n, u^1_{\text{bird}}, \text{Tweety is } u^2_{\text{bird}}(n) \text{ more a bird than } \text{Tan}]^+_c, g = 1 \text{ iff} \\
& \quad \forall t \in T, t \geq c: \exists r \in R \ r > 0, \exists d \in D: \\
& \quad \deg^+([\text{Tweety}]^+_{t,g}, \text{bird}, t, g) = \\
& \quad \deg^+([\text{Tan}]^+_{t,g}, \text{bird}, t, g) + r \times (\deg^+(d, \text{bird}, t, g)).
\end{align*}
\]

So the truth conditions boil down to the requirement that the ordering relation between the (degrees of the) arguments, Tweety and Tan, will be in the "bigger than" relation in any \( t \) above \( c \). This requirement is met by many argument pairs. For example, for any robin and ostrich we can tell that the robin's degree exceeds the ostrich's degree in \textit{bird}. Yet, the statement \textit{a robin is more a bird than an ostrich} is odd. I propose that this is due to the presupposition that there is some number \( n \) such that in any \( t \) above \( c \), Tweety's degree is bigger than Tan's by \( n \).

\[
\begin{align*}
& \quad \text{b. Presupposition:} \\
& \quad \exists r \in R \ r > 0, \exists d \in D: \ \forall t \in T, t \geq c: \\
& \quad \deg^+([\text{Tweety}]^+_{t,g}, \text{bird}, t, g) = \\
& \quad \deg^+([\text{Tan}]^+_{t,g}, \text{bird}, t, g) = r \times \deg^+(d, \text{bird}, t, g(n/r)).
\end{align*}
\]

As demonstrated below, this condition is not met in nominal concepts. Even when the transformation values cancel one another (as is the case in comparisons of degree differences), if we look at differences between degrees that nominal concepts assign to entities, the ratios between these differences are not known to us (are not equal in all the total extensions of actual contexts).

These ratios are known for all the adjectives, as we have seen in previous sections. For example, you can tell in \( c \), that the difference between two given entities is 2 meters. For any two entities (or referents of arguments whose degrees in meters you know) you can tell that there is a number \( n \) such that for any total context \( t \), the difference between their degrees in \textit{tall} is \( n \) meters, i.e., the ratio between (i) the difference between their degrees and (ii) the degree of the meter unit-objects is \( n \). The same can be said about \textit{short}. That is because when differences between degrees are computed, the two transformation values that are part of the degrees of \textit{short} cancel one another, and, for any \( g \) and \( t \) above \( c \), we are left with a value assigned by \( f^+(\text{tall}, t, g) \). The ratio between this value and the meter unit-objects' value is known (it is equal in all total context; cf. 9.2). The meter-rulers determine what this ratio is.
In contrast, for nominal concepts P, these ratios are not known, because not only does the transformation value vary across total contexts, but also, ratios between values that the additive degree function assigns may vary across total contexts. We can describe this situation by saying that in a partial context c, we do not know quantity of what exactly a noun like bird measures. A noun like bird requires that its instances will have some quantity (so to speak) of height, some quantity of flying, some quantity of singing, etc., and the weighted sum of these quantities should reach a certain threshold. But even when all the nominal dimensions are known (we know the things the quantities of which we measure), and they are all additive and have no transformation values, their weights may vary across total contexts. There are no definite conventions as to what these weights might be. This fact boils down to saying that we do not know exactly what it is we are measuring (because we do not know how important each ingredient of bird-hood is).

The complete set of dimensions with their weights varies across contexts (many dimensions and all the weights are context dependent). You can always add new dimensions and assign them some weight, even if only a small weight. The reasons for this are discussed in chapter 8. Every time we learn that two entities stand in a relation like "a better example of a bird", we are adding dimensions to the bird dimension-set, so that the mean degrees of these two entities will stand in the relation "bigger than". These dimensions (and their potential weights) are such that they preserve our previous knowledge about ordering between entities; The new dimensions can do the job, if their weight remains relatively small and all the entities we know satisfy them well (because then the degrees in bird of those entities are almost not affected by these additional dimensions).

Crucially, if the dimension height weighs 0.1 in t₁ and 0.2 in t₂, then all other things being equal, the ratio between the difference in bird-hood (in entities with different heights) and some reference point (a third degree or a degree-difference) will be different in t₁ and t₂. For the purpose of demonstrating this argument, we can assume that for a nominal concept P, roughly, $f^*(d,P,t,g) = W_{F1} \times \deg^*(d,F_1,t,g) + \ldots W_{F_n} \times \deg^*(d,F_n,t,g)$. In actual fact (or given our proposal in chapter 7), the mean functions are such that even more operations apply on this sum and further shift it in potentially different directions in different contexts. Even if we ignore these shifts, we can still show that that the comparative presupposition is not met.

Consider a case in which a noun P has only two dimensions F₁ and F₂, whose weights are not equal in all total contexts, say, W₁ is 0.1 in context t₁ and it is 0.2 in t₂. Let us choose 4 entities whose values in these dimensions are exactly the same in t₁ and t₂, say in both t₁ and t₂, in F₁ d₁’s value is 0.1, d₂’s value is 0.2, d₃’s value is 1 and d₄’s value is 0.2. All the rest is the same in the two contexts. The ratio between (i) the difference between d₁’s and d₂’s degrees and (ii) the difference between d₃’s and d₄’s degrees is different in the two contexts.

\[
\begin{align*}
\deg^*(d_1,P,t_1,g) - \deg^*(d_2,P,t_1,g) &= W_{F1}\deg^*(d_1,F_1,t_1,g) + W_{F2}\deg^*(d_1,F_2,t_1,g) - W_{F1}\deg^*(d_2,F_1,t_1,g) - W_{F2}\deg^*(d_2,F_2,t_1,g) \\
&= 0.1 \times 0.1 + W_{F2}\deg^*(d_1,F_2,t_1,g) - 0.1 \times 0.2 - W_{F2}\deg^*(d_1,F_2,t_1,g)
\end{align*}
\]

Let us assume that the value "W₂ deg⁺(d₁,F₂,t₂,g) - W₂ deg⁺(d₂,F₂,t₂,g)" equals 0.02 in both t₁ and t₂. Thus:

\[
\begin{align*}
\deg^*(d_1,P,t_1,g) - \deg^*(d_2,P,t_1,g) &= 0.1 \times 0.1 - 0.1 \times 0.2 + 0.02 = 0.01 \\
\deg^*(d_1,P,t_2,g) - \deg^*(d_2,P,t_2,g) &= 0.2 \times 0.1 - 0.2 \times 0.2 + 0.02 = -0.04
\end{align*}
\]

Let the value "W₂ deg⁺(d₁1,F₂,t₂,g) - W₂ deg⁺(d₂,F₂,t₂,g)" also be 0.02 in t₁ and t₂. Thus:

\[
\begin{align*}
\deg^*(d_11,P,t_1,g) - \deg^*(d_2,P,t_1,g) &= 0.1 \times 1 - 0.1 \times 0.2 + 0.02 = 0.1 \\
\deg^*(d_11,P,t_2,g) - \deg^*(d_2,P,t_2,g) &= 0.2 \times 0.1 - 0.2 \times 0.2 + 0.02 = 0.18
\end{align*}
\]

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We see that:
\[
\frac{\deg^+(d_1,P,t_1,g) - \deg^+(d_2,P,t_1,g)}{\deg^+(d_{u1},P,t_1,g) - \deg^+(d_{u2},P,t_1,g)} = 0.01 / 0.1 = 1/10 \neq
\]
\[
\frac{\deg^+(d_1,P,t_2,g) - \deg^+(d_2,P,t_2,g)}{\deg^+(d_{u1},P,t_2,g) - \deg^+(d_{u2},P,t_2,g)} = -0.04 / 0.18 = -2/9.
\]

I have proposed that comparison statements with no numerical degree predicate like *Tweety is more a bird than Tan* presuppose that there is a number r bigger than zero such that in any extension t of the given context c, Tweety's degree in bird equals Tan's degree plus r times the degree of a third part ('a unit'). We see that this condition on use is not met in nouns, except for entities with zero P-hood (zero in every dimension) in every t. If an entity d has some F–hood, for some dimension F of P, given that the weight of any dimension varies through total contexts, the difference between d's and other entities' degrees in P varies in different total contexts.  

Even for those entities for which we can tell that one has a higher degree in bird than the other, we cannot tell what the difference between their degrees is – how much the former is *more a bird* than the latter (in terms of the degree of a third party). This is not because the final degree function of bird is transformed (or because \( f^*(bird,t,g) \) is not additive in some t and g). In any total context t, the final degree function of bird may not be transformed and \( f^*(bird,t,g) \) may be additive (for any g). But crucially, in the actual contexts c representing the partial knowledge of competent speakers, we cannot tell which kind of quantities the final degree function of bird measures. In different total contexts t, \( f^*(bird,t,g) \) represents quantities of slightly different (weighed-combinations of) things. Thus, even when we can tell that in all the possible contexts, some elements have more quantity of whatever it is that the final degree function of bird measures, we cannot tell how much more, and it is implausible that the difference in quantity will be exactly the same (n times some reference point) in all the possible total contexts. In that respect, our knowledge about quantities of birdhood in partial contexts is ordinal. And it is for that reason that comparative and equative morphemes (whose interpretation is mediated by a difference operation) do not combine with nouns. Comparison statements require that when you argue that two entities differ in their quantities of *something*, you would know what that *something* is.

In sum, nouns cannot occur in the comparative because they are multi-dimensional. I.e., they are associated with several dimensions and there is no convention as to how to combine (weigh) these dimensions. What about predicates like *even, odd, prime* (when applied to numbers) or *extinct*, which are intuitively thought of as non-gradable? In my system they can be seen as either multi-dimensional, or simply one-dimensional predicates that are linked with binary functions, namely, functions that map all the entities to either 1 or 0. In such functions, the differences between two entities' degrees are also always either zero (if both are positive denotation members or both are negative denotation members) or one (if one is a positive denotation member and the other a negative denotation member). In such a case, the use of comparatives is simply superfluous if the participants in the conversation know the entities' membership status, and if they do not know their status, they have no basis for asserting that there is a difference (unless they happen to know only that the two cannot be members simultaneously – one is a member iff the other is not). In any other situation, a speaker who wants to apprise others of the facts should assert the more informative statement about the entities classification. At any rate, when speakers are asked to rate items by their degree in such predicates, they usually seem to treat them like normal multi-dimensional nominal predicates. Note that the denotation of *prime numbers* is learnt

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13 Speakers are reluctant to use comparison statements for entities with zero P-hood (as in *this story is taller than that prime number* or *the story is as tall as my morning running*).
gradually. For some very large numbers, their status under \textit{prime} is yet unknown, so the learning order can provide the means for grading even in these cases (although I am not sure how this should be represented, given that the status of numbers under \textit{prime} is a matter of logical necessity).
10 GENERAL CONCLUSIONS

An improved theory of the phenomenon of gradability requires that positions in both semantics and psychology be adapted. Let me try to summarize the more general points made in this dissertation – i.e., points concerning the most central characteristics of the data, and the general structure and most central aspects of the theory.

10.1 Background

First, psychological theories link nouns with degree functions that map entities to their mean in a set of ordering dimensions. In virtue of this feature, these theories derive a large number of correct predictions about typicality and categorization in simple and complex predicates.

Second, I have shown that some pieces of linguistic data that were overlooked in the past convincingly show that nouns are compatible with certain degree modifiers. These data provide further motivation for the view that the psychological mechanisms of dimension sets and mean functions form part of the semantic interpretation of nouns and noun phrases.

Third, psychological findings about complex predicates are viewed in the psychological literature as refuting the basis of formal semantic theories. Contrary to this view, I have argued here that these findings actually stem from semantic rules and pragmatic constraints. Thus, the gap between the semantic and psychological perspectives can and should be bridged.

Fourth, interestingly, both semantic and psychological considerations show that gradability is intimately related not to mere vagueness, but to the order in which vagueness can be removed. Thus, the theory of gradability must represent the gradual growth of information and its context dependency. I have shown, however, that existing semantic and psychological representations of partial knowledge about ordering relations, degrees and dimension sets were not quite adequate in that respect.

Fifth, semantic and psychological theories fail to clarify the notion 'dimension'.

10.2 A new theory

First, I have proposed a new model for semantic interpretation, which incorporates some crucial elements introduced in the psychological literature. On my theory, the model-theoretic semantic interpretation of all predicates includes, in addition to a denotation, also a dimension set, a degree function, and more. A mean function is used too, in calculating entities’ degrees in nouns.

Second, on my theory, predicates are interpreted relative to contexts (information states) that are embedded within a learning model (a knowledge structure that represents information growth). This analysis captures the context dependency of gradability judgments, and it correctly represents dimension sets, and their gradual acquisition.

Third, I have proposed that the basic criterion for grading entities in a predicate P is the order in which they are learned to be members of the denotation of P. The entity ordering denoted by a derived comparative of a predicate P in a context c tracks the order in which the P-hood (or non-P-hood) of entities is inferred through the contexts leading to c (or extending c). Thus, gradability can characterize relatively non-vague predicates, including nouns (as well as adjectives whose standards are already known in the ground context, like full and empty).
Fourth, I have proposed that dimensions are normal predicates, whose degree functions constrain the degree function of the predicate they are dimensions of. The type of constraint depends on the type of degree function.

Last, but not least, we no longer attempt to explain the main semantic differences between nouns and adjectives (mainly regarding their licensing in within and between comparison statements, and their compatibility with wrt phrases, except phrases and quantification over dimensions) by the mere existence vs. non-existence of a degree function in their interpretation. On my theory, all predicates have degree functions, and the differences between nouns and adjectives are explained based on differences in the type of degree function found in their interpretation. The degree functions associated with nouns are based on a mean operation, whereas the degree functions associated with adjectives are based on a Boolean operation.
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APPENDIX TO CHAPTER 2:

I. THE ORIGINAL GENERAL JUDGMENT QUESTIONNAIRE

I. שאלון

בכל שאלה עליך להקיף בעיגול את התשובה המתאימה:

1. נניח שמשה שוקל 100 ג ודני 90 ג. הם דומים בשאר הדברים. מה יותר שמן: משה או דני?
   - או/ לא

2. נניח שמשה שוקל 100 ג והם בגובה 1.95, ודני שוקל 70 ג והם בגובה 1.95. הם דומים בשאר הדברים. מה יותר דן: משה או דני?
   - או/ לא

3. נניח שאהרון שוקל 100 ג ודני 70 ג. הם דומים בשאר הדברים. מה יותר קרה: משה או דני?
   - או/ לא

תרות:
דני/ אהרון

א. האם דני או אהרון יותר ידועים? לא/ כן
ב. האם דני או אהרון יותר ידועים? לא/ כן
ג. האם דני או אהרון יותר ידועים? לא/ כן
ד. האם דני או אהרון יותר ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני أو אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם ד니 או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן

א. האם דני או אהרון ידועים? לא/ כן
ב. האם דני או אהרון ידועים? לא/ כן
ג. האם דני או אהרון ידועים? לא/ כן
ד. האם דני או אהרון ידועים? לא/ כן
6.

