## After the exam of 19.09.2014: Typical errors, comments etc.

#### QUESTION 1

ERROR: Calculations not justified. (Nothing at all, or something like "no problem, since  $r^{a+b-1} \neq r^{-1}$ "; "these Gamma-functions are well-defined, since a, b > 0".)

CLARIFICATION: "Each step must be justified." Use results of Sections 9f and 9i. Or alternatively, use a (correct!) exhaustion of the domain.

PENALTY: 15 points.

ERROR: Incorrect treatment of the unbounded integrand (like "no problem, since the function is integrable" or excluding a neighborhood of (0,0) only).

PENALTY: 10 points.

### QUESTION 2

Item (a)

ERROR: Convergence of integrals derived just from *pointwise* convergence of functions.

CLARIFICATION: if you split the horizontal segment in three, you get a *non-uniform* convergence on each part.

PENALTY: 5 points.

#### QUESTION 3

### Item (b)

ERROR: Exactness of  $\omega_2$  not treated, or treated incorrectly.

CLARIFICATION:  $\omega_2$  is not exact, since its integral over the sphere is not zero. You cannot apply the divergence theorem or Stokes' theorem to the ball, since its center does not belong to G.

PENALTY: 5 points.

REMARK: every closed 1-form is indeed exact in this simply connected domain; but 2-forms are another story. Every "1-loop" is null homotopic, but "2-loop" (the sphere) is not.

# QUESTION 4

### Item (b)

ERROR: An example of  $N, n, M, x_0$  and f.

CLARIFICATION: you are not asked to prove existence of  $N, n, M, x_0$  and f (which is easy). Rather, you are given  $N, n, M, x_0$  and asked to prove (the first item and) existence of f.

PENALTY: 13 points.

ERROR: "There exists  $x_1 \in \mathbb{R}^N$  such that  $|x_1 - x_0| = \min_{x \in M} |x_1 - x| > 0$ ." CLARIFICATION: did you prove this? In fact, this holds for manifolds of class  $C^2$  but fails for manifolds of class  $C^1$ . A counterexample:  $\mathbb{R}^2 \supset M = \{(x, y) : y = |x|^{3/2} \operatorname{sgn} x\}$  near (0, 0).

PENALTY: 10 points.

#### GRADES STATISTICS

Total	Question 1	Question 2	Question 3	Question 4
110	35	40	35	
109		40	34	35
103	33		35	35
95		40	30	25
92	35		35	22
92	23	35	34	
87	35		30	22
87	35		30	22
73	23		25	25
73	25		30	18
72	20		30	22
63	20		35	8
50	20	5	25	
50	20		30	0
50	10		27	13
48	20		10	18
25	0		13	12
0	0	0		