After the exam of 01.07.2015: Typical errors, comments etc.

Question 1

Error: solution for the Euclidean norm only.

Penalty: 15 points.

Item (a)

Error: the special basis (according to the hint) is used, but its existence is not proved.

Penalty: 7 points.

Item (b)

Error: no valid proof that φ is a diffeomorphism. (No proof at all, or only a proof that φ is a *local* diffeomorphism.)

Clarification: for example, the mapping $(x, y) \mapsto (\mathrm{e}^x \cos y, \mathrm{e}^x \sin y)$ is a local diffeomorphism near every point of $\mathbb{R}^2$, however, it is neither one-to-one nor onto (see Sect. 4b). True, a *bijection* of class C^1 is a diffeomorphism; but see the next error...

Penalty: 7 points.

Error: no valid proof that φ is bijective (that is, one-to-one and onto). In particular, wrong claims about its inverse (such as $\varphi^{-1}(x) = x \cdot \|x\|^\alpha$, $\varphi^{-1}(x) = x/\|x\|^\alpha$, etc).

Penalty: 5 points.

Remark: it is not at all hard, to find x such that $x/\|x\|^\alpha = y$; strangely, very few students did it correctly.

Question 2

Item (c)

Error: one implication is proved instead of the eqivalence.

Penalty: 7 points.

Error: unexplained transition from $\int f = \int g$ to $f(t) = g(t)$ (given $f \le g$).

Clarification: do not forget to mention continuity of f, g.

Penalty: 3 points.

Question 3

Error: the hint is proved, but the goal is not reached.
Penalty: 20 points.

Error: incorrect treatment of the singularity at $z = 0$.
Penalty: 10 points.

Error: incorrect treatment of the shifted ball.
Penalty: 10 points.

Question 4

Error: a "proof" that $f \notin C^1(M)$.
Clarification: in fact, $f \in C^1(M)$.
Penalty: 20 points.

Error: unexplained use of only one chart $t \mapsto (t, t^3)$ instead of an arbitrary chart.
Penalty: 3 points.

Grades statistics

Total	Question 1	Question 2	Question 3	Question 4
100	35	35		30
100		35	35	30
100	30	35	35	
100	35	35		30
98	28	35	35	
97		35	35	27
97		35	35	27
95	30	35		30
95	30	35		30
93	28	35		30
90	28	32		30
90	35	35		20
88	23	35		30
88	23	35		30
87		35	25	27
86	21	35		30
83	18	35		30
80		35	15	30
80		35	15	30
80	23	32		25
80		35	15	30
80	30	35		15
79	21	28		30
79	32	32	15	
79		32	20	27
77	15	35		27
76	13	35		28
73	28	35		10
73	28	35		10
73	11	32		30
72	10	32		30
70	28	32		10
70	11	32		27
67	20	35		12

Total	Question 1	Question 2	Question 3	Question 4
53	21	22		10
53		28	0	25
51	13	28		10
43		18	0	25
42		32	0	10
42	0	32		10
41	11		0	30
38		28	0	10
25	0	25	0	
22		12	0	10
12		12	0	0