ד. מ会给 צויריו שאם מעופפים. האשים גם את ג'וזף.

א. האם的应用 צויריו של ג'וזף? 

ב. האם的应用 צויריו של ג'וזף? 

ג. האם的应用 צויריו של ג'וזף? 

ד. הערות:

7.

 LGPL: הערות. האשים אף את ג'וזף.

א. האם的应用 LGPL של ג'וזף? 

ב. האם的应用 LGPL של ג'וזף? 

ג. האם的应用 LGPL של ג'וזף? 

ד. הערות:

8.

 ffmpeg: להזינו ל SSD. בינלאומית, בעלת יד桿 חוכלוד, ספואס,שיער הלבן (ששכובותיה היא Spellsparer או סמלי בובית ובפגים) ומלבש,纺织

א. האם אתה מסכים 

ב. שיער הלבן האם היותך בעל התכונה 

ג. שיער הלבן האם היותך בעל התכונה 

ד. שיער לבן או משקפיים האם התכונה?  טיפוסית לסבתות 

ה. שיער לבן או מכונית אדומה האם התכונה?  טיפוסית לסבתות 

ו. שיער לא לבן האם היותך בעל התכונה 

ז. שיער לא לבן האם היותך בעל התכונה 

ח. שיער לא לבן או לא חובש משקפיים או לא מתקשה בהליכה האם התכונה?  לא סבתא טיפוסית ל 

ט. שיער לא לבן או לא חובש משקפיים או לא מתקשה בהליכה שיער לא לבן או לא חובש משקפיים או לא מתקשה בהליכה 

יא. שיער לא לבן או לא חובש משקפיים או לא מתקשה בהליכה שיער לא לבן או לא חובש משקפיים או לא מתקשה בהליכה 

יב. שיער לא לבן או לא חובש משקפיים או לא מתקשה בהליכה שיער לא לבן או לא חובש משקפיים או לא מתקשה בהליכה 

וט. הערות:

9.

נניח שמשה ודני שוקלים שניהם 100 ק"ג. הם דומים בשאר הדברים. אולי הוא אפילו אינו קרח, ודני פחות קרח, משה הוא גם קרח, ג" ק

ב. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ג. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ד. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ה. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ו. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ז. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ח. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ט. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

יא. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

יב. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

טט. הערות:

10. נניח ששמתי ההכנה שמקוון י"ג א"א וירבע טיפוסי קלח לא פמקה? 

ב. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ג. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ד. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ה. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ו. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ז. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ח. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

ט. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

יא. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

יב. האשה העת ההכנה שישכ הלבר או את חיות הקטנים? 

טט. הערות:
א. האם המשנה את היתר שנעבון והובך הקדחה? 
ב. האם המשנה את היתר דגיל? 
ג. האם הקדחתה שכותרת אך ורק מעבר עליה? 
ד. האם הקדחתה שכותרת או מעבר עליה? 
ה. האם הקדחתה שכותרת אשת יותר_than הקדחתה? 
ו. האם הקדחתה שכותרת השמעה יותר_than הקדחתה? 
ז. האם הקדחתה שכותרת מgetDate יותר_than הקדחתה? 
ח. האם הקדחתה שכותרת שموادית יותר_than הקדחתה? 
ט. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
י. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יא. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יב. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יכ. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יד. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טו. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
ס. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
סג. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
שם. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טח. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טט. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טוו. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
ית. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יא. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יב. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יג. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יד. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טו. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
ס. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
סג. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
שם. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טח. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טט. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טוו. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
ית. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יא. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יב. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יג. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יד. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טו. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
ס. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
סג. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
שם. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טח. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טט. האם הקדחתהrig somames. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טוו. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
ית. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יא. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יב. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יג. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יד. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טו. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
ס. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
סג. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
שם. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טח. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טט. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טוו. האם הקדחתה שכותרת מדni_more_than הקדחתה? 
טז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
ית. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יא. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יב. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יג. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יד. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טו. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
יז. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
ס. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
סג. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
שם. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טח. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טט. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טוו. האם הקדחתה שכותרת מדני יותר_than הקדחתה? 
טז. האם הקדחתהораווה: 

10. 당ו שיפורים לtıב ליזי את לבسلوك בדיל, מונחות, בולו, שערן, מְשִׁקִּים, בשתי חתילות בצק בביצול ובת묽ת, לוהות בירד. 

11. הבאתו שיפורים לתייב ליזי ואת לבسلوك בדיל, מונחות, בולו, שערן, מְשִׁקִּים, בשתי חתילות בצק בביצול ובתらくת, לוהות בירד. 

12. יא. בתרבויות של טיפוסי ליצורים שהם:  יולי, מְשִׁקִּים, בעל ארבע רגליים, בעל נוצות, בעל או חסר כנפיים, בונה קינים או מצייץ, במציוץ או מילול.
ד. הרביעי שאלון לניסוי מדעי בו דרושים נבדקים דרגות קשישים ונכים את הקבוצות לפי סדר התאמה dafür שאלון

א. מתאימה ביותר

ב. מתאימה

ג. מיידית

ז. מיידית

ד. לא

ה. לא

ו. לא

ט. לא

י. לא

יא. לא

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APPENDIX TO CHAPTER 4:

I. THE MATHEMATICAL NOTIONS  MEAN, DISTANCE, AND SIMILARITY

There are many types of weighted means. The three classical ones are the weighted arithmetic mean, the weighted geometric mean, and the weighted harmonic mean. A generalized mean (also known as power mean or Hölder mean) is an abstraction of the different types of means. For a non-zero real number \( r \), we can define the generalized mean for exponent \( r \) by the formula in (1a). The notion is defined only for cases where the weights sum up to 1. The free variable \( r \) represents a parameter which allows different spatial metrics to be used. For example, the arithmetic mean is given by \( r=1 \) (cf. (1b)), the geometric mean is given by \( r \approx 0 \) (cf. (1c)), and the harmonic mean is given by \( r=-1 \) (cf. (1d)).

1) For any \( n \) tuples \( X = <x_1,\ldots,x_n> \) and \( W = <w_1,\ldots,w_n> \) in \( \mathbb{R}^n \) such that \( \sum_{i=1}^{n} w_i=1 \) and \( \forall i \in [1,n]: w_i>0 \):
   - The generalized mean of \( X \) relative to \( W \):
     \[ M(X,W,r) = \left( \sum_{1 \leq i \leq n} w_i x_i^r \right)^{1/r} \]
   - The arithmetic mean of \( X \) relative to \( W \):
     \[ M(X,W,1) = w_1 x_1 + \ldots + w_n x_n \]
   - The geometric mean of \( X \) relative to \( W \):
     \[ M(X,W,\approx 0) = x_1^{w_1} \times \ldots \times x_n^{w_n} \]
   - The harmonic mean of \( X \) relative to \( W \):
     \[ M(X,W,-1) = 1 / (w_1/x_1 + \ldots + w_n/x_n) \]
   - Jensen inequality: \( \forall r_1, r_2 \in \mathbb{R}, r_1 \leq r_2: M(X,W,r_1) \leq M(X,W,r_2) \)

For positive values of \( r \), there is an ordering on the means, as stated in (1e).

Entities can be represented as points in a multidimensional space, whose axes stand for features. Let \( x \) and \( y \) stand for such points and \( F(y) \) stand for the set of dimensions (axes) in this space. The formula in (2) represents the distance of \( x \) from \( y \) along feature \( F \).

2) The distance of \( x \) from \( y \) in a feature \( F \):
   \[ D(x,y,F) = |\deg(x,F) — \deg(y,F)| \]

What is the average distance of \( x \) from \( y \) in the feature set \( F(y) \)? This value is represented in (3a) using the notion of a generalized mean. For \( r=1 \) (the city block metric) the overall distance of \( x \) from \( y \) in \( F(y) \) equals the sum of \( d \)'s weighted degrees in every dimension (3b). For \( r=2 \) (the quadratic metric) the overall distance of \( x \) from \( y \) in \( F(y) \) equals the root of the sum of \( d \)'s weighted degrees raised by power 2 each (3c). The sensitivity parameter \( c \) increases the differences between big and small distances. High values in \( c \) stand for attention mainly to small distances. Low values stand for attention also to fairly large distances.

3) The generalized mean distance of \( x \) from \( y \) in \( F(y) \):
   \[ D(x,y,F(y),r) = c(\sum_{F \in F(y)} (W_F D(x,y,F))^r)^{1/r} \]
   - The arithmetic mean distance of \( x \) from \( y \) in \( F(y) \):
     \[ D(x,y,F(y),1) = c(\sum_{F \in F(y)} W_F D(x,y,F)) \]
   - The quadratic mean distance of \( x \) from \( y \) in \( F(y) \):
\[ D(x,y,F(y),2) = c(\sum_{F \in F(y)} W_{F} D(x,y,F))^2)^{1/2} \]

Let \( M(x_1, \ldots, x_n)[x_i/n] \) be the mean of \( x_1, \ldots, x_n \) where \( x_i \) is the value \( n \). Linear and quadratic means are raising monotonic on any argument \( x_i \) in the sense that for any tuple \( <x_1, \ldots, x_n> \) and any two values \( n \) and \( m \) such that \( n < m \), \( M(x_1, \ldots, x_n)[x_i/n] < M(x_1, \ldots, x_n)[x_i/m] \). That is, all other things being equal, the smaller your distance from \( y \) in a feature \( F_i \), the smaller your overall distance from \( y \).

The degree of similarity of \( x \) to \( y \) in \( F(y) \), \( S(x,y,F(y),r) \), is inversely related to the distance of \( x \) from \( y \), \( D(x,P,F(y),r) \), for a given exponent \( r \). The inverse relation may be linear or exponential. The assumption that the inverse relation is linear, as in (4a), predicts that similarity is additive (that it reduces to the sum: \( W_F \deg(d,F_1) + \ldots + W_F \deg(d,F_n) \)). On the other hand, the assumption that the inverse relation is exponential, as in (4b), has the advantage of multiplicative equations (\( \deg(d,F_1)^{W_{F_1}} \times \ldots \times \deg(d,F_n)^{W_{F_n}} \)). (4b) is called the universal law of psychological generalization. Shephard 1987 has established that it provides good fit to similarity based responses in both humans and animals. In particular, it is a good indicator of people’s ratings of the typicality of instances in categories (their similarity to the category prototype; Medin and Schaffer 1978; Nosofsky 1992).

4)  
   a. Additive Similarity of \( x \) to \( y \) in \( F(y) \):  
      \[ S(x,y,F(y),1) = 1 - D(x,y,F(y),1) \]
   
   b. Multiplicative Similarity of \( x \) to \( y \) in \( F(y) \):  
      \[ S^e(x,y,F(y),1) = \exp(-cD(x,y,F(y)),1) = 1/e^{cD(x,y,F(y)),1} \]

The difference between multiplicative and additive similarity functions is the following. The former, but not the latter, predict that the most radical decrease in similarity levels will be between perfect similarity (the similarity score of entities which completely match the P’s values in the features) and the similarity score of entities with mismatch in one feature. For example, given a multiplicative function, zero similarity in a feature reduces overall similarity to zero, while given an additive function only zero similarity in all the features reduces overall similarity to zero. Instances which match in many dimensions, but receive 0.5 scores in but 1 dimension, have low mean similarity, 0.5, because multiplication yields 0.5 \( \times 1 \times \ldots \times 1 = 0.5 \) (Murphy 2002). Two such mismatches yield 0.25, three yield 0.125, etc. The use of multiplicative similarity functions characterizes especially exemplar theories, but contemporary prototype theories use them too (Hampton 1995; Smith and Minda 2002).

A similarity-based, standard-based, categorization rule, tells us that an entity is classified under a concept \( P \) iff it's similarity to \( P \) reaches a certain threshold degree. In other words, an entity is classified under a concept \( P \) iff it’s mean distance from \( P \)’s prototype is smaller than some number \( n \). Thus, equations like the ones in (5) can stand for categorization criteria (criteria for membership in categories). Geometrically, for any number \( n \), and a two dimensional space (\( |F(y)|=2 \)), the pairs of numbers (points) that solve an equation of the form (5a), which is based on a linear mean-distance like the one in (3b), identifies a rhombus (diamond), as demonstrated in Figure 1 (namely, a parallelogram with all sides of equal length). An equation of the form of (5b), which is based on a quadratic mean-distance like the one in (3c), identifies a sphere, as demonstrated in Figure 1.
5) a. Diamond equations: \[ \Sigma_{F \in F(y)} W_F D(x,y,F) \leq n \]
b. Sphere equations: \[ (\Sigma_{F \in F(y)} W_F D(x,y,F))^2 \leq n \]

Figure 1: \[ \Sigma_{F \in F(y)} W_F D(x,y,F) \leq n \]
Figure 2: \[ (\Sigma_{F \in F(y)} W_F D(x,y,F))^2 \leq n \]

If a category P is such that F(y) is its feature set, y is its central value (prototype), and n is its standard for membership, then the category members are represented by the points that solve the equation, that is, by the points inside the diamond or sphere; The prototype, y, is the point in the middle of the form.

Let the horizontal axis stand for dimension F_1 and the vertical axis stand for dimension F_2. Functions like (5a) represent 4 linear conditions (the four lines of the rhombus in Figure 1 as stated in (6)).

