Analysis-III

# After the exam of 03.03.2015: Typical errors, comments etc.

### QUESTION 1

ERRORS:

The goal is not reached, that is, the equality  $\max_A(\ldots) = \max_B(\ldots)$  is not proved, though the equality  $3x_i^2 = \lambda + 2\mu x_i$  is reached: 20 points.

It is claimed (ridiculously) that  $\max_A(...)$  is reached at a point with all coordinates equal: 15 points.

(Indeed, such point never belongs to A.)

No attention to linear independence of  $\nabla g_1, \nabla g_2$ : 7 points.

Wrong explanation of this linear independence: 5 points.

(No explanation; linear independence confused with  $\nabla g_1 \neq \nabla g_2$ , or with  $\nabla g_1 \neq 0, \nabla g_2 \neq 0$ .)

No attention to compactness of A: 2 points.

#### QUESTION 2

Error:

Implicit (or inverse) function theorem is applied without checking that the relevant determinant is not zero: 25 points.

## QUESTION 3

FATAL ERROR:<sup>1</sup> the integral convergence confused with pointwise (or uniform) convergence.

ERRORS:

Incorrect estimation of AB - ab for  $A \ge a \ge 0$ ,  $B \ge b \ge 0$ : 10 points. I mean something like  $AB - ab \le (A - a)(B - b)$  or  $AB - ab \le A(B - b)$ .

Spaces confused: 5 points.

I mean,  $\int_{B_1 \times B_2} f(x) dx dy$  is confused with  $\int_{B_1} f(x) dx$ ; or a partition of  $B_1 \times B_2$  is confused with a partition of  $B_1$ ; etc.

Implicit assumption that  $f \ge 0, g \ge 0$ : 5 points.

Incorrect reduction of the general case to the case  $f \ge 0, g \ge 0$ : 5 points. I mean, fg = (f + a)(g + b) - ab, etc.

Use of results from Section 7: 5 points.

The inequality  $\sup h \leq \sup f + 17$  assumed to follow from  $\int h \leq \int f + \varepsilon$ ,  $f \leq h$ : 3 points.

(Do not ask me, why "17"; I have no idea.)

<sup>&</sup>lt;sup>1</sup>It means, no points for this question!

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# Question 4

Errors:

Item (b): ridiculous combinations of symbols: 15 points. I mean something like  $\frac{\partial}{\partial t} \int_0^\infty x f(x+t) dt$ , or  $\frac{d}{dy} \iint f(x+y) dx dy$ ; how could we differentiate in t the function  $x \mapsto \int_0^\infty x f(x+t) dt$  of x only?

Item (b): unclear notation: 5 points.

For example,  $\int \frac{\partial}{\partial t} \int f_t(x) x \, dt dx$ ; does it mean  $\int \left(\frac{\partial}{\partial t} \int f_t(x) x \, dt\right) dx$ , that is,  $\int dx \frac{\partial}{\partial t} \int dt f_t(x) x$ ? Or rather,  $\int \left(\frac{\partial}{\partial t} \int f_t(x) x \, dx\right) dt$ , that is,  $\int dt \frac{\partial}{\partial t} \int dx f_t(x) x$ ?

Item (c): a counterexample: 10 points.

In fact, the claim is true; thus, all counterexamples are erroneous.

#### GRADES STATISTICS

Total	Question 1	Question 2	Question 3	Question 4
110	35	35		40
103	35	38		30
97	35	37	25	
93	33	35	25	
90	30	35	25	
76	13		28	35
70	33		10	27
69	21		23	25
65	28		22	15
65	15		20	30
61	28	20		13
60		10	21	29
60	28	10		22
60	33	10	17	
60	25		17	18
53	30		23	0
52	23	29	0	
50	7	10		33
48	18		0	30
48	33		15	
46	13		25	8
28	28		0	0
28	13	0	15	
20	10		10	0