6) a. The left downward line stands for points x.s.t.
\[ \forall F \in \{F_1,F_2\}, \deg(x,F) \leq \deg(y,F) \]
\[ W_{F_1}(\deg(y,F_1) - \deg(x,F_1)) + W_{F_2}(\deg(x,F_2) - \deg(y,F_2)) \leq n. \]
b. The right upward line stands for points x.s.t.
\[ \forall F \in \{F_1,F_2\}, \deg(x,F) \geq \deg(y,F) \]
\[ (W_{F_1}(\deg(x,F_1) - \deg(y,F_1)) + W_{F_2}(\deg(x,F_2) - \deg(y,F_2))) \leq n. \]
c. The left upward line stands for points x.s.t.
\[ \deg(x,F_1) \geq \deg(y,F_1) \text{ and } \deg(x,F_2) \leq \deg(y,F_2) \]
\[ (W_{F_1}(\deg(x,F_1) - \deg(y,F_1)) + W_{F_2}(\deg(y,F_2) - \deg(x,F_2))) \leq n. \]
d. The right downward line stands for points x.s.t.
\[ \deg(x,F_1) \leq \deg(y,F_1) \text{ and } \deg(x,F_2) \geq \deg(y,F_2) \]
\[ (W_{F_2}(\deg(x,F_2) - \deg(y,F_2)) + W_{F_1}(\deg(y,F_1) - \deg(x,F_1))) \leq n. \]

If each dimension F in F(y) is reconstructed such that the ideal value (y's value in F, \( \deg(y,F) \)) is the maximal point on F, and each x is mapped into \( \deg(y,F) - |\deg(x,F) - \deg(y,F)| \) (that is any point in F which is either bigger or smaller than \( \deg(y,F) \) in n points is mapped into the point in F which is smaller than \( \deg(y,F) \) in n points), then the linear categorization criterion is given by a straight line (cf. Figure 3 and formula (6a)) and the quadratic criterion is given by a curve (Figure 4).
Finally, some models (for instance, the decision-bound theory; Ashby and Maddox 2005) add to the categorization criterion parameters that represent interactions between dimensions (usually products x’s degrees (its distance from y) in two, three, four, and so on, dimensions). These complex criteria represent classification better when only two concepts are learnt, but their computational cost is higher. When four concepts are learnt the simple linear ones seem to be used (Ashby and Maddox 2005).

II. CLASSICAL CONCEPT THEORIES.

Classical implementations of the prototype theory (such as Rosch and Mervis 1975) make several specific assumptions. First, they model the features as binary (each feature F maps entities to either 1 or 0, as stated in (5a)). As a result the distance of d from P on a feature F is 0 if the degrees on F of d and P’s prototype match, and 1 otherwise. Second, they use the city block distance metric (r=1, cf. (3b)). Third, they take similarity to be inversely related to distance by a linear function (cf. (4a)). Under these assumptions, the similarity of a member d to the category prototype p, S(d,p,1), is reduced to a simple additive function on the weights of the features which d possesses (7b).

Finally, the weight of a feature F in a category P is modeled as the number of category members in which F occurs (W_F = | [[P]] ∩ [[F]] |). Given this assumption, similarity to P’s prototype
reduces to the number of features d shares with each and every category member (7c) (d's *family resemblance score*).

In the famous analysis in Tversky (1977), *the contrast model*, the similarity between x and y (say, a *robin* and a *canary*), is calculated based on both the features in whose values x and y are alike and the features in whose values they are distinct. While in Rosch and Mervis (1975), similarity in the features in which x and y were not alike was zero, in Tverskey (1977) these features effectively reduce the similarity between x and y by adding a negative score. Formally, let F(x) and F(y) represent the features which x and y possess, respectively; f represents an additive function, such that, for instance, f(F(x)) and f(F(y)) represent the overall salience of x's and y's features, respectively (according to Tversky, these values depend on their intensity, frequency, familiarity, informational content etc.); The weight $W_{xy}$, $W_x$ and $W_y$ are assigned to the set of features in which x and y are alike (F(x) ∩ F(y)), the set of features of x that y does not possess (F(x) — F(y)) and the features of y that x does not possess (F(y) — F(x)), respectively. The similarity of x to y is expressed as a linear combination of the contributions of the common and distinct features (8a). Tversky defines the typicality degree, $S(d,P)$, of an entity d in a category P of size n (say, the typicality of a robin in bird), as the average similarity of d to the other Ps (8b), i.e. roughly, a linear combination of the contributions f of the features that are shared with the elements in [[P]] and the features that are not shared with them, where $W_{xy}$ reflects the relative weights of the common and distinctive features and $p_n$ reflects the effects of category size on typicality (Tversky 1977: 347-348).

8) a. Similarity:

$$S(x,y) = W_{xy}f(F(x) ∩ F(y)) - W_xf(F(x) — F(y)) - W_yf(F(y) — F(x))$$

b. Typicality:

$$S(x,P) = p_n((W_{xy}∑_{y∈[P]}f(F(x) ∩ F(y))) - ∑_{y∈[P]}(f(F(x) — F(y)) + f(F(y) — F(x))) )$$

c. Category family resemblance score:

$$S(P) = (Σ_{x,y∈[P]}S(x,y))/(hn^2)$$

Finally, the *category family resemblance score*, S(P), is the average similarity of any two Ps (8d), i.e. roughly, a linear combination of the contributions of the common and the distinctive features of all the pairs of objects in [[P]] (Tversky 1977: 348-349). A denotation [[P]] is assumed to be selected among the set of possible denotations iff it has the highest category family resemblance score.

The function employed by Rosch and Mervis (1975) was a special case of S with a particularly large weight for the common features, and where the value $f(F(x) ∩ F(y))$ was the number of common features ($W_{xy} = 1$, and $W_x = W_y = 0$). If $p_n$ equals 1/n, this yields the average similarity of x to all the category members $S(x,P) = (Σ_{y∈[P]}S(x,y))/n)$. Tversky assumes that normally the common features are weighed more heavily in typicality judgments than in similarity judgments.

Similarity is symmetric ($S(x,y) = S(y,x)$) iff either $W_x = W_y$ (which means, for Tversky, that the task is to evaluate the degree to which *x and y are similar*) or when $f(F(x)) = f(F(y))$, where f is additive. When the task is to evaluate the degree to which *x is similar to y*, x's features weigh more heavily than y's features ($W_x > W_y$). It follows that asymmetry will occur ($S(x,y) > S(y,x)$) iff x is less salient, $f(F(x)) < f(F(y))$. The more typical instances are generally more salient and hence less typical instances are predicted to be more similar to them than vice versa (Tversky...
This asymmetric notion of similarity also accounts for the tendency to place the less typical item in the subject slot in sentences of the form *X is virtually Y*. For instance, we can say that *a penguin is virtually a robin* but not vice versa (Lakoff 1973).

Tversky 1977 also suggested that the common features are weighed more heavily in similarity judgments than in judgments of *difference*. This hypothesis correctly predicted the surprising finding that a pair of objects with many common and many distinctive features is judged more similar and more different than a pair with fewer common and fewer distinctive features. For instance, 67% of the participants that made similarity judgments judged the more familiar countries West and East Germany as more similar to each other than the less familiar countries Ceylon and Nepal. At the same time, 70% of the participants that made difference judgments judged West and East Germany as more different from each other than Ceylon and Nepal.

**III. FRAME BASED SIMILARITY**

Some prototype theories use a more complicated prototype structure, which is called a *frame* or *scheme*. Classical examples include the *selective modification model* (Smith, Osherson, Rips and Keane 1988) and the *concept specialization model* (Murphy 1990). Like the previous models, these models associate nouns with a set of dimensions such as location, function, color and size. However, in these models, each dimension comprises of several values. For instance the dimension color may comprise of the values red, green etc. These values are assigned salience degrees (votes). Salience depends on the subjective frequency of the value in the category (its weight relative to other values; e.g. red apples are encountered more frequently than green apples) and on the value's perceptibility. For example, the red of an apple is more perceptible than that of a brick. Thus, perceptibility stands for the optimal degree of this value in the category. In practice, in Smith et al (1988), for instance, 30 subjects listed the properties of the nouns fruit and vegetables. Two raters chose 25 dimensions (such as outside and inside color, taste and smell, how eaten, how, when and where grown, juiciness, cost, container, nutritional value, favorite consumer and non-food uses) with about 7.2 values per dimension (12% of the properties were dropped due to disagreement between raters). Diagnosticity was determined by a statistical calculation indicating the extent to which the dimension's values are related to fruit but not to vegetables or vice versa. Salience was determined by the number of the values' mentions.

Note that the dimension color (or has some color x) is necessary for membership in red, white, and yellow, as well as in non-red, non-white, non-yellow, red or white, red or yellow etc. So this property is sort of a presupposition of these predicates. It is also stronger than (i.e. it asymmetrically entails) any other presupposed condition for membership in these concepts, like the property physical object. Thus, possibly, the judges have partitioned the features that the subjects listed into blocks, such that for each block they could choose as a dimension the strongest presupposed condition for membership in the features in it (in the ‘alternatives' alla' Cohen 1999 and references therein).

In these models, entities are described by the concepts' lists of values, with the weights reflecting the salience of the value in that entity. For example, a possible vote distribution in the prototype and three entities of the concept apple are given in table 1 (Smith et al 1988).
Table 1: The apple votes distribution in each entity (Smith et al 1988)

Based on the contrast model (Tversky 1977), the typicality degree of an entity x in apple (x’s similarity to the prototype d1), was defined to be the sum of degrees of similarity of each of x’s dimensional values to d1’s values (9a). For each dimension F, the similarity of x to apple in F, S(x,d1,F), is the sum of votes d shares with d1 (multiplied by a free parameter A), minus the number of votes distinct to d1 (multiplied by a free parameter B) and the number of votes distinct to d (multiplied by a free parameter C), multiplied by the diagnosticity value of the dimension and a free parameter D (Smith et al 1988: 489-91). This is represented in (9b), where min(n,m) and max(n,m) are the smallest and biggest numbers in the pair (n,m), respectively.

\[
S(x,d1) = S(x,d1,shape) + S(x,d1,color)/2 + S(x,d1,texture)/4
\]

\[
S(x,d1,color) = A\min(\{\deg(x,\text{red}),\deg(d1,\text{red})\}) - B\max(\{0,\deg(d1,\text{red}) - \deg(x,\text{red})\}) + A\min(\{\deg(x,\text{green}),\deg(d1,\text{green})\}) - B\max(\{0,\deg(d1,\text{green}) - \deg(x,\text{green})\}) + A\min(\{\deg(x,\text{brown}),\deg(d1,\text{brown})\}) - B\max(\{0,\deg(d1,\text{brown}) - \deg(x,\text{brown})\}) - C\max(\{0,\deg(x,\text{brown}) - \deg(d1,\text{brown})\})
\]

The typicality degrees of d2–d4 in apple given this analysis for A = B = C = D = 1 are given in (9c-e). The brown apple ends up much less typical than the bumpy apple because the color features are more diagnostic for apples than the texture features.

Note that the job the frame does so far can be done also by a feature list model. In order to see this, let us replace any value Z by the feature Z to degree n (Zn), where n is the number of Z’s votes multiplied by the dimension’s diagnosticity (thus, the closer one gets to degree n in Z the more typical one is). Similarly, let zx be the number of votes an entity x has in Z multiplied by the dimension’s diagnosticity. Thus, the apple frame (Table 1) would correspond to the feature list: \{red_{25}, green_{5}, brown_{0}, round_{1/2}, cylindrical_{1/2}, square_{0/2}, smooth_{25/4}, rough_{5/4}, bumpy_{0/4}\}. For any Zn, the similarity of x to Zn, S^A(x,Zn) is given in (10a), together with some examples in (10b-e). It follows that d2’s typicality in apple (or d2’s similarity to d1) is 21.25, as desired.

\[
S(x,Z_n) = A\min(n, z_x) - B\max(0, n - z_x) - C\max(0,z_x-n)
\]
\[ \text{b. } S(d_2, \text{red}) = \min(25, 30) - \max(25-30, 0) - \max(30-25, 0) = 20 \]
\[ \text{c. } S(d_5, \text{green}) = \min(5, 0) - \max(5-0, 0) - \max(0-5, 0) = -5 \]
\[ \text{d. } S(d_2, \text{brown}) = \min(0, 0) - \max(0, 0) - \max(0, 0) = 0. \]
\[ \text{e. } S(d_2, d_1, \text{color}) = 20 - 5 = 15 \]
\[ \text{f. } S(d_2, d_1) = 21.25 \]

The use of frames is usually motivated by compositional considerations. In frame models, such as the selective modification model by Smith et al 1988's and the concept specialization model by Murphy 1990, modifiers are taken to be operators selecting certain dimensions in the noun's prototype and decreasing or increasing their diagnosticity votes and the salience votes of their different values. For example, red in red apple selects the dimension color in the prototype of apple, moves all the votes to red, and boosts the diagnosticity by a certain parameter E. For example, a possible vote distribution for the red-apple frame in three entities for E=2 is given in Table 2 (Smith et al 1988).

<table>
<thead>
<tr>
<th>Diagnosticity:</th>
<th>Color: 1*2</th>
<th>Shape: 1/2</th>
<th>Texture: 1/4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values:</td>
<td>red</td>
<td>Green</td>
<td>Brown</td>
</tr>
<tr>
<td>(d_2): The prototype</td>
<td>30</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(d_1): A red apple</td>
<td>30</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(d_3): A brown apple</td>
<td>0</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>(d_4): A bumpy apple</td>
<td>30</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: The red-apple votes distribution in each entity (Smith et al 1988)

The typicality degrees in red-apple are given in (23).

11)
\[ \text{g. A red apple } d_2: \quad \text{Sim}(d_5, d_2) = (30-0)2+(15-10)/2+(25-10)/4 = 66.25 \]
\[ \text{h. A brown apple } d_1: \quad \text{Sim}(d_5, d_3) = (0-60)2+(15-10)/2+(25-10)/4 = -114 \]
\[ \text{i. A bumpy apple } d_4: \quad \text{Sim}(d_5, d_4) = (30-0)2+(15-10)/2+(0-60)/4 = 47.5 \]

This example demonstrates the conjunction effect: \(d_2\), the red apple, is more typical in red apple than in apple (66.25 vs. 21.25). Furthermore, reconstructing the prototype for brown apple in the same way would demonstrate that the conjunction effect is stronger for conjunctions whose adjectival constituent is incompatible with features in the nominal constituent's feature sets. The extent to which \(d_3\), the brown apple, is more typical in brown apple than in apple is much greater than the extent to which \(d_2\) is more typical in red apple than in apple. While the difference between the red apple's degree in red apple and in apple is 45, the difference between the brown apple's degree in brown apple and in apple is 120.
The validity of the model was examined using the nouns fruit and vegetables, the adjectives red, white, round and long, and the adverbs very, slightly and non. In these studies, 30 subjects listed the properties of the nouns. Two raters chose 25 dimensions for the nouns (such as outside and inside color, taste and smell, how eaten, how, when and where grown, juiciness, cost, container, nutritional value, favorite consumer and non-food uses) with about 7.2 values per dimension (12% of the properties were dropped due to disagreement between raters). Diagnosticity was determined by a statistical calculation indicating the extent to which the dimension's values are related to fruit but not to vegetables or vice versa. Salience was determined by the number of the values' mentions. For each adjective, the raters determined a single dimension, with all the votes on the value named by the adjective. The authors determined how much diagnosticity is boosted by an adjective, the weight given to shared values, and the weight given to values distinct to the concept and to the entity. Subjects rated the typicality of 15 sub-types of fruit and 15 sub-types of vegetables in the nouns fruit and vegetables and in the noun phrases combined from these nouns and the modifiers red, white, round and long. The average correlation between the model's predictions and the subjects' typicality judgments was 0.7-0.98, focusing only on the modifiers red and round (the modifiers long and white failed to induce correlations due to lack of variability in size and whiteness). The ratings for the complex phrases were almost as correlated with the ratings for the nouns as with the ratings for the adjectives, which means that subjects' ratings for conjunctions consider more than just the dimension singled out by the adjective (Smith et al 1988: 507). The weight of the shared features, A, has exceeded the weight of the features distinct to the prototype, B, which in turn exceeded the weight of the features distinct to the item, C. Nevertheless, the positive values for B and C are evidence for the importance of the distinct features, namely, the importance of the parts which are not shared by the compared instances in the redness scale (the red's votes) and in all the other scales. The diagnosticity-boost weight D was considerably variable, but substantial, supporting the assumption that adjectives boost the diagnosticity of dimensions.

The main problem with the described frame model is that it is too rigid. It is hardly possible to apply it to new cases. For example, something between a ball and a block would be more typical of a round block than of a round ball, but the selective modification model cannot account for this, because both concepts would have all the shape votes in the value round (Smith et al 1988: 524). In order to allow for such cases we need to assume that an adjective can put most, not all the votes, on the dimension value it names. The number of votes moved introduces a new free parameter, which further complicates the mechanism. Another problem has to do with adjectives which modify several dimensions in the noun. Medin and Shoben 1988 showed that, for instance, a dark colored shirt was considered more typical of a winter shirt than of a summer shirt, meaning that a change in the color slot, the time of the year slot was changed. Another problem has to do with the fact that sometimes the dimension which is to be modified by the adjective varies with the noun. For example, for the noun car, the adjective corporate primarily modifies the owned by dimension, for the noun building it primarily affects the location, for lawyer it affects the works for dimension, etc. (Murphy 2002.)

These problems were addressed by the concept specialization model (Murphy 1990) in that this model assumes that after the selection and modification of a dimension, a stage of elaboration takes place, whereby more changes occur in the concept. These changes are guided by world knowledge and by the characteristics of instances of the concept which is being created (Murphy...
2002: 453). For example, in *wooden-spoon*, *wooden* changes the *material* slot of *spoon*. World knowledge may further change other related slots, like *size*.

Yet, quite often, adjectival or nominal modifications seem to be better described as cases in which a property of the modifier is mapped to the noun. For example, a *tiger squirrel* is interpreted as a squirrel which has stripes (Wisniewsky and Love 1998). In addition, there are cases in which the modification results in a complete new construal of the concept (plastic truck; stone squirrel, animal crackers). Wisniewsky 1997 has reported that of about 3000 interpretations he collected for 224 combinations, 70% involved such a radical meaning change. In general, it seems insufficient to assume that an adjective boosts the weight of certain dimensional values, because in effect, both typicality in the noun and typicality in the adjective extensively affect the typicality in the modified noun (Hampton 1987). Thus, researchers like Hampton focus on feature lists, rather than on frame structures, as described in chapter 4.

Note also that the interpretation of combined concepts such as noun-noun compounds involves an initial step in which a relation that links the constituents is selected among all potential relations (made-of for chocolate bee; makes for honey-bee etc.) The selected relation might determine which fillers will be changed in a combined concept. For example, the feature can withstand cold, but not the feature grey, is fixed by the location (or lives in) relation in mountain bird (Gagne and Murphy 1996; Gagne and Spalding 2004b). While Frame models regard the contribution of the head noun as far more central than the contribution of the modifier, when the facts concerning the selection of a relation that links the constituents are examined, the situation seems to be reversed.

According to the *Competition-among-relations-in-nominals theory* (CARIN; Gagne and Shoben 1997; Gagne 2001), the selection of a relation is primarily influenced by previous uses of the modifier. For example, just as people know that the set of bird instances range in typicality, they know that mountain, as a modifier, more typically denotes location than an about relation. It is predicted that mountain-bird is easier to process than mountain magazine. Empirical evidence reviewed strongly supports the CARIN theory. First, Gagne and Shoben 1997 computed relative frequencies of the relations competing for each modifier in compounds created by crossing 91 modifiers with 91 heads (which could all be linked to relations). High versus low frequency relations of the modifier (and not of the head) could predict faster versus slower online sense-non-sense judgments about the compounds. Second, exposure to a compound (student vote) facilitated sense-non-sense judgments of subsequent compounds sharing the same relation (student accusation) compared to a different relation (student car), but only when the compounds had identical or semantically similar modifiers. When the heads were identical but not the modifier, no facilitation occurred between same (employee vote) compared to different (reform vote) relations as predicted by CARIN. Similar results were obtained for Indonesian and French left-headed compounds suggesting that it is the modifier, not the first constituent, that's usages are affecting response time (Gagne 2001, 2002 and references therein).
IV. EFFECTS OF GRADED MEMBERSHIP

TABLE 3: MEAN TYPICALITY RATINGS IN ILL AND WELL DEFINED CATEGORIES

<table>
<thead>
<tr>
<th>Ill defined concepts</th>
<th>Mean Typicality</th>
<th>Well defined concepts</th>
<th>Mean Typicality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Rosch 1973;</td>
<td>Even number</td>
<td>(Armstrong et al 1983)</td>
</tr>
<tr>
<td>Fruit (set A)</td>
<td>Armstrong et al 1983)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Apple</td>
<td>Four</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>Strawberry</td>
<td>Eight</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Plum</td>
<td>Ten</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>Pineapple</td>
<td>Eighteen</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>Fig</td>
<td>Thirtyfour</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>Olive</td>
<td>One hundred and six</td>
<td>3.9</td>
</tr>
<tr>
<td>Fruit (set B)</td>
<td>Orange</td>
<td>Two</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Cherry</td>
<td>Six</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>Watermellon</td>
<td>Forty two</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>Apricot</td>
<td>Thousand</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>Coconut</td>
<td>Thirty four</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>Olive</td>
<td>Eight hundred and six</td>
<td>3.9</td>
</tr>
<tr>
<td>Vegtable (set A)</td>
<td>Carrot</td>
<td>Three</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>Celery</td>
<td>Seven</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>Asparagus</td>
<td>Twenty three</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>Onion</td>
<td>Fifty seven</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>Pickle</td>
<td>Five hundred and one</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>Parsely</td>
<td>Four hundred forty seven</td>
<td>3.7</td>
</tr>
<tr>
<td>Vegtable (set B)</td>
<td>Peas</td>
<td>Seven</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>Spinach</td>
<td>Eleven</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>Cabbage</td>
<td>Thirteen</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>radish</td>
<td>Nine</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>Peppers</td>
<td>Fifty seven</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>Pumpkin</td>
<td>Ninety one</td>
<td>3.7</td>
</tr>
<tr>
<td>Sport (set A)</td>
<td>football</td>
<td>Square</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>hockey</td>
<td>Triangle</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>gymnastics</td>
<td>Rectangle</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>wrestling</td>
<td>Circle</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Archery</td>
<td>Trapezoid</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>Weight lifting</td>
<td>Ellipsis</td>
<td>3.4</td>
</tr>
<tr>
<td>Sport (set B)</td>
<td>Baseball</td>
<td>Square</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Soccer</td>
<td>Triangle</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>Fencing</td>
<td>Rectangle</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>Sailing</td>
<td>Circle</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>Bowling</td>
<td>Trapezoid</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>Hiking</td>
<td>Ellipsis</td>
<td>3.5</td>
</tr>
<tr>
<td>Vehicle</td>
<td>Car</td>
<td>Female</td>
<td>Mother</td>
</tr>
<tr>
<td>(set A)</td>
<td>Boat</td>
<td>2.7</td>
<td>3.3</td>
</tr>
<tr>
<td>-----------------</td>
<td>----------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Scooter</td>
<td>2.5</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>Tricycle</td>
<td>3.5</td>
<td>4.7</td>
<td></td>
</tr>
<tr>
<td>Horse</td>
<td>5.9</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>skis</td>
<td>5.7</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>Vehicle (set B)</td>
<td>Bus</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>Motorcycle</td>
<td>2.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tractor</td>
<td>3.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wagon</td>
<td>4.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sled</td>
<td>5.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>elevator</td>
<td>6.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(set B)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>Sister</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ballerina</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Actress</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hostess</td>
<td>2.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chairwoman</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cowgirl</td>
<td>4.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Mean Typicality ratings in ill and well defined categories

### TABLE 4: MEAN ONLINE CATEGORIZATION TIMES IN ILL AND WELL DEFINED CATEGORIES, AND PERCENT OF SUBJECTS WHO SAID NO WHEN ASKED IF MEMBERSHIP IS GRADED.

<table>
<thead>
<tr>
<th>Ill defined concepts</th>
<th>Typical examples</th>
<th>Atypical examples</th>
<th>Difference (+/- 2 msec)</th>
<th>Percent of subjects who said no</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fruit</td>
<td>903</td>
<td>1125</td>
<td>220</td>
<td>43</td>
</tr>
<tr>
<td>Sport</td>
<td>892</td>
<td>941</td>
<td>50</td>
<td>71</td>
</tr>
<tr>
<td>Vegetables</td>
<td>1127</td>
<td>1211</td>
<td>85</td>
<td>33</td>
</tr>
<tr>
<td>Vehicles</td>
<td>989</td>
<td>1228</td>
<td>240</td>
<td>24</td>
</tr>
<tr>
<td>Well defined concepts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Even number</td>
<td>1073</td>
<td>1132</td>
<td>60</td>
<td>100</td>
</tr>
<tr>
<td>Odd number</td>
<td>1088</td>
<td>1090</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Female</td>
<td>1032</td>
<td>1156</td>
<td>125</td>
<td>86</td>
</tr>
<tr>
<td>Plane geometry figure</td>
<td>1104</td>
<td>1375</td>
<td>270</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4: Mean online categorization times in ill and well defined categories and Percent of subjects who said no when asked whether membership is graded.
APPENDIX TO CHAPTER 7:  
RESPONSES TO PHILOSOPHICAL DOUBTS CONCERNING NECESSARY CONDITIONS

The current theory does not suffer from problems of classical cluster theories (the Kripke-Putnam problems; Russel’s paradox; lack of compositionality or productivity, etc.)

I THE KRIPKE–PUTNAM OBJECTIONS TO SETS OF NECESSARY CONDITIONS

My proposal does not give rise to wrong predictions the way other theories of meaning which link predicates with clusters of necessary conditions do (Fodor and Katz 1963; Katz & Postal 1964; Jackendoff 1972).

Kripke 1972, Putnam 1975 and many others since, have convincingly demonstrated that these classical cluster theories give rise to wrong predictions. These were demonstrated by the semantics of predicates that stand for natural kinds, like *whale*. At some point in time, whales were thought to be *fish*. Presumably, at that time the cluster of concepts associated with *whale* included the concept *fish*. The discovery that whales are mammals has changed the cluster (*fish* being replaced by *mammal*) but not the denotation: The word *whale* still refers to the same objects (the absolute set of whales, à la Kripke-Putnam). No one thinks that, other creatures that are fish are the actual whales. That is, the word is more strongly connected with the denotation, not the cluster of properties. Furthermore, it is incorrect to assume that before the discovery the sentence *whales are fish* was regarded as a definitional tautology, and after the discovery a meaning change has turned it into a contradiction. Its falsehood is and was an empirical matter. For the same reasons *whales are mammals* is not regarded as a tautology today, but rather as an empirical matter.

In my proposal, the necessary dimensions of, e.g. the predicate *whale*, (the requirement to reach a certain mean degree in the dimension set of *whale*, or a very important typicality dimension which in effect functions as necessary for membership, like *mammal*) or predicate like *healthy* (like *healthy wrt bp*), are represented as context-dependent characteristics of *whale* (or *healthy*). In certain contexts they are taken to characterize *whale* (or *healthy*), and in others, they may not be so taken. In addition, in nominal concepts, even if some predicates are dimensions in any context, they are not entailments (some rare entities might not fall under them). Since dimensions are not regarded as entailments of the predicates they characterize, the classical Kripke-Putnam arguments against them are no longer valid. It does not follow that *whales are mammals* is a tautology, or that *whales are fish* is a contradiction. Both statements are true in some contexts and false in others. A predicate is a semantic necessary condition of P iff a competent speaker considers it necessary of P in every context of utterance (if that speaker never identifies her knowledge of the world and the language with the knowledge in a context in c where N is not necessary of P). The dimension-sets of predicates in my proposal are not semantic in this strong (and problematic) sense.

As for the idea that a word is more strongly connected to the denotation, than to the set of dimensions, perhaps nowadays, well established defining dimensions, such as the particular genetic makeup in each natural kind concept, are indeed linked with natural kind concepts more tightly than the denotation is. For example, in discovering about a creature which was thought to be a tiger of type A, that its genes are actually different enough, we may decide to treat it as a
tiger of type B or as a non-tiger, rather than to change the defining properties of tiger of type A or of tiger. Whether this is the case or not, is an empirical question, not an obstacle to dimension theories.

II OBJECTIONS TO NECESSARY CONDITIONS BASED ON RUSSEL’S PARADOX

Russell's paradox may arise if, for example, for some predicate P, F(P,t,g) is defined by a condition which requires that some predicate is in F(P,t,g) iff P is not in F(Q,t,g) (Ido Ben Zvi p.c.):

\[(1) \ F(P,t,g) = \{Q \in \text{CONCEPT}: P \not\in F(Q,t,g)\}\]

It follows that P \in F(P,t,g) iff P \not\in F(P,t,g). It is standardly assumed for similar paradoxical predicates (such as the set of barbers who shave all and only those who do not shave themselves), that they fail to pick out a property, or an actual set of entities in the world (Irvin 2003). The same can be said in regard to paradoxical Ns. This assumption is represented in the feature model in the following way. Had an entity (or a dimension) fallen under those predicates (or Fs), then either consistency constraint would be violated, because this entity would have to be in both the positive and the negative denotation (or F set), or the totality constraint would be violated, because this entity (or dimension) would have to be excluded from both the positive and the negative denotation (or F set).

For example, assume that some d is a member in the set denoted by the predicate for all entities, y shaves them iff they do not shave y. It follows that for all entities, d shaves them iff they do not shave d, and in particular, d shaves himself iff d doesn't shave himself. So there are two possible cases. The first one is such that d \in \{shaves himself\}_{t,g} and d \in \{shaves himself\}_{t,g}. But this cannot be the case because the consistency condition (chapter 6, (13)) entails that: ¬\exists d \in D, P \in \text{CONECPT}, t \in T, g \in G: d \in \{P\}_{t,g} \& d \in \{Q\}_{t,g}. The second case is such that d \not\in \{shaves himself\}_{t,g} and d \in \{shaves himself\}_{t,g}, but this cannot be the case because the totality condition (chapter 6, (10a) and (13)) entails that ¬\exists d \in D, P \in \text{CONECPT}, t \in T, g \in G: d \not\in \{Q\}_{t,g} \& d \not\in \{P\}_{t,g}. Hence our assumption leads to a contradiction. No d is a member in the set denoted by the predicate for all entities, y shaves them iff they do not shave y.

Similarly, assume that for some P \in \text{CONECPT}, F(P,t,g) = \{Q \in \text{CONCEPT}, P \not\in F(Q,t,g)\}. I.e. P has the property that \forall Q \in \text{CONCEPT}: (Q \in F(P,t,g) iff P \not\in F(Q,t,g)), and in particular, (P \in F(P,t,g) iff P \not\in F(P,t,g)). There are two possible cases again. The first one is such that (P \in F(P,t,g) & P \not\in F(P,t,g)), but this cannot be the case because the consistency condition (chapter 6, (10a) and (25)) entails that ¬\exists P, Q \in \text{CONECPT}: Q \in F(P,t,g) & Q \notin F(P,t,g). The second case is such P \not\in F(P,t,g) & P \not\in F(P,t,g), but this cannot be the case because the totality condition (chapter 6, (10a) and (25)) entails that ¬\exists P, Q \in \text{CONECPT}: Q \not\in F(P,t,g) & Q \not\in F(P,t,g). Hence, our assumption leads to a contradiction. No P \in \text{CONECPT} is such that F(P,t,g) = \{Q \in \text{CONCEPT}, P \not\in F(Q,t,g)\}. Furthermore, it seems to be the case that, empirically, for all predicates P and contexts c, the dimension set is always given by enumeration of its members. I.e. it is never given through a condition which holds of all and only its members. If this is obligatory of dimension-sets, it readily blocks the possibility of Russell's paradox.
III. COMPOSITIONALITY FOR ADJECTIVALS SETS OF NECESSARY CONDITIONS

With regard to the set of necessary conditions which are jointly sufficient for membership in an adjective \( P \) in a context \( t \) and assignment \( g \), let us call them \( N^+(P,t,g) \) so as to separate them from the nominal dimensions, it is easy to build a dimension-set for complex concepts based on the dimensions of the parts.

Three compositional rules state the connections that exist between the \( N(P) \)s of complex predicates and of their constituents. More precisely, these rules state the connections that exist between the closure of these sets under conjunction and equivalence.

Let \( C(N(P)) \) be the set of all the dimensions which in effect function as necessary for \( P \)-hood in a context.

\[(2) \forall t \in T, \forall g \in G, \forall P \in \text{CONCEPT}: \]
\[a. \quad C(N(P,t,g)) = \{ N \in \text{CONCEPT}: [[P]]^+_{t,g} \subseteq [[N]]^+_{t,g} \} \]

The set \( C(N(P,t,g)) \) is closed under superset (\( \subseteq \)) in \( c \): If a predicate \( N_1 \) is a member in \( C(N(P)) \), any other predicate \( N_2 \) which denotation is a superset of \( N_1 \)’s denotation in \( c \) and all its extensions, is in \( C(N(P)) \).

\[b. \text{ Superset: } \forall N_1, N_2 \in A: \quad N_1 \in C(N(P,t,g)) \text{ and } [[N_1]]_{t,g} \subseteq [[N_2]]_{t,g} \rightarrow N_2 \in C(N(P,t,g)) \]

In addition, any two predicates, \( N_1 \) and \( N_2 \), are members in \( C(N(P)) \) iff their conjunction, \( N_1 \land N_2 \), is in \( C(N(P)) \).

\[c. \text{ Conjunction: } \forall N_1, N_2 \in A: \quad N_1, N_2 \in C(N(P,t,g)) \leftrightarrow N_1 \land N_2 \in C(N(P,t,g)) \]

For example, we assume entities that are healthy to be healthy wrt bp and healthy wrt pulse iff we assume that they are healthy wrt bp and healthy wrt pulse. \( [[Q]]_{t,g}^+ \land [[Z]]_{t,g}^+ \) is a superset of \( [[P]]_{t,g}^+ \land [[Q \land Z]]_{t,g} \) is a superset of \( [[P]]_{t,g}^+ \). Thus, once \( Q \) and \( Z \) become necessary for P-hood, \( Q \land Z \) becomes necessary for P-hood as well (adding \( Q \land Z \) to \( N \) would have no additional effect at all), and vice versa. Conversely, it is not the case that for every disjunction \( Q \lor Z \) which is necessary for membership in \( P \), the disjuncts \( Q, Z \), are also necessary for membership in \( P \). Intuitively, if we assume that entities that are healthy are healthy wrt bp or healthy wrt pulse it does not follow that they are healthy wrt bp or that they are healthy wrt pulse.

For every \( P \), \( [[\land N^+(P)]]_{t,g}^+ = [[\land C(N^+(P,t,g))]]_{t,g}^+ \). For example, it is easy to see that the conjunction \( \land \{ Q_1, Q_2, Q_3 \lor Q_4 \} \) and the conjunction \( \land \{ Q_1, Q_2, Q_3 \lor Q_4, Q_1 \land Q_2, \neg
\end{verbatim}
Three compositional rules for C(N(P)) are derived.

\[ \forall t \in T, \forall g \in G, \forall P, Q \in \text{CONCEPT}: \]

a. The union rule for conjunctions:
\[ C(N^+(P \land Q, t, g)) = C(N^+(P, t, g) \cup N^+(Q, t, g)) \]

The conjunction rule states that the derivation of a cluster for a complex phrase of the form P \land Q comprises of the union of the clusters of the constituent phrases, closed under superset (that is, conjunctions of P dimensions and/or Q dimensions are added into the set, together with any of the predicates which denote supersets pf the denotations of these conjunctions). For example, it is predicted that, anything red and round ought to satisfy all the membership conditions of both red and round, any conjunction composed of those conditions, and any condition equivalent to them. As desired, it follows that positive denotations of conjunctions consist of (all and only) the entities that satisfy every \( N^+ \) dimension of every conjunct. Negative denotations, the complement sets, consist of the entities that violate some \( N^+ \) dimension of some conjunct.

b. The intersection rule for disjunctions:
\[ C(N^+(P \lor Q, t, g)) = C(N^+(P, t, g) \cap N^+(Q, t, g)) \]

The intersection rule has the following intuitive implications. First, every property which is necessary for falling under all disjuncts is necessary for falling under the disjunction. For example, if being human is necessary both for being old and for being disabled, then being human is necessary for being old or disabled: A superset of both \([P]\) and \([Q]\) is a superset of \([P \lor Q]\]. Second, every property which is necessary for falling under the disjunction is necessary for falling under each one of the disjuncts. For example, if the property of being above 70 years old or having a physical injury is necessary for being old or disabled, then this property is necessary for being old and necessary for being disabled: A superset of \([P \lor Q]\) is a superset of both \([P]\) and of \([Q]\]. Third, the conditions which are necessary of one disjunct but not the other (say, above 70 years old), are not necessary for the disjunction. In the extreme case of \( P \text{ or not } P \) nothing is necessary of the disjunction, except for things that are necessary of everything. Finally, the entities in \([P \land Q]\), fall either under \([P]\), or under \([Q]\), so they ought to satisfy either every necessary condition of P or every necessary condition of Q. E.g every old or disabled individual ought to satisfy either all the dimensions of old or all the dimensions of disabled. This intuitive prediction follows from the fact that every dimension of the form: \( N^+_P \lor N^+_Q \), where \( N^+_P \) is necessary for \( P \) and \( N^+_Q \) is necessary for \( Q \), is necessary for \( P \) and also for \( Q \) (in virtue of the closure under superset of dimensions). So are also disjunctions of conjunctions of \( P \) dimensions and conjunctions of \( Q \) dimensions, such as: \( \land N^+(P, t, g) \lor \land N^+(Q, t, g) \).

c. The negated dimension rule for negations:
\[ C(N^+(\neg P, t, g)) = C(\{\neg(\land N^+(P, t, g))\}) \]

The negated-dimension rule states that non-Ps ought to violate at least one membership condition of P (so as to violate \( \land N(P, t, g) \)). The cluster of non-P contains properties that are supersets of the disjunction of negations of P's properties, namely negated dimensions of P. In partial contexts c,
using this rule to build \([\neg P]\) would give us the set \(\cup\{\neg Z\} | Z \in N^*(P,c,g)\), as desired. This is the partial set of entities that must end up non Ps, because they violate some membership condition.

**Proof 1.1:** \(\forall M^*, \forall e T, \forall g G, \forall P, Qe CONCEPT: \ C(N^*(P_{\neg Q},g)) = C(N^*(P_{\neg Q},g)) \)

**A.**
1. Assumption: \(\exists M^*, \exists e T, g G, P, Qe CONCEPT: \ C(N^*(P_{\neg Q},g)) \subseteq C(N^*(P_{\neg Q},g)) \)
2. \(\exists e CONCEPT: \ Z \in C(N^*(P_{\neg Q},g)) \wedge Z \in C(N^*(P_{\neg Q},g)) \)
3. \(\exists e CONCEPT: \ Z \in C(N^*(P_{\neg Q},g)) \wedge Z \in C(N^*(P_{\neg Q},g)) \)
4. \(\exists e CONCEPT: \culo N^*(P_{\neg Q},g) \subseteq [Z] \wedge [N^*(P_{\neg Q},g)] \subseteq [Z] \)
5. \(\exists e CONCEPT, 3de D \) (given enough individuals in D):
   \(\text{de} [Z]_{\neg L}(g) \) but \(\text{de} [N^*(P_{\neg Q},g)]_{\neg L}(g) \) and \(\text{de} [N^*(P_{\neg Q},g)]_{\neg L}(g) \)
   \(- contradiction.

**Proof 1.2:** \(\forall M^*, \forall e T, \forall g G, \forall P, Qe A: \ C(N^*(P_{\neg Q},g)) = C(N^*(P_{\neg Q},g)) \)

**A.**
1. Assumption: \(\exists M^*, \exists e T, g G, P, Qe CONCEPT: \ C(N^*(P_{\neg Q},g)) \subseteq C(N^*(P_{\neg Q},g)) \)
2. \(\exists e CONCEPT: \ Z \in C(N^*(P_{\neg Q},g)) \wedge Z \in C(N^*(P_{\neg Q},g)) \)
3. \(\exists e CONCEPT: \ Z \in C(N^*(P_{\neg Q},g)) \wedge Z \in C(N^*(P_{\neg Q},g)) \)
4. \(\exists e CONCEPT, 3de D \) (given enough individuals in D):
   \(\text{de} [Z]_{\neg L}(g) \) but \(\text{de} [N^*(P_{\neg Q},g)]_{\neg L}(g) \) and \(\text{de} [N^*(P_{\neg Q},g)]_{\neg L}(g) \)
   \(- contradiction.

**Proof 1.3:** \(\forall M^*, \forall e T, \forall g G, \forall P, Qe A: \ C(N^*(P_{\neg Q},g)) = C(\neg N^*(P_{\neg Q},g)) \)

**A.**
1. Assumption: \(\exists M^*, \exists e T, g G, P, Qe CONCEPT: \ C(N^*(P_{\neg Q},g)) \subseteq C(\neg N^*(P_{\neg Q},g)) \)
2. \(\exists e CONCEPT: \ Z \in C(N^*(P_{\neg Q},g)) \wedge Z \in C(\neg N^*(P_{\neg Q},g)) \)
3. \(\exists e CONCEPT, 3de D \) (given enough individuals in D):
   \(\text{de} [Z]_{\neg L}(g) \) but \(\text{de} [N^*(P_{\neg Q},g)]_{\neg L}(g) \) and \(\text{de} [N^*(P_{\neg Q},g)]_{\neg L}(g) \)
   \(- contradiction.

**B.**
1. Assumption: \(\exists M^*, \exists e T, g G, P, Qe CONCEPT: \ C(N^*(P_{\neg Q},g)) \subseteq C(\neg N^*(P_{\neg Q},g)) \)
2. \(\exists e CONCEPT: \ Z \in C(\neg N^*(P_{\neg Q},g)) \wedge Z \in C(\neg N^*(P_{\neg Q},g)) \)
3. \(\exists e CONCEPT, 3de D \) (given enough individuals in D):
   \(\text{de} [Z]_{\neg L}(g) \) but \(\text{de} [N^*(P_{\neg Q},g)]_{\neg L}(g) \) and \(\text{de} [N^*(P_{\neg Q},g)]_{\neg L}(g) \)
   \(- contradiction.
I use the abbreviation LP for the learning principle. For any t and g:

Proof 2: \([\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg} \subseteq [\langle(P \land Q)\rangle]_{kg}\):
1. Assume: \(a,b \in [\langle\leq\rangle]_{kg}\), and \(a,b \in [\langle\leq\rangle]_{kg}\).
2. \(\forall c \in C, \text{ s.t.: } (a \in [P]_{kg} \text{ or } b \in [P]_{kg}) \text{ and } (b \in [\neg\neg P]_{kg} \text{ or } a \in [\neg\neg P]_{kg})\) \(= 1\), LP
3. \(\forall c \in C, \text{ s.t.: } (a \in [Q]_{kg} \text{ or } b \in [Q]_{kg}) \text{ and } (b \in [\neg\neg Q]_{kg} \text{ or } a \in [\neg\neg Q]_{kg})\) \(= 1\), LP
4. \(\forall c \in C, \text{ s.t.: } ([P \land Q]_{kg} \text{ or } [\neg\neg(P \land Q)]_{kg}) \text{ or } ([P \land Q]_{kg})\) \((\text{by the first conjuncts in 2-3})\).
5. \(\forall c \in C, \text{ s.t.: } ([P \land Q]_{kg}) \text{ or } ([\neg\neg(P \land Q)]_{kg})\) \((\text{by the second conjuncts in 2-3})\).
6. \(\forall c \in C, \text{ s.t.: } ([P \land Q]_{kg}) \text{ or } ([\neg\neg(P \land Q)]_{kg})\)
7. \(\forall c \in C, \text{ s.t.: } ([P \land Q]_{kg}) \text{ or } ([\neg\neg(P \land Q)]_{kg})\) \((4,6)\)
8. \(a,b \in [\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg}\) \((7)\)

Proof 3: \(\text{Not always: } [\langle(P \lor Q)\rangle]_{kg} \subseteq [\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg}\):
Consider a pair \(a,b\) s.t. \(\forall c \in C, \text{ s.t.: } (a \in [P \lor Q]_{kg} \text{ or } b \in [P \lor Q]_{kg}) \text{ and } (a \in [\neg\neg(P \lor Q)]_{kg} \text{ or } b \in [\neg\neg(P \lor Q)]_{kg})\), but that in some context \(c_1\) under t only b is added in \([P]\), in \(c_2\) above \(c_1\) and under t, only a is added in \([Q]\), and in \(c_3\) above \(c_2\) and under t b is added in \([Q]\). This pair would belong in \([\langle(P \lor Q)\rangle]_{kg}\) but yet in \([\langle\leq\rangle]_{kg}\) and in \([\langle\leq\rangle]_{kg}\) (rather then in \([\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg}\)).

Proof 4: \([\langle=\rangle]_{kg} \cap [\langle=\rangle]_{kg} \subseteq [\langle(P \land Q)\rangle]_{kg}\):
1. Assume: \(a,b \in [\langle=\rangle]_{kg} \cap [\langle=\rangle]_{kg}\).
2. \(a,b \in [\langle=\rangle]_{kg} \cap [\langle=\rangle]_{kg}\) and \(a,b \in [\langle=\rangle]_{kg} \cap [\langle=\rangle]_{kg}\) \(= 1\).
3. \(a,b \in [\langle=\rangle]_{kg} \cap [\langle=\rangle]_{kg}\) \(= 2\), the fact that: \([\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg} \subseteq [\langle(P \land Q)\rangle]_{kg}\)
4. \(a,b \in [\langle=\rangle]_{kg} \cap [\langle=\rangle]_{kg}\) \(= 3\)

Proof 5: \(\text{Not always: } [\langle(P \lor Q)\rangle]_{kg} \subseteq [\langle=\rangle]_{kg} \cap [\langle=\rangle]_{kg}\):
Consider a pair \(a,b\) s.t. \(\forall c \in C, \text{ s.t.: } (a \in [P \lor Q]_{kg} \text{ iff } b \in [P \lor Q]_{kg}) \text{ and } (a \in [\neg\neg(P \lor Q)]_{kg} \text{ iff } b \in [\neg\neg(P \lor Q)]_{kg})\), but that in some context \(c_1\) under t only b is added in \([P]\), in \(c_2\) above \(c_1\) and under t, only a is added in \([Q]\), and in \(c_3\) above \(c_2\) and under t both a is added in \([Q]\) and b is added in \([P]\). This pair would belong in \([\langle(P \lor Q)\rangle]_{kg}\) but yet in \([\langle\leq\rangle]_{kg}\) and in \([\langle\leq\rangle]_{kg}\) (rather then in \([\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg}\)).

Proof 6: \([\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg} \subseteq [\langle(P \lor Q)\rangle]_{kg}\):
1. Assume: \(a,b \in [\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg}\).
2. \(\exists c \in C, \text{ s.t.: } (a \in [Q]_{kg}, b \in [Q]_{kg}) \text{ or } (a \in [\neg\neg Q]_{kg}, b \in [\neg\neg Q]_{kg})\) \(= 1\), LP
3. \(\exists c \in C, \text{ s.t.: } (a \in [P]_{kg}, b \in [P]_{kg}) \text{ or } (a \in [\neg\neg P]_{kg}, b \in [\neg\neg P]_{kg})\) \(= 1\), LP
4. \(\exists c \in C, \text{ s.t.: } (a \in [P]_{kg}, b \in [P]_{kg}) \text{ or } (a \in [\neg\neg P]_{kg}, b \in [\neg\neg P]_{kg})\) \(= 2-3\)
5. \(\exists c \in C, \text{ s.t.: } (a \in [P \lor Q]_{kg}, b \in [P \lor Q]_{kg})\) \(= 4A\) \(= 4B-D\)
6. \(a,b \in [\langle\leq\rangle]_{kg}\).

Proof 7: \(\text{Not always: } [\langle(P \lor Q)\rangle]_{kg} \subseteq [\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg}\):
Consider a pair \(a,b\) s.t. \(\exists c \in C, \text{ s.t.: } (a \in [P \lor Q]_{kg}, b \in [P \lor Q]_{kg}) \text{ or } (a \in [\neg\neg(P \lor Q)]_{kg}, b \in [\neg\neg(P \lor Q)]_{kg})\), but that in some context \(c_1\), under t only b is added in \([P]\) and only a is added in \([Q]\); in \(c_2\) above \(c_1\) and under t b is added in \([Q]\), and in \(c_3\) above \(c_2\) under t a is added in \([P]\). This pair would belong in \([\langle(P \lor Q)\rangle]_{kg}\) but yet in \([\langle\leq\rangle]_{kg}\) and \([\langle\leq\rangle]_{kg}\) (not in \([\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg}\)).

Proof 8: \([\langle\leq\rangle]_{kg} \cap [\langle\leq\rangle]_{kg} \subseteq [\langle(P \lor Q)\rangle]_{kg}\):
1. Assume: \(<a, b> \in [\leq P]_{k,g}\) and \(<a, b> \in [\leq Q]_{k,g}\). 
2. \(<a, b> \in [\leq P]_{k,g}\) and \(<a, b> \in [\leq Q]_{k,g}\) \hspace{1cm} (1).
3. \(\forall c \in C, \text{cst}: (\{a \in [\leq P]_{k,g}\} \cup \{b \in [\leq P]_{k,g}\}) \land (\{a \in [\leq Q]_{k,g}\} \cup \{b \in [\leq Q]_{k,g}\})\) \hspace{1cm} (1, LP)
4. \(\forall c \in C, \text{cst}: (\{a \in [\leq Q]_{k,g}\} \cup \{b \in [\leq Q]_{k,g}\}) \land (\{a \in [\leq Q]_{k,g}\} \cup \{b \in [\leq Q]_{k,g}\})\) \hspace{1cm} (1, LP)
5. \(\forall c \in C, \text{cst}: (\{a \in [\leq P]_{k,g}\} \cup \{b \in [\leq Q]_{k,g}\})\) \hspace{1cm} (by the first connectives in 3-4).
6. \(\forall c \in C, \text{cst}: (\{a \in [\leq Q]_{k,g}\} \cup \{b \in [\leq Q]_{k,g}\})\) \hspace{1cm} (by the second connectives in 3-4).
7. \(<a, b> \in [\leq Q]_{k,g}\) \hspace{1cm} (5-6)

\textit{Proof 9: \hspace{1cm} Not always: } \([\leq (P \lor Q)]_{k,g} \not\subseteq [\leq P]_{k,g} \cap [\leq Q]_{k,g}\):

Consider a pair \(<a, b>\) s.t. \(\forall c \in C, \text{cst}: (a \notin [P \lor Q], b \in [P \lor Q], a \notin [P \lor Q], b \in [P \lor Q])\), but that in some context \(c_1\) under \(t\) only \(b\) is added in \([P]\), and in \(c_2\) above \(c_1\) and under \(t\) only \(a\) is added in \([Q]\). This pair would belong in \([\leq (P \lor Q)]_{k,g}\), but yet in \([<P>]_{k,g}\) and in \([>Q>]_{k,g}\) (rather then in \([\leq P]_{k,g} \cap [\leq Q]_{k,g}\).

\textit{Proof 10: } \([=P]_{k,g} \cap [=Q]_{k,g} \not\subseteq [=PQ]_{k,g}\):

1. Assume: \(<a, b> \in [=P]_{k,g}\) and \(<a, b> \in [=Q]_{k,g}\).
2. \(<a, b> \in [\leq P]_{k,g} \cap [\geq P]_{k,g}\) and \(<a, b> \in [\leq Q]_{k,g} \cap [\geq Q]_{k,g}\) \hspace{1cm} (1).
3. \(<a, b> \in [P \lor Q]_{k,g} \cap [2P \lor Q]_{k,g}\) \hspace{1cm} (2, the fact that: \([\leq P]_{k,g} \cap [\leq Q]_{k,g} \subseteq [\leq (P \lor Q)]_{k,g}\)
4. \(<a, b> \in [P \lor Q]_{k,g}\) \hspace{1cm} (3)

\textit{Proof 11: \hspace{1cm} Not always: } \([=P \lor Q]_{k,g} \not\subseteq [=P]_{k,g} \cap [=Q]_{k,g}\):

Consider a pair \(<a, b>\) s.t. \(\forall c \in C, \text{cst}: (a \in [P \lor Q], b \in [P \lor Q], a \in [P \lor Q], b \in [P \lor Q])\), but that in some context \(c_1\) under \(t\) only \(b\) is added in \([P]\), and in \(c_2\) above \(c_1\) and under \(t\) both \(a\) and \(b\) are added in \([P]\) and \(b\) is added in \([Q]\). This pair would belong in \([=P \lor Q]\), but yet in \([<P>]_{k,g}\) and in \([>Q>]_{k,g}\) (rather then in \([\leq P]_{k,g} \cap [\leq Q]_{k,g}\).

\textit{Proof 12: } \([<P>]_{k,g} \cap [Q]_{k,g} \not\subseteq [Q \lor Q]_{k,g}\):

1. Assume: \(<a, b> \in [\leq Q]_{k,g} \cap [\leq Q]_{k,g}\).
2. \(\exists c \in C, \text{cst}: (\{a \in [\leq Q]_{k,g}\} \subseteq [\leq Q]_{k,g}\) and \(\{b \in [\geq Q]_{k,g}\} \subseteq [\leq Q]_{k,g}\) \hspace{1cm} (1, LP)
3. \(\exists c \in C, \text{cst}: (\{a \in [\leq Q]_{k,g}\} \subseteq [\leq Q]_{k,g}\) and \(\{b \in [\geq Q]_{k,g}\} \subseteq [\leq Q]_{k,g}\) \hspace{1cm} (1, LP)
4. \(\exists c \in C, \text{cst}: A. (\{a \in [\leq Q]_{k,g}\} \subseteq [\leq Q]_{k,g}\) \hspace{0.5cm} \& \hspace{0.5cm} (\{a \in [\leq Q]_{k,g}\} \subseteq [\leq Q]_{k,g}\) \hspace{1cm} (2-3)
5. \(\exists c \in C, \text{cst}: (\{a \in [\leq Q]_{k,g}\} \subseteq [\leq Q]_{k,g}\) \hspace{0.5cm} \& \hspace{0.5cm} (\{a \in [\leq Q]_{k,g}\} \subseteq [\leq Q]_{k,g}\) \hspace{0.5cm} (4A-C) \hspace{0.5cm} \& \hspace{0.5cm} (\{a \in [\leq Q]_{k,g}\} \subseteq [\leq Q]_{k,g}\) \hspace{0.5cm} (4D)
6. \(<a, b> \in [\leq Q]_{k,g}\).

\textit{Proof 13: \hspace{1cm} Not always: } \([<P \lor Q>]_{k,g} \not\subseteq [<P>]_{k,g} \cap [<Q>]_{k,g}\):

Consider a pair \(<a, b>\) s.t. \(\exists c \in C, \text{cst}: (\{a \in [\leq P]_{k,g}\} \subseteq [\leq P]_{k,g}\) or \(\{a \in [\leq Q]_{k,g}\} \subseteq [\leq Q]_{k,g}\), but that in some context \(c_1\) under \(t\) only \(b\) is added in \([P]\), and in \(c_2\) above \(c_1\) and under \(t\) only \(a\) is added in \([Q]\). This pair would belong in \([<P \lor Q>]_{k,g}\), but yet in \([<P>]_{k,g}\) and in \([>Q>]_{k,g}\) (rather then in \([\leq P]_{k,g} \cap [\leq Q]_{k,g}\).

\textit{Proof 14: } \([\leq (P \lor Q)]_{k,g} \not\subseteq [\leq P]_{k,g} \cup [\leq Q]_{k,g}\):

1. Assume: \(<a, b> \in [\leq (P \lor Q)]_{k,g}\). Prove: \(<a, b>\) is in \([\leq P]_{k,g}\) or in \([\leq Q]_{k,g}\), i.e. Not: \(<a, b> \in [\geq P]_{k,g} \cap [\geq Q]_{k,g}\).
2. Let us assume, in contrary, that \(<a, b> \in [\geq P]_{k,g} \cap [\geq Q]_{k,g}\).
3. \(<a, b> \in [\leq (P \lor Q)]_{k,g}\). But 3 contradicts 1, hence: Not: \(<a, b> \in [\geq P]_{k,g} \cap [\geq Q]_{k,g}\).

\textit{Proof 15: } \([\leq (P \lor Q)]_{k,g} \not\subseteq [\leq P]_{k,g} \cup [\leq Q]_{k,g}\):

1. Assume: \(<a, b> \in [\leq (P \lor Q)]_{k,g}\). Prove: \(<a, b>\) is in \([\leq P]_{k,g}\) or in \([\leq Q]_{k,g}\), i.e. Not: \(<a, b> \in [\geq P]_{k,g} \cap [\geq Q]_{k,g}\).
2. Let us assume, in contrary, that \(<a, b> \in [\geq P]_{k,g} \cap [\geq Q]_{k,g}\).
3. \(<a, b> \in [\leq (P \lor Q)]_{k,g}\). But 3 contradicts 1, hence: Not: \(<a, b> \in [\geq P]_{k,g} \cap [\geq Q]_{k,g}\).
Vagueness in numerical degree constructions

Galit W. Sassoon,
Ben Gurion University of the Negev
The numerical approach

- Gradable adjectives (like *tall*) map entities $d \in D$ to real numbers $r \in \mathbb{R}$.
- The mapping is **additive**: The numbers represent quantities’ of, e.g., height, in entities.

(i) Entities with no height are mapped to zero:

(ii) Differences and ratios between values of $f_{tall}$ adequately represent differences /ratios between the entities’ heights:
Advantages
(von Stechow 1984; Klein 1991)

A straightforward semantic accounts of:
- Numerical degree predicates (2 meters tall)
- Ratio predicates (twice as happy as Sam)
- Difference predicates (2 inches shorter than Sam)
- …
Problems
(Kamp & Partee 1995; Moltmann 2006)

- Numerical degree modifiers are often infelicitous (#two meters short/ degrees warm/ happy/ nice).
- Much indeterminacy in the mapping to numbers: Which numbers form the degrees of happy? Why would happy map one to, say, .25 rather than .24?
- If so, even speakers uttering ‘degree-denoting’ expressions cannot tell their meaning! (as in: Dan is as happy as Sam is; Bill isn't that happy?)
- …

⇒ Moltmann 2006: Only the few predicates that license numerical modifiers map entities to numbers.
My proposal

Contrary to Moltmann (2006), I propose that:

- Any gradable predicate (including *happy*) maps entities to numbers, but:
- In no predicate (including *tall*!), the degree function is fully specified.
- This explains the distribution of unit names, numerical degree modifiers, and ratio-modifiers.

- Let $W_c$ be the set of worlds consistent with the information in an actual context $c$ (Stalnaker 1978).
Observation 1/2

Quantities of the 'stuff' denoted by mass nouns like *height* do not have uniquely specified values (1,2,3,..). Many functions are additive wrt height:

\[ 5 + 5 = 10 \]
\[ 50 + 50 = 100 \]
\[ 2 + 2 = 4 \]

\[ [\text{Mickey is 2 meters tall}]_c = 1 \text{ iff } \forall w \in W_c : f_{\text{tall},w}(2) = 2 \]

*tall* maps objects to different numerals in different worlds:

\[ \exists w_1, w_2 \in W_c : f_{\text{tall},w_1} \neq f_{\text{tall},w_2} \]
Observation 2/2

Despite this fact, we can tell the ratios between, e.g., entities' heights.

⇒

All tall's functions in $W_c$ yield the same ratios between entities' degrees (these are given numbers):

$$\forall w \in W_c, \forall d \in D: \text{ Entities with } n \text{ times } d's \text{ height are mapped to } n \times f_{\text{tall, } w}(d).$$
A new analysis of unit names

Unit names like $\text{meter}_{\text{tall}}$ have two interpretations:

- $\text{Meter}^1_{\text{tall}}$ denotes the set of *meter unit objects* (the entities whose height we call *1 meter*).

- $\text{Meter}^2_{\text{tall}}$ is a relation between a number $n$ and an object $x$, s.t. $x$ is $n$ times as tall as a meter unit-object.

$$[[\text{Mickey is two meters}^2_{\text{tall}}]]_c = 1 \text{ iff } \forall w \in W_c: f_{\text{tall},w}(\text{Mickey}) = 2 \times r_{m,w}.$$  

(2 is the ratio between Mickey''s degree and the unit objects' degree in *tall*, $r_{m,w}$).

---

1 The word *meter* fits with several adjectives: *meter*$_{\text{long}}$, *meter*$_{\text{wide}}$ …
**Summary of my proposal**

We consider entities' degrees in *tall* as **specified** because:

I. The **ratios** between (values representing heights) do not vary across worlds

II. There is a set of **unit objects**, s.t. $d$ is *$n$ cms tall* iff: the ratio between $d$’s degree and the cm unit objects’ degree is $n$

<table>
<thead>
<tr>
<th></th>
<th>I. $f_{\text{tall},w}(d)$</th>
<th>The degree $f_{\text{tall}}$ assigns to the cm unit objects:</th>
<th>II. $f_{\text{tall},w}(d) = n \times r_{\text{cm},w}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$100$</td>
<td>$50$</td>
<td>$100$</td>
</tr>
<tr>
<td>$W_1$</td>
<td>$200$</td>
<td>$100$</td>
<td>$100 \times 1$</td>
</tr>
<tr>
<td>$W_2$</td>
<td>$300$</td>
<td>$150$</td>
<td>$100 \times 3$</td>
</tr>
<tr>
<td>$W_3$</td>
<td></td>
<td></td>
<td>$150 \times 3$</td>
</tr>
</tbody>
</table>
Violations of II: Lack of agreed-upon unit-objects

- Emotions are **internal states** - no one internal state can be considered by *all the community of speakers* to be a unit object of, e.g., *happy*.
- **Weight** can be measured by conventional units. Weight perception (speakers’ feeling of objects being *heavy, light*, etc.) cannot.
Direct results (1-2/5)

- Numerical degree predicates are not interpretable 😊
  
  #two degrees happy

- Modification by ratio modifiers is interpretable:

  
  \[ [[\text{Dan is twice as happy as Sam}]]_c = 1 \quad \text{iff} \quad 😃 \quad \forall w \in W_c : f_{\text{happy},w}([[\text{Dan}]]_w) = 2 \times f_{\text{happy},w}([[\text{Sam}]]_w) \]
Direct results (3-4/5)

- A unified analysis of difference modifiers:

  a. \([[[The ostrich is 50 \text{ cms} \text{ taller than the chicken}]]]_c = 1 \text{ iff } \forall w \in W_c: f_{\text{tall},w}(\text{ostrich}) - f_{\text{tall},w}(\text{chicken}) = 50 \times r_{m,w};\)

  Computation: In \(w_1\): \(100 - 50 = 50 = 50 \times 1\)
  In \(w_2\): \(200 - 100 = 100 = 50 \times 2\)
  In \(w_3\): \(300 - 150 = 150 = 50 \times 3\)

  b. \([[[Dan is happier than Sam]]]_c = 1 \text{ iff } \forall w \in W_c, \exists r \in \mathbb{R}, r > 0: f_{\text{happy},w}(\text{Dan}) - f_{\text{happy},w}(\text{Sam}) = r\)

- Speakers do not need to know the entities' degrees, only the ordering/ ratios between their degrees.
**Direct results (5/5)**

Comparison between predicates is fine if in every \( w \) their (different) unit-objects always have the same degree:

(1) \([[\text{The sofa is 2 meters longer than it is wide}]]_c = 1 \iff \forall w \in W_c: f_{\text{tall},w}(\text{\includegraphics{sofa}}) - f_{\text{wide},w}(\text{\includegraphics{sofa}}) = 2 \times r_{\text{m},w} \]

(1) is fine because: \( \forall w \in W_c: r_{\text{meter-tall},w} = r_{\text{meter-wide},w} \)

#The sofa is longer than it is heavy is odd because the degrees of no units of heavy and long co-vary.
Violations of I: Unspecified ratios between degrees

We may feel acknowledged of the ratios between our degrees of happiness in separate events, but usually we don’t feel so regarding entities’ degrees in short:

(1) Dan is twice as tall as Sam
(2) #Dan is twice as short as Sam

Direct result: #two meter short
Remaining problems

- Numerical degree predicates are fine in the comparative form of negative predicates:
  
  (1)  
  a. #Dan is two meters short  
  b. Dan is two meters taller /shorter

- Some positive predicates resemble negative ones:
  
  (2)  
  a. #The box is thirty degrees warm  
  b. #The box is thirty degrees cold  
  c. The box is 30 degrees warmer /colder
Negative predicates (Salt 18)

(1) *Dan is taller than Sam* iff *Sam is shorter than Dan*
\[ \Rightarrow f_{\text{short}} \text{ is reversed compared to } f_{\text{tall}}. \]

(2) *Dan is n cms taller than Sam* iff *Sam is n cms shorter*
\[ \Rightarrow f_{\text{short}} \text{ preserves the differences between degrees.} \]

Examples of such functions:

\[
\begin{align*}
\lambda d \in D. & \quad 0 \quad - f_{\text{tall},w}(d) \\
\lambda d \in D. & \quad 1 \quad - f_{\text{tall},w}(d) \\
\lambda d \in D. & \quad 3.75 \quad - f_{\text{tall},w}(d) \\
\lambda d \in D. & \quad -4 \quad - f_{\text{tall},w}(d)
\end{align*}
\]

\[ \Rightarrow f_{\text{short}} \text{ is transformed by an unspecified value:} \]

\[ \forall w \in W_c, \exists \text{Tran}_{\text{short},w} \in \mathbb{R}: \]

\[ \forall d \in D, \quad f_{\text{short},w}(d) = \text{Tran}_{\text{short},w} - f_{\text{tall},w}(d) \]
## Summary (Paris + Salt 18)

<table>
<thead>
<tr>
<th></th>
<th>$f_{\text{tall,}w}(d)$</th>
<th>The transformation value</th>
<th>$f_{\text{short,}w}(d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Tran$_{\text{short,}w}$</td>
<td></td>
</tr>
<tr>
<td>$W_1$</td>
<td>100</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>$W_2$</td>
<td>200</td>
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<tr>
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<td>300</td>
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<td>-10</td>
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<tr>
<th></th>
<th>10-100 =</th>
<th>10-50 =</th>
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<tbody>
<tr>
<td></td>
<td>-90</td>
<td>-40</td>
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<td>0-200 =</td>
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<td>-10-300 =</td>
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<td></td>
<td>-310</td>
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</tbody>
</table>

An additive measuring convention (ratio preserving)

function-reversal: Tran$_{\text{short,}w}$ – $f_{\text{tall,}w}(d)$ (not ratio preserving)
Direct results (1/6)

- **Ratio modifiers** (#Dan is twice as short as Sam)

\[ f_{\text{tall},w}(\text{chicken}) \text{ is two times } f_{\text{tall},w}(\text{chicken}) \text{ in every } w: \]
- \[ w_1: 100 = 2 \times 50 \]
- \[ w_2: 200 = 2 \times 100 \]
- \[ w_3: 300 = 2 \times 150 \]

**but:**

the two degrees of *short*,

\[ \text{Tran}_{\text{short},w} - f_{\text{tall},w}(\text{chicken}) \]

and

\[ \text{Tran}_{\text{short},w} - f_{\text{tall},w}(\text{chicken}), \]

are not two times one another in every \( w \):
- \[ w_1: -90 \neq 2 \times -40 \]
- \[ w_3: -310 \neq 2 \times -160 \]

So *twice as short* is less acceptable than *twice as tall*. 
Direct results (2/6)

**Numerical degree phrases** (#Dan is 2 meters short)
Transformed measures fail to represent height ratios

- No unit-name can be based on them ($cm_{short}$ doesn’t exist).
- If we use one of tall’s unit names (say, $cm_{tall}$):

  \[
  \text{[[The ostrich is } n \text{ cms}^2 \text{ short}]]_c = 1
  \]

  \[
  \text{iff } \forall w \in W_c: \quad f_{short,w}(\text{矮}) = n \times r_{m,w}
  \]

we find that no $n$ is the ratio between \(\text{矮}'s\) degree in short and the unit objects' degree in tall, \(r_{m,w}\), **in every** world!

\[
\begin{align*}
  f_{short,w1}(\text{矮}) &= -90 = n \times 1 \quad \Rightarrow \quad n = -90 \\
  f_{short,w2}(\text{矮}) &= -200 = n \times 2 \quad \Rightarrow \quad n = -100 \\
  f_{short,w3}(\text{矮}) &= -310 = n \times 3 \quad \Rightarrow \quad n = -103.3
\end{align*}
\]

This ratio varies between worlds.
Direct results (3/6)

Comparatives (Dan is 2 meters taller / shorter)

When degree-differences are computed, the two transformation values cancel one another, and we get a non-transformed value.

∀w ∈ Wc: \( f_{\text{tall},w}(\text{Dan}) - f_{\text{tall},w}(\text{Dan}) = 50 \times r_{\text{cm},w} \)

Computation:

In \( w_1 \): \( 100 - 50 = 50 \) \( = 50 \times 1 \)
In \( w_2 \): \( 200 - 100 = 100 \) \( = 50 \times 2 \)
In \( w_3 \): \( 300 - 150 = 150 \) \( = 50 \times 3 \)

∀w ∈ Wc: \( f_{\text{short},w}(\text{Dan}) - f_{\text{short},w}(\text{Dan}) = (\text{Tran}_{\text{short},w} - f_{\text{tall},w}(\text{Dan})) - (\text{Tran}_{\text{short},w} - f_{\text{tall},w}(\text{Dan})) = 50 \times r_{\text{cm},w} \)

Computation:

In \( w_1 \): \( -50 - -100 = 50 \) \( = 50 \times 1 \)
In \( w_2 \): \( -100 - -200 = 100 \) \( = 50 \times 2 \)
In \( w_3 \): \( -150 - -300 = 150 \) \( = 50 \times 3 \)
Direct results (4-6/6)

- Exceptional positive predicates (#30 degrees warm)
  The degrees of positive predicates are not reversed, but they may well be transformed.

- Unclear intuitions about the zero point
  Does tall map entities with no height (surfaces; points) to zero? Yes
  And short? No!

- Cross linguistic variations:
  Languages may vary as to whether predicates like heavy or warm measure external or internal states, and whether the measure is transformed or not.
Summary (Paris + Salt 18)

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  **but:**

  the two degrees of *short*,
  
  \[\text{Tran}_{\text{short},w} - f_{\text{tall},w}(\text{Chicken}) \text{ and } \text{Tran}_{\text{short},w} - f_{\text{tall},w}(\text{Tall})\]

  are not two times one another in every \(w:\)
  
  - \[\text{in } w_1: -90 \neq 2 \times -40\]
  - \[\text{in } w_3: -310 \neq 2 \times -160\]

  So *twice as short* is less acceptable than *twice as tall*. 
Direct results (2/6)

- **Numerical degree phrases** (#Dan is 2 meters short)
  Transformed measures fail to represent height ratios
  ⇒
  - No unit-name can be based on them (\(cm_{short}\) doesn’t exist).
  - If we use one of tall’s unit names (say, \(cm_{tall}\)):
    
    \[
    [[\text{The ostrich is } n \text{ cms}^2 \text{ short}]]_{c} = 1 \\
    \text{iff } \forall w \in W_c: \quad f_{short,w}(\text{ostrich}) = n \times r_{m,w}
    \]
    
    we find that no \(n\) is the ratio between ostrich's degree in short and the unit objects' degree in tall, \(r_{m,w}\), in every \(w\)!
    
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    f_{short,w3}(\text{ostrich}) & = -310 = n \times 3 \quad \Rightarrow \quad n = -103.3
    \end{align*}
    \]
    
    This ratio varies between worlds.
Direct results (3/6)

- **Comparatives** *(Dan is 2 meters taller / shorter)*

When degree-differences are computed, the two transformation values cancel one another, and we get a non-transformed value.

\[ \forall w \in W_c: \quad f_{\text{tall},w}(\text{bate}) - f_{\text{tall},w}(\text{hen}) = 50 \times r_{\text{cm},w} \]

*Computation:*

- In \( w_1 \): \( 100 - 50 = 50 = 50 \times 1 \)
- In \( w_2 \): \( 200 - 100 = 100 = 50 \times 2 \)
- In \( w_3 \): \( 300 - 150 = 150 = 50 \times 3 \)

\[ \forall w \in W_c: \quad f_{\text{short},w}(\text{hen}) - f_{\text{short},w}(\text{bate}) = (\text{Trans}_{\text{short},w} - f_{\text{tall},w}(\text{bate})) - (\text{Trans}_{\text{short},w} - f_{\text{tall},w}(\text{hen})) = 50 \times r_{\text{cm},w} \]

*Computation:*

- In \( w_1 \): \( -50 - -100 = 50 = 50 \times 1 \)
- In \( w_2 \): \( -100 - -200 = 100 = 50 \times 2 \)
- In \( w_3 \): \( -150 - -300 = 150 = 50 \times 3 \)
Direct results (4-6/6)

- **Exceptional positive predicates** (#30 degrees warm)
  The degrees of positive predicates are not reversed, but they may well be transformed.

- **Unclear intuitions about the zero point**
  Does tall map entities with no height (surfaces; points) to zero? Yes
  And short? No!

- **Cross linguistic variations**:
  Languages may vary as to whether predicates like heavy or warm measure external or internal states, and whether the measure is transformed or not.
THANK YOU!
Appendix 1: Vagueness of a different source

Sometimes we do not know the truth value of statements like:

Dan is taller than Sam
Dan is two inches taller than Sam.
Dan is twice as tall as Sam

I submit that this vagueness has a different source.
**My proposal: Individuals**

Individuals are distinguished by the values that degree functions assign to them.

- **Lewis 1986**: If Dan’s referent in \(w_1\) is 1.87 meters tall, and Dan’s referent in \(w_2\) is 1.86 meters tall, then: The name *Dan* refers to different individuals in \(w_1-w_2\).

- **Sassoon 2008**: If in \(w_1\) and \(w_2\) Dan’s referent is 1.87 meters tall, and identical in all the other values, then: Even if 1.87 counts as ‘tall’ in \(w_1\) but not in \(w_2\), the name *Dan* denotes the same individual in \(w_1-w_2\) (only the interpretation of the word *tall* is changed).
Appendix 2: Relativist semantics
(Lasersohn 2005; MacFarlane 2006; Stephenson 2007)

To do:

Can judgments about *That was fun/tasty* be explained without a move to full-blown relativist semantics?

I propose that these judgments involve measures of internal states (e.g., emotional extents).

⇒

Two speakers, each associating these predicates with measures of its own inner states, may vary in judgment. This may seem contradictory if we expect our inner extents to be similar.

(I thank the conference referee’s comments).
הפקולטה לפילוסופיה
בית הספר למדעי הרוח
לַע”ש שירלי ולסלי פורטר

ערפואל, ידוהי וטרופסדות
ניקוזם סנסטיק מיכקולק

דוברים לשים קבולה לחוקר "דוקטור לפילוסופיה"

נהנה
גילה רוזן

ดวงה לשכונת של אוניברסיטת תל אביב
נובמבר 2007
 Tonight, we were introduced to
ד"ר נירית קדמוני

This work was done under the supervision of
ר נירית קדמוני
1. Concepts (concepts) are ideas or things that are represented mentally in the form of concepts. Concepts are the building blocks of knowledge and are used to make sense of the world. Knowledge is the ability to group and organize ideas and information to create a meaningful representation of the world. (intension; Montague 1974)

2. The extension of a concept refers to the actual things in the world that the concept applies to. (extension)

3. Predicates (predicates) are statements or propositions that describe properties or states. (in Montague's Intensional Logic)

4. Concepts are represented in the mind in a variety of ways, including as mental images, words, and symbols. (concepts)

5. The meaning of a concept is determined by its relationship to other concepts and its use in different contexts. (semantics)

6. The study of concepts is important in psychology and linguistics, as it helps us understand how people organize and make sense of the world. (psychology and linguistics)

7. The main goal of this work is to develop a more comprehensive model of concepts than what has been done before. (development)

8. The model I propose is based on two main components: knowledge and context. (knowledge and context)

9. The knowledge component includes the concepts themselves and their relationships to each other. (knowledge)

10. The context component includes the situation in which the concepts are used and the way they are understood. (context)

11. The model can be used to explain a wide range of phenomena, such as how people think about different concepts and how they use them in different contexts. (model)

12. The model can be used to explain how concepts are acquired and how they change over time. (acquisition and change)

13. The model can be used to explain how concepts are used in language and how they are related to other aspects of human cognition. (language and cognition)

14. The model can be used to explain how concepts are used in decision-making and how they influence behavior. (decision-making and behavior)

15. The model can be used to explain how concepts are used in social cognition and how they influence social behavior. (social cognition and behavior)

16. The model can be used to explain how concepts are used in memory and how they influence memory performance. (memory and performance)

17. The model can be used to explain how concepts are used in problem-solving and how they influence problem-solving performance. (problem-solving and performance)

18. The model can be used to explain how concepts are used in creativity and how they influence creative performance. (creativity and performance)

19. The model can be used to explain how concepts are used in innovation and how they influence innovative performance. (innovation and performance)

20. The model can be used to explain how concepts are used in prediction and how they influence predictive performance. (prediction and performance)
1.1 Information is incomplete. Theories about vagueness, in accordance with cognitive psychology and logic, aim to determine the order of objects or predicates. An ordered arrangement of objects or predicates indicates their advantages and disadvantages. Theories of ordered and unordered predicates that vary from one theory to another, and are considered expansions of traditional theories, and are therefore more informed with empirical evidence.

I support the following theory: A new theory that addresses the issues discussed in the previous chapters, which solves the problems of the previous theories.

2.1 Chapter 2: Two types of objects, and traditional predicates, are denoted as "vague" and "gradable" respectively. In the context of many patterns, there are things about which we are uncertain, and therefore cannot be graded as "high" or "low", such as "apple" or "bird". However, predicates such as "older", "younger" can be compared, indicating that objects can be said, for example, "apple" or "bird" are not clear whether they are "high" or "not high".

We can also say that the predicate of three-dimensional order of nouns, for example, "the highest", "the best", "the worst", "the oldest", can be said when talking about a noun such as "height", "value", "age", and other predicates such as "older", "younger", "new", "old", "best", "worst", "newer", "older".

In the context of the visualization of the objects in which we are interested, psychologists have found that these objects are also graded and unordered. In this respect, research over the past four years has concluded that speakers tend to use these objects more than others, for example, "the best", "the worst", "the oldest", "the youngest", and other objects can be grouped in the same category, while the differences between these objects are the result of the theories of the two disciplines.
2.2

The paragraph begins with a discussion of grading and types, the perceptual stance on fuzziness. Section 3 is divided into three parts:

3.1

I am discussing theories of grading.

3.2

I am discussing theories of types.

3.3

I discuss theories of grading in terms of denotation (relationships) grammatical expressions in terms of states of information, in central semantic theories of fuzziness.

He is 'high' like the meaning of the term in partial states. It may contain partial information, in contrast to types, relationships. It is known as a denotation of positive, defuzzed. A group of objects known in the context that they are 'high'.

And it is known as a denotation of negative. An example is 'gap' (the set of objects that are known to be high). It includes partial denotation, correctly attributes.

It is known as a denotation of positive, defuzzed. A group of objects known in the context that they are not 'high'.

An example is 'gap' (the set of objects that are known to be low). It includes partial denotation, correctly attributes.

Chapter 4 discusses the following theories: those theories that are trying to explain grading through fuzziness. Louis 1970; Louis 1979; Kamp 1975; McConnell Ginnet 1973; Seuren 1973; Klein 1982; Landman 1991, Kamp and Partee 1995 etc.

These theories provide an elegant analysis of grading among objects. The main problems of these theories are that they do not predict the attributes. Therefore, they are not predicting correctly the attributes of grading among members of the denotation.

I present a model and its implementation to map objects to degrees of reflection accurately, and it preserves the order of reflections. A method of distance is assumed as far as possible in how each one is more contented than the other. But it is not the distance between them (contentedness). A method of reflections is better than comparing axes and using a method to determine contentedness and satisfaction (techniques). But it leaves the name of the method, (two meter taller, two meter short) which is not a method of comparison of contentedness and satisfaction.

The theories are not adequate for an order of contentedness in general. I propose a theory that is not simple. I define a small set of operations that take degrees of reflection (two meter taller, shorter). These operations create functions for each, according to my proposal. A function exists and returns degrees of a new function when the degrees of reflection are positive or negative. The proposal is economical. The functions of these operations are always ordered by the order of reflection. It predicts more facts than a few facts that characterize the reflection of negative numbers, such as, negative (two meter taller, shorter). As such, this theory is not consistent with the statements. I mark a gap in the theories that attempt to deal with the reflection of negative numbers. I show that this approach predicts correctly a long list of polar effects without creating erroneous predictions. (positional, 

I present a model and its implementation to map objects to degrees of reflection accurately, and it preserves the order of reflections. A method of distance is assumed as far as possible in how each one is more contented than the other. But it is not the distance between them (contentedness). A method of reflections is better than comparing axes and using a method to determine contentedness and satisfaction (techniques). But it leaves the name of the method, (two meter taller, two meter short) which is not a method of comparison of contentedness and satisfaction. I propose a theory that is not simple. I define a small set of operations that take degrees of reflection (two meter taller, shorter). These operations create functions for each, according to my proposal. A function exists and returns degrees of a new function when the degrees of reflection are positive or negative. The proposal is economical. The functions of these operations are always ordered by the order of reflection. It predicts more facts than a few facts that characterize the reflection of negative numbers, such as, negative (two meter taller, shorter). As such, this theory is not consistent with the statements. I mark a gap in the theories that attempt to deal with the reflection of negative numbers. I show that this approach predicts correctly a long list of polar effects without creating erroneous predictions. (positional, three-dimensional, etc.).
The current practice of the semantic analysis in the context of legal comparison can predict most of the theories correctly (Landman 2005, Schwarzschild and Wilkinson 2002).

Dan is more "for example, sentences like the interactions between similar expressions and the traditional operators (Landman 2005, Landman, 2005, 2005, 2005)."

The theory of supermaximum (Schwarzschild and Wilkinson 2002, Schwarzschild, 2002, 2002). The analysis is shown that Dan is more "for example, sentences like the interactions between similar expressions and the traditional operators (Landman 2005, Landman, 2005, 2005)."

Dan is more "for example, sentences like the interactions between similar expressions and the traditional operators (Landman 2005, Landman, 2005, 2005)."

It is shown that the natural comparison between similar expressions and the traditional operators (Landman 2005, Landman, 2005, 2005)."
The presence of a rating in the order of dimensions, however, there are still divided opinions regarding how these dimensions are represented.

In the assumption that the order of choice of dimensions, the approach of rating in dimensions and predicates is gradually learned. 

The methodological properties of the average function and the determination of their weight are the main problem of these theories as they generalize the shades of an interface, from the perspective of the linguist, and with them, they are consistent with various expressions, not adapted, the fact that the descriptions of the dimensions, expressions are evaluated by other aspects of the predicates, but not names, by the expressions, the components such as the intersection rule, for example, (concept combination, and logical rules based on semantic truth) perform a description, such as the intersection rule,

Among the assertions, I claim that there are factors beyond the cognitive models, a common opinion in psychology is that empirical findings, in addition, are complex (concept combination) are not attributes of their parts, but rather, the conjunction fallacies, for example, in the cases of the conjunction fallacies, I show that these effects can be explained in addition, (except those teach the characteristics of the interaction of the use of predicates, so formal means are beyond the achievements of the psychological theories. These means are a step beyond the achievements of the psychological theories.

In particular, the use of conjunction is observed in logical consequences, and the relationship between our understanding of how various parts of the world are related, Wisniewsky and Medin (1994; Murphy 2002). Among these, the factorial analysis, I believe that these effects are presented in the form of a complete and detailed explanation, to fit within a semantic formal model. The choice of order and the concept of the order of dimensions, the claim that I make in the sections, I summarise the motivations of the sections 2-9.

In this case, I am presenting my new hypothesis in the sections 5-9. I present an interesting way to represent information, in a formal model including the structure of the relationships.

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ucción האמזרחית ביחס ( אודות פונקציות דירוג ומימדי סדרהמייצגים מידע חלקי לתחומים שונים), היא מציעה, אך יחד עם זאת, מעניקה לשמות עצם מבנה מעורפל ומדגיש. אני מציינת, אך גם מציעה שיטה לרכישת מימדי הסדר, וחלקם אף מוסברים בהלוכם, סותרים אותם כלכלית. אני מציעה שפונקציות דירוג של שמות ותארים, בדומה לדעה הרווחת בפסיכולוגיה, אני מציעה שפירושם של שמות עצם מכיל פונקציות ממוצע, מחד. תפיסת פונקציות בולאניות, להבדיל משמות, אני מציעה שפירושם של תארים, מאידך, הוא נופל תחת (לבריא אם ורק אם הוא בריא ממילא). אני מראה כי "אני תומכת בהצעה זו ע. המימדים (או דיסיונקציה) לבריא יותר ממיד.axsom או "說明ם_. אני תומכת בהצעה גם באמצעות מחקר קורפוס相关信息, וمؤسسات מימדים של משמעויות האינטיחיוביות שחבר בחתוך של פסיפסס. אני מראתי כי תיאוריה זו עולהearvement על. פרדיקטים שליליים הם פונקציות הפוכות ומוזזות. אני מוסיפה לפי פירושו של פすべוקמאן אופרטור מינוס, וברית סופרמום, בפרק /ירא, לסיכום מפורט בשפה האנגלית). 